INFLUENCE OF COUPLING IN ANTENNA ARRAYS ON THE SAGE ALGORITHM

Jo Verhaevert[(1)], **Emmanuel Van Lil**[(2)], **Antoine Van De Capelle**[(3)]

[(1)]*K.U.Leuven, Div. ESAT/TELEMIC, Kasteelpark Arenberg 10,*
B-3001 Leuven - Heverlee, Belgium, Tel:+32/16 32 11 51, Fax:+32/16 32 19 86,
Email: jo.verhaevert@esat.kuleuven.ac.be
[(2)]*As (1) above, but Email: emmanuel.vanlil@esat.kuleuven.ac.be*
[(3)]*As (1) above, but Email: antoine.vandecapelle@esat.kuleuven.ac.be*

ABSTRACT

In this study, we focus on the Space Alternating Generalised Expectation maximisation (SAGE) algorithm, which allows for high-resolution determination of the Direction of Arrival (DOA) parameters. This algorithm is used to replace the high-dimensional optimisation procedure necessary to compute the joint maximum likelihood estimate of the parameters by several separate, sequentially performed, procedures.

Using different antenna elements causes that the radiation from one element couples to its neighbours. In this paper, this mutual coupling will be integrated in the SAGE algorithm. Convergence and performance analysis are based on numerical simulations and will be compared with the case without coupling.

INTRODUCTION

The technology of smart or adaptive antennas for mobile communications has received enormous interest worldwide in recent years. Different levels of intelligence can be introduced, ranging from simple switching between predefined beams to optimum beamforming. Numerous approaches have been considered in order to exploit the antenna structure. Null steering to isolate co-channel users, optimum combining to reduce multipath fading and suppress interference, and beam steering to focus energy toward desired users.

By separating the users in space (both in direction of arrival and distance), the capacity of the system can be increased significantly. To obtain this Direction of Arrival (DOA) information, a lot of algorithms have been investigated the last decades. The spatial resolution is investigated mostly by incidence Direction of Arrival (DOA) algorithms, where the different users are located by distance and angle. We will focus on a technique derived from the Maximum Likelihood (ML) principle which allows for high resolution determination of the incident angle, the propagation delay and the complex amplitude. The Space Alternating Generalised Expectation maximisation (SAGE) Algorithm [1] and [2] updates the parameters sequentially by replacing the high dimensional optimisation process necessary to compute the joint maximum likelihood estimate of the parameters, by several separate, low dimensional maximisation procedures, which are performed sequentially.

Using different antenna elements in an array causes that the radiation from one element couples to its neighbours, as do currents that propagate along the surface of the array. The real current on each array element is the sum of the values due to the excitation plus all the contributions from the various coupling sources from each of the neighbours. This mutual coupling is in most cases not desired, but will often be a significant factor in the total radiation characteristics.

The coupling for an antenna array is simulated with NEC (Numerical Electromagnetic Code) [3]. Earlier studies integrated mutual coupling in the MUSIC Algorithm [4]. The main purpose of this study is to integrate the mutual coupling into the SAGE Algorithm. Convergence and performance analysis are based on numerical simulations and will be compared with the case without coupling.

MODEL DESCRIPTION

In this part, we will describe the model used in the SAGE algorithm. Each user sends a known training sequence containing N_b bits. One bit corresponds with five periods of the carrier frequency. The carrier signal of the Binary Phase Shift Keying (BPSK) modulated signal is 900 MHz. The N_u users are spread in a two dimensional space. Because we consider no local scatterers, the number of users N_u is also the number of paths L. In the channel, the signal is delayed and attenuated, resulting in a relative delay and a complex amplitude at the receiver antennas. To reconstruct the original signal, the incident signals have to be downconverted by multiplying the signal with the signal of a local oscillator. After the downconversion, the signal is filtered in order to get a baseband result. A schematic view can be seen in Fig. 1.

