EFFECT OF A GRAIN SIZE DISTRIBUTION ON THE RESPONSE OF A DUSTY PLASMA TO A MOVING TEST CHARGE

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ABSTRACT

Dust in a plasma modifies the response to a moving test charge. In general, the dust has a distribution of grain sizes. The linear kinetic behaviour is then quite different to the case with single sized grains. For a reasonable choice of the size distribution the contribution of the dust component to the dispersion relation is equivalent to that for a kappa (generalised Lorentzian) distribution of single-sized particles. Expressions for the potential of a slowly moving test charge, correct to second order in the velocity, are found.

TEST CHARGE POTENTIAL IN DUSTY PLASMA

The linearised potential due to a test charge q moving with velocity $\mathbf{v}_q$ in a non-magnetised plasma is (cf [1]):

$$\phi = q/4\pi\varepsilon_0(2\pi^2)^{-1}\int \exp(ik.x)/\{k^2\varepsilon_p(k,\mathbf{k}.\mathbf{v}_q)\}d^3k \tag{1}$$

where ε_p is the plasma dielectric response function. The dust grains are assumed to have a thermal velocity distribution with temperature T, and size distribution $h(a)$ which has a power law for small radii, a, and for large masses, $m(a)$, decreases with an exponential factor [2]:

$$dn_d = f(a,v)dadv = h(a)f_T(v)dadv = Ca^\beta e^{-\alpha^3 a^3}\exp\left[-m(a)v^2/2kT\right]dadv \tag{2}$$

The dust response term is then equivalent to that of a single-sized component with a Lorentzian distribution [2] with $\kappa = (2\beta+5)/6$ and a characteristic velocity v_T equal to the thermal velocity of a grain with radius $a^*=\kappa^{1/3}\alpha^{-1}$ ($a^*\to a_m$, the radius for maximum $h(a)$ as α and $\beta\to\infty$). Known results [3] may be used to get an analytical expression for this. If the test charge velocity $\mathbf{v}_q$ is sufficiently smaller than v_T, a power series expansion to second order gives:

$$k^2\varepsilon_p \approx k^2 + k_{eff}^2 + iA(\beta)k_d^2 w - B(\beta)k_d^2 w^2, \quad \text{where } w = \hat{\mathbf{k}}.\mathbf{v}_q/v_T \text{ and } k_{eff}^2 = k_e^2 + k_i^2 + k_d^2 \tag{3}$$

The only effect of the ions and electrons is to contribute to the total effective inverse Debye length k_{eff}. Following [1] but with important corrections and generalizations, the response potential may be written as:

$$\phi(r) = (q/4\pi\varepsilon_0 r)\left[\exp(-k_{eff}r) + (v_q/v_T)A(\beta)f_1 + (v_q/v_T)^2\{A(\beta)^2 f_2 + B(\beta)f_3\}\right] \tag{4}$$

where the case for a Maxwellian single-sized distribution (given by $h(a)$ as α and $\beta\to\infty$) is recovered by setting $A=\sqrt{\pi}$ and $B=2$. Introducing λ the angle between the velocity $\mathbf{v}_q$ and the radial vector $\mathbf{r}$ the strength functions f_j are :

$$f_1 = (2/\pi)(k_d r)^2 I_{21}(k_{eff}r)\cos(\lambda), \quad f_2 = -(k_d r)^2(k_{eff}r)^2\{[I_{30}(k_{eff}r) + I_{32}(k_{eff}r)]/3 - I_{32}(k_{eff}r)\cos^2(\lambda)\},$$

$$f_3 = (k_d r)^2\{[I_{20}(k_{eff}r) + I_{22}(k_{eff}r)]/3 - I_{22}(k_{eff}r)\cos^2(\lambda)\} \tag{5}$$

The functions $I_{mn}(s)$ are defined as weighted integrals of spherical Bessel functions:

$$I_{mn}(s) = \int_0^\infty t^2(t^2+s^2)^{-m} j_n(t)\,dt \tag{6}$$

and are of the same type as used previously [1]. f_2 and f_3 are expressible in terms of exponential functions and powers, and for f_1 the exponential integrals $E_i(s)$ and $E_1(s)$ are also required. The mass distribution effects are given through:

$$A(\beta) = \{\sqrt{6\pi}/\sqrt{2\beta+5}\}\Gamma[(2\beta+11)/6]/\Gamma[(\beta+4)/3] \text{ and } B(\beta) = 4(\beta+4)/(2\beta+5) \tag{7}$$

The structure of the potential $\phi(\mathbf{r})$ changes as the relative sizes of A and B vary with the index β.

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