DUAL-SURFACE INTEGRAL EQUATIONS IN ELECTROMAGNETIC SCATTERING[1]

Robert A. Shore and Arthur D. Yaghjian
Air Force Research Laboratory, Hanscom AFB, MA 01731, USA
robert.shore@hanscom.af.mil

1. INTRODUCTION

Computational electromagnetics relies heavily on surface integral equations for the efficient calculation of scattering from perfectly electrically conducting (PEC) bodies. The magnetic-field integral equation (MFIE) and electric-field integral equation (EFIE) are applied only to the surface current of the scatterer and thus require a number of unknowns proportional to the surface area in square wavelengths of a three-dimensional (3-D) scatterer. Unfortunately both the MFIE and EFIE have a serious limitation: they fail to produce a unique solution for the current on a PEC scatterer at frequencies equal to the resonant frequencies of the interior cavity formed by the surface of the scatterer [1]. In numerical practice this lack of uniqueness is reflected in erroneous currents and scattered fields, known as spurious resonances, occurring over finite bandwidths about the cavity resonant frequencies. Since the density of the cavity resonant frequencies increases rapidly with frequency beyond the first resonance, which for many cavities occurs when the maximum linear dimension is approximately one wavelength, the numerical calculation of scattering from 3-D multiwavelength bodies is severely hampered by these spurious resonances.

A variety of remedies has been developed to deal with the spurious resonance problem. The best known method is the combined-field integral equation (CFIE) which uses a linear combination of the MFIE and EFIE to provide a unique stable solution for closed scatterers. Less well known is the dual-surface integral equation method which makes use of the vanishing of the electromagnetic field inside the surface of a closed PEC object to impose an appropriate boundary condition on the electric or magnetic field on a fictitious surface located inside the actual surface. The resulting integral equation expressing this boundary condition, multiplied by a constant, is then added to the corresponding original MFIE or EFIE to obtain the dual-surface magnetic-field (DSMFIE) or dual-surface electric-field integral equation (DSEFIE). The DSMFIE and DSEFIE have a unique solution for the current at all frequencies provided that the multiplier constant has an imaginary part, and that the separation of the original and dual surface is less than a half-wavelength. In contrast to the CFIE, the dual-surface integral equations require only one type of integral operator, either the MFIE or EFIE operator. The DSMFIE has been used for a number of years [2][3][4], but it is only very recently that the authors have solved and programmed the DSEFIE for a PEC body of revolution [5].

Since the CFIE is so widely used it may seem unnecessary to have other surface integral equation formulations available to perform scattering calculations that are free from spurious resonances. In fact, however, surface integral equation solvers are far from being "turn-the-crank" procedures. For example, a recent investigation of the convergence properties of the CFIE has shown [6] that the accuracy of results obtained depends very strongly on both the value of the combination parameter weighting the EFIE and MFIE contribution, and on the fineness of discretization (points per wavelength). Similar considerations apply to our own experience with the influence of the combination parameter, the separation distance between the original and dual surface, and the grid point density, on the results obtained with dual-surface integral equations. Furthermore, for scatterers containing narrow-angle wedges and tips, the MFIE operator becomes unstable so that the CFIE (which contains an MFIE component) cannot be relied on to give accurate results. Because spurious resonance-free scattering calculations performed with the same surface integral equation operator or with different surface integral equation operators can differ significantly, careful checking of the results of scattering calculations is very important. Such checking can be done more easily and rigorously if two independent surface integral equation formulations are available for comparison purposes.