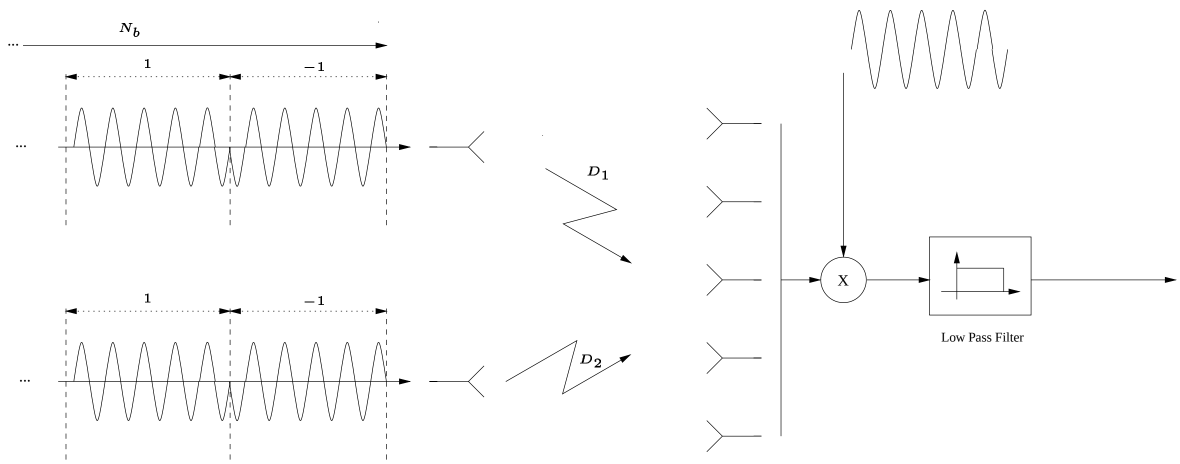

Figure 1: The model definition.

THE SAGE ALGORITHM

All the parameters to be estimated (the relative delay $\hat{\tau}_l$, the azimuth of the incident wave $\hat{\phi}_l$ and the complex amplitude $\hat{\gamma}_l$), are placed together in a vector $\hat{\zeta}_l$. The different incident waves form L different $\hat{\zeta}_l$, because the number of paths is L. These vectors are all stacked together in the matrix $\hat{\zeta}$. Each step of the SAGE algorithm is an estimate of a subset of the components of $\hat{\zeta}$, while keeping the estimates of the other components fixed. We define one iteration cycle of the SAGE algorithm as L consecutive iteration steps for updating the parameters of all waves once.

Because we know at the receiver side the training sequence that has been sent, it is possible to calculate the correlation between the received signal (corrected for one user) and the calculated signal $u(t)$ (dependent of the relative delay, the azimuth angle and the complex amplitude). If we maximise this cost function, we can extract the correct parameters. The correlation function $z(\tau, \phi, \hat{x}_l) = [c(\phi)]^H \int u^*(t' - \tau)\hat{x}_l(t'; \hat{\zeta})dt'$ between the calculated and the received signal, is the cost function that has to be maximised as follows:

$$\hat{\tau}_l' = \arg\max_{\tau}\{|z(\tau, \hat{\phi}_l, \hat{x}_l(t; \hat{\zeta}))|^2\} \quad (1)$$

$$\hat{\phi}_l' = \arg\max_{\phi}\{|z(\hat{\tau}_l', \phi, \hat{x}_l(t; \hat{\zeta}))|^2\} \quad (2)$$

where the prime variables denote the new values after the iteration. The complex amplitude can be calculated with the closed formula: $\hat{\gamma}_l' = \frac{1}{N_a T_a}|z(\hat{\tau}_l', \hat{\phi}_l', \hat{x}_l(t; \hat{\zeta}))|$. The received signal $\hat{x}_l(t)$ of one user, is calculated based on the received global signal $y(t)$ and estimates of the signals $s(t; \hat{\zeta}_l')$ of the different users, with $c(\phi)$ the steering vector:

$$\hat{x}_l(t; \hat{\zeta}) = y(t) - \sum_{l'=1; l' \neq l}^{L} s(t; \hat{\zeta}_{l'}) \quad (3)$$

$$s(t; \hat{\zeta}_l) = \gamma_l \cdot c(\phi_l) \cdot u(t - \tau_l) \quad (4)$$

ADDING COUPLING

In a practical antenna array, using different antenna elements causes that the radiation from one element couples to its neighbours, as do currents that propagate along the surface of the array. The real current on each array element is the sum of the values due to the excitation plus all the contributions from the various coupling sources from each of the neighbours. This mutual coupling is in most cases not desired, but will often be a significant factor in the total radiation characteristics. These coupling values can be integrated into the SAGE Algorithm, as follows:

$$Y_{\text{coupling}}(f_c) = (1 + S) \cdot Y_{\text{nocoupling}}(f_c) \quad (5)$$

with $Y_{\text{nocoupling}}(f_c)$ the frequency domain signal before demodulation and filtering, and $Y_{\text{coupling}}(f_c)$ the frequency domain received signal with coupling.

The mutual coupling between two antenna elements not only contains an absolute value, but also a phase. For an antenna array of $N_a = 5$ elements, the S-parameter coupling matrix S, can be defined as follows:

$$S = \begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} & s_{15} \\ s_{21} & 0 & s_{23} & s_{24} & s_{25} \\ s_{31} & s_{32} & 0 & s_{34} & s_{35} \\ s_{41} & s_{42} & s_{43} & 0 & s_{45} \\ s_{51} & s_{52} & s_{53} & s_{54} & 0 \end{bmatrix} \quad (6)$$

PERFORMANCE EVALUATION

For the performance evaluation of the coupling, we use a linear antenna array, consisting of 5 omnidirectional antennas. The distance between two neighbouring elements is $d = \lambda/4$. We will envisage a scenario with 2 mobile users and no reflections. Hence the number of multipaths equals 2. The performance is evaluated based on the geometrical configuration plotted in Fig. 2. Note that the Direction of Arrival of the two users have small angular separation. For user 1 the exact values can be calculated: $\tau = 1.341 \cdot 10^{-8}$ sec, $\phi = 40.26°$ and $\gamma = -0.0523 - 0.1127i = 0.1243\angle -114.89°$ V. For user 2 is that $\tau = 2.029 \cdot 10^{-8}$ sec, $\phi = 39.47°$ and $\gamma = -0.0818 + 0.0076i = 0.0821\angle 174.68°$V.

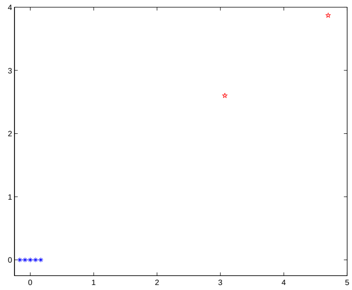

Figure 2: The geometrical configuration of the simulation.

For those simulations, the absolute value of the mutual coupling between element x and y is fixed to $\frac{ad}{d_{xy}}$ with d_{xy} the distance between both elements and with d the distance between two neighbouring elements and a the absolute value of the coupling of between two neigbouring elements. The phase of the mutual coupling between those elements is assumed to be $e^{-j \cdot \frac{2\pi}{\lambda} d_{xy}}$. The resulting coupling matrix S as follows:

$$S = \begin{bmatrix} 0 & a \cdot e^{-j\cdot\frac{\pi}{2}} & \frac{a}{2} \cdot e^{-j\pi} & \frac{a}{3} \cdot e^{-j\cdot\frac{3\pi}{2}} & \frac{a}{4} \cdot e^{-j\cdot 2\pi} \\ a \cdot e^{-j\cdot\frac{\pi}{2}} & 0 & a \cdot e^{-j\cdot\frac{\pi}{2}} & \frac{a}{2} \cdot e^{-j\cdot\pi} & \frac{a}{3} \cdot e^{-j\cdot\frac{3\pi}{2}} \\ \frac{a}{2} \cdot e^{-j\cdot\pi} & a \cdot e^{-j\cdot\frac{\pi}{2}} & 0 & a \cdot e^{-j\cdot\frac{\pi}{2}} & \frac{a}{2} \cdot e^{-j\cdot\pi} \\ \frac{a}{3} \cdot e^{-j\cdot\frac{3\pi}{2}} & \frac{a}{2} \cdot e^{-j\cdot\pi} & a \cdot e^{-j\cdot\frac{\pi}{2}} & 0 & a \cdot e^{-j\cdot\frac{\pi}{2}} \\ \frac{a}{4} \cdot e^{-j\cdot 2\pi} & \frac{a}{3} \cdot e^{-j\cdot\frac{3\pi}{2}} & \frac{a}{2} \cdot e^{-j\cdot\pi} & a \cdot e^{-j\cdot\frac{\pi}{2}} & 0 \end{bmatrix} \quad (7)$$

The received signal with coupling is calculated with Formula 5. Finally the SAGE algorithm calculates an estimate of the relative delay, the azimuth angle of the incident waves and the complex amplitude of the different paths.

Errors of the SAGE Algorithm

In Table 1, the absolute errors for 2 mobile users for the two different cases are shown: without and with mutual coupling

Absolute errors	$a = -20$ dB		no coupling	
	user 1	user 2	user 1	user 2
$\lvert\tau - \tau_{real}\rvert$ (sec)	$2.597 \cdot 10^{-11}$	$2.831 \cdot 10^{-11}$	$9.188 \cdot 10^{-12}$	$1.584 \cdot 10^{-12}$
$\lvert\phi - \phi_{real}\rvert$ (degrees)	1.1714	0.2786	0.4125	1.2862
$\lvert\gamma - \gamma_{real}\rvert$ (V)	0.0089	0.0046	0.0057	0.0042

Table 1: Absolute errors for the 2 mobile users (with $a = -20$ dB coupling and without coupling).