2. THE DUAL-SURFACE MAGNETIC AND ELECTRIC FIELD INTEGRAL EQUATIONS

In this section we derive the DSMFIE and DSEFIE starting with the MFIE and EFIE. The MFIE and EFIE

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are

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}^{inc}(\mathbf{r}) = \mathbf{J}(\mathbf{r})/2 - \hat{\mathbf{n}}(\mathbf{r}) \times \int_S \mathbf{J}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') dS', \tag{1a}$$

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{E}^{inc}(\mathbf{r}) = \hat{\mathbf{n}}(\mathbf{r}) \times \frac{j}{\omega\epsilon_0} \int_S [k^2 \mathbf{J}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') - \nabla'_S \cdot \mathbf{J}(\mathbf{r}') \nabla' G(\mathbf{r}, \mathbf{r}')] dS', \tag{1b}$$

where $\mathbf{J}(\mathbf{r})$ is the surface current density on the surface S of the scatterer ($\mathbf{r} \in S$), $\nabla_S \cdot$ is the surface divergence [7], $\hat{\mathbf{n}}$ is the outward unit normal from the surface S, ϵ_0 is the permittivity of free space, and $(\mathbf{E}^{inc}, \mathbf{H}^{inc})$ are the electric and magnetic fields incident upon the scatterer. Harmonic time dependence of the form $\exp(j\omega t)$ has been suppressed, k is the wavenumber, and the free-space Green's function is given by

$$G(\mathbf{r}, \mathbf{r}') = \frac{\exp(-jk|\mathbf{r} - \mathbf{r}'|)}{4\pi|\mathbf{r} - \mathbf{r}'|}. \tag{2}$$

To obtain the corresponding dual-surface integral equations the surface S is assumed to be closed so that for points inside S the total magnetic and electric fields vanish, and hence for $\mathbf{r}$ inside S

$$\mathbf{E}^{sc}(\mathbf{r}) = -\mathbf{E}^{inc}(\mathbf{r}), \tag{3a}$$

$$\mathbf{H}^{sc}(\mathbf{r}) = -\mathbf{H}^{inc}(\mathbf{r}), \tag{3b}$$

where $(\mathbf{E}^{sc}, \mathbf{H}^{sc})$ are the scattered electric and magnetic fields inside S. Then for $\mathbf{r}$ inside S

$$\mathbf{H}^{inc}(\mathbf{r}) = -\int_S \mathbf{J}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') dS', \tag{4a}$$

and

$$\mathbf{E}^{inc}(\mathbf{r}) = \frac{j}{\omega\epsilon_0} \int_S \left[k^2 \mathbf{J}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') - \nabla'_S \cdot \mathbf{J}(\mathbf{r}') \nabla' G(\mathbf{r}, \mathbf{r}')\right] dS'. \tag{4b}$$

In (4a,b) we can let $\mathbf{r}$ lie on a surface S^d parallel to, and a small distance $\delta > 0$ inside the actual surface S. Letting $\mathbf{r}^d(\mathbf{r})$ denote the point on the dual surface S^d corresponding to a point $\mathbf{r}$ on the actual surface S and adding the cross product of $\hat{\mathbf{n}}(\mathbf{r})$ with (4a,b) multiplied by a constant α to (1a,b) and we then obtain the dual-surface magnetic and electric field integral equations

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}^{d,inc}(\mathbf{r}) = \mathbf{J}(\mathbf{r})/2 - \hat{\mathbf{n}}(\mathbf{r}) \times \int_S \mathbf{J}(\mathbf{r}') \times \nabla' G^d(\mathbf{r}, \mathbf{r}') dS', \tag{5a}$$

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{E}^{d,inc}(\mathbf{r}) = \hat{\mathbf{n}}(\mathbf{r}) \times \frac{j}{\omega\epsilon_0} \int_S \left[k^2 \mathbf{J}(\mathbf{r}') G^d(\mathbf{r}, \mathbf{r}') - \nabla'_S \cdot \mathbf{J}(\mathbf{r}') \nabla' G^d(\mathbf{r}, \mathbf{r}')\right] dS', \tag{5b}$$

where $\mathbf{H}^{d,inc}(\mathbf{r})$, $\mathbf{E}^{d,inc}(\mathbf{r})$, and $G^d(\mathbf{r}, \mathbf{r}')$ are defined as