($a = -20$ dB). For the time delay τ (or distance: 10 picosec $= 3$ mm), there is a small difference. For the azimuth angle ϕ and the complex amplitude γ, there is a bigger difference. Although, a correct estimate is still possible.

Errors as function of the coupling

Here, we calculate the influence of the coupling on the different errors. For every value of the coupling between $a = -40$ dB and $a = 0$ dB, the cost function of the SAGE Algorithm is maximised and the results are compared with the exact values. These absolute errors are plotted in Fig. 3.

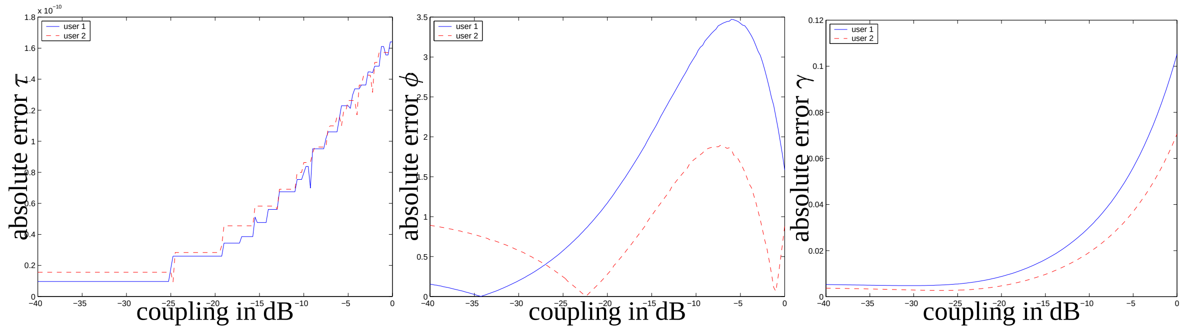

Figure 3: The absolute errors as function of the coupling.

In Fig. 3(a) the absolute error of τ is plotted as function of the coupling, in Fig. 3(b) the absolute error of ϕ is plotted as function of the coupling and in Fig. 3(c) the absolute error of γ is plotted as function of the coupling. For coupling values lower then -25 dB, the SAGE Algorithm is able to detect the exact distance and the exact complex amplitude of the users. The detection of the azimuth angle is a little bit more difficult. For coupling values higher than -25 dB, the error for estimating the time delay grows with the coupling. The same can be said from the error for estimating the azimuth angle. Note that for some values of the coupling (the nulls in Fig. 3(b)), the coupling even cancels the small differences in amplitudes and phases of the incident spherical waves.

In Fig. 3(c), the absolute error γ is plotted as function of the coupling. The received power doesn't differ that much between the different antenna elements. The SAGE Algorithm automatically compensates for the higher received power levels, by adapting the complex amplitude γ, resulting in a bigger difference with the exact values. But hence the model definition fits better the received signal with coupling (important for the SAGE Algorithm), and results in a more accurate estimate for time delay τ and azimuth angle ϕ.

CONCLUSIONS

In this paper, the Space Alternating Generalised Expectation maximisation (SAGE) algorithm is explained very briefly. After including mutual coupling in the model definition, the influences of these coupling values on the SAGE Algorithm are shown. It can be easily seen that the SAGE Algorithm is still able to detect very accurately the different users, for reasonable coupling values between the antenna elements.

REFERENCES

[1] J. Verhaevert, E. Van Lil, and A. Van de Capelle, "Optimal Three Dimensional Antennas for Adaptive Applications (such as the SAGE Algorithm)", in *4th European Personal Mobile Communications Conference (EPMCC 2001)*, 20 - 22 February 2001, pp. 13.5.1–8, Vienna, Austria.

[2] J. Verhaevert, E. Van Lil, and A. Van de Capelle, "The SAGE Algorithm with a Three Dimensional Antenna Array", in *COST273, Towards Mobile Broadband Multimedia Networks*, 17-18 January 2002, pp. TD(02)020.1–10, Guildford, UK.

[3] J. Verhaevert, E. Van Lil, and A. Van de Capelle, "Study and implementation of a uniform spherical distributed antenna array", in *Open Symposium on Propagation and Remote Sensing, URSI Commission F*, 12-15 February 2002, pp. P022.1–9, Garmisch-Partenkirchen, Germany.

[4] B. Friedlander and A.J. Weiss, "Direction Finding in the Presence of Mutual Coupling", *IEEE Transactions on Antennas and Propagation*, vol. 3, pp. 273 –284, March 1991.