$$\mathbf{H}^{d,inc}(\mathbf{r}) = \mathbf{H}^{inc}(\mathbf{r}) + \alpha \mathbf{H}^{inc}(\mathbf{r}^d), \tag{6a}$$

$$\mathbf{E}^{d,inc}(\mathbf{r}) = \mathbf{E}^{inc}(\mathbf{r}) + \alpha \mathbf{E}^{inc}(\mathbf{r}^d), \tag{6b}$$

$$G^d(\mathbf{r}, \mathbf{r}') = G(\mathbf{r}, \mathbf{r}') + \alpha G(\mathbf{r}^d, \mathbf{r}'), \tag{6c}$$

with $\mathbf{r} \in S$ and $\mathbf{r}^d$ lying on the dual surface S^d. The DSMFIE and DSEFIE in (5a,b) although identical in form and comparable in complexity to the original MFIE and EFIE, each have a unique solution for $\mathbf{J}(\mathbf{r})$ at all real frequencies provided that the combination parameter α has an imaginary part and that the separation constant δ is less than half a wavelength [2].

3. SOLUTION OF THE DUAL SURFACE INTEGRAL EQUATIONS FOR A BODY OF REVOLUTION

For a body of revolution (BOR), the DSMFIE and DSEFIE are conveniently solved by the method of moments in almost exactly the same way as are the MFIE and EFIE. We let

$$\mathbf{J}(t,\phi) = \sum_{n=-N}^{N} \sum_{j} \left[I_{nj}^{t} \mathbf{J}_{nj}^{t}(t,\phi) + I_{nj}^{\phi} \mathbf{J}_{nj}^{\phi}(t,\phi) \right] e^{jn\phi} \tag{7}$$

where $\mathbf{J}_{nj}^{t}$ and $\mathbf{J}_{nj}^{\phi}$ are the expansion (basis) functions of the BOR surface coordinates t (measured along the generating curve of the BOR) and ϕ (the angle of rotation of the generating curve around the BOR axis)

$$\mathbf{J}_{nj}^{p} = \hat{\mathbf{p}} f_j(t) e^{jn\phi}, n = 0, \pm 1, \cdots, \pm N,\ p = t \text{ or } \phi, \tag{8}$$

and the I_{nj}^{p}, $p = t$ or ϕ, are coefficients to be determined. N is set equal to the number of Fourier modes sufficient to represent, to the desired accuracy, the ϕ variation of the incident tangential magnetic or electric field on the surface of the BOR[5]. For the DSMFIE the $f_j(t)$ in (8) are chosen to be pulse functions. Simple point matching in the variable t, following the multiplication of (5a) by $\exp(-jm\phi)$ and performing the ϕ and ϕ' integrations using the orthogonality of the Fourier functions, then yields a matrix equation for each Fourier mode that is solved to obtain the coefficients of the basis functions. Once the current on the surface of the BOR is then calculated from (7) it is straightforward to obtain the scattered field.

For the DSEFIE the $f_j(t)$ in (8) are chosen to be four-impulse approximations to triangle functions. Then, using the Galerkin method, the dot product of (5b) with each one of a collection of testing functions $\mathbf{W}_{mi}^{t}, \mathbf{W}_{mi}^{\phi}$ defined by

$$\mathbf{W}_{mi}^{p} = \hat{\mathbf{p}} f_j(t) e^{-jm\phi}, m = 0, \pm 1, \cdots, \pm N,\ p = t \text{ or } \phi, \tag{9}$$

is integrated over S and the orthogonality of the Fourier functions is used to obtain a matrix equation for each Fourier mode that can be solved for the coefficients of the basis functions. The only way in which this procedure differs from that of Mautz and Harrington [8] is that the device they use to transfer the differential operator acting on the Green's function in (5b) to the testing function and thereby avoid increasing the self-term singularity of the Green's function cannot be used when the observation point lies on the dual surface rather than on the actual surface of the BOR. This, however, does not create numerical difficulties because the distance between points on the actual surface and points on the dual surface never vanishes.

4. RESULTS

Since the DSMFIE has been successfully applied over the past ten years to multiwavelength rectangular boxes and to BORs [2][3][4], we focus our attention here on the DSEFIE. Calculations performed with a computer program of the DSEFIE solution demonstrate the removal of spurious resonances that appear in calculations of the radar cross sections (RCSs) of spheres, spheroids, and finite cylinders made with the conventional EFIE. As an example, in Fig. 1 we show the backscatter RCS of a PEC sphere as a function of its normalized radius ka in the vicinity of the interior cavity resonance value of $ka = 6.062$. It is seen that the spurious resonance displayed by the EFIE curve at $ka = 6.08$ is eliminated by the DSEFIE curve which agrees closely with the exact Mie solution curve.

A second example of the elimination of spurious resonances by the DSEFIE is provided by calculations of scattering by a finite cylinder. Fig. 2 shows the monostatic E-plane RCS for a cylinder with height and radius equal to 0.587λ and $\lambda = 1$, very close to the dimensions of a cylindrical cavity that supports the dominant TE cavity mode [9]. The striking spurious resonance in the EFIE pattern is not present in either the DSEFIE or CFIE patterns which are in close agreement with each other.

Calculations were also performed to calculate the RCS of a cone-sphere with a narrow cone semi-vertex angle of 5°. This is a shape for which the MFIE and hence the DSMFIE is ill suited because portions of the surface close to the cone-tip are separated by much less than a wavelength. The CFIE, since it contains an MFIE "component" can also not be relied on to yield highly accurate results. Application of the DSEFIE to the narrow-tip cone-sphere requires careful consideration of the placement of the dual surface. If the dual surface is taken to be simply another cone-sphere "parallel" to the actual cone-sphere surface with a small separation distance δ, then points close to the actual cone vertex can be considerably further apart than δ from the corresponding points on the dual surface. For example, the distance between the actual and dual vertex points is $\delta/\sin\zeta$ where ζ is the cone semi-angle; for $\delta = \lambda/4$ and $\zeta = 5°$ this separation distance is almost 3λ. The boundary condition of zero tangential electric field on the dual surface is then insufficient to guarantee that the currents close to the cone-tip will be correct. To overcome this difficulty we not only place the dual surface closer to the actual surface but also insert an auxiliary dual surface close to the vertex of the cone-sphere and use an increased density of grid points in the t direction a wavelength or so from the tip. When these precautions are taken the DSEFIE yields accurate spurious-resonance free results.

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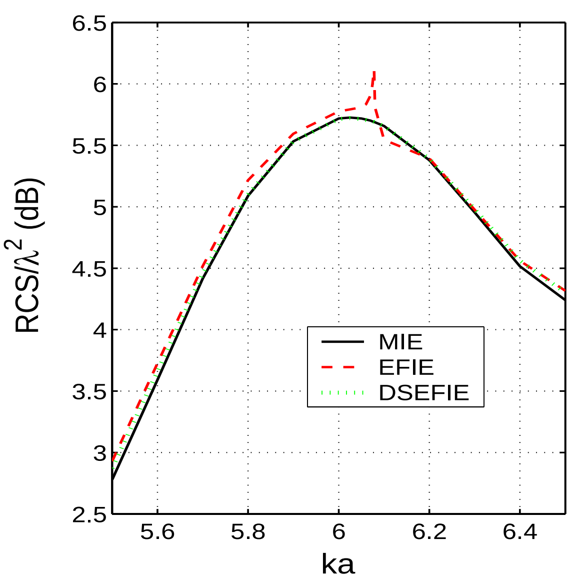

Figure 1: Backscatter RCS for a PEC sphere.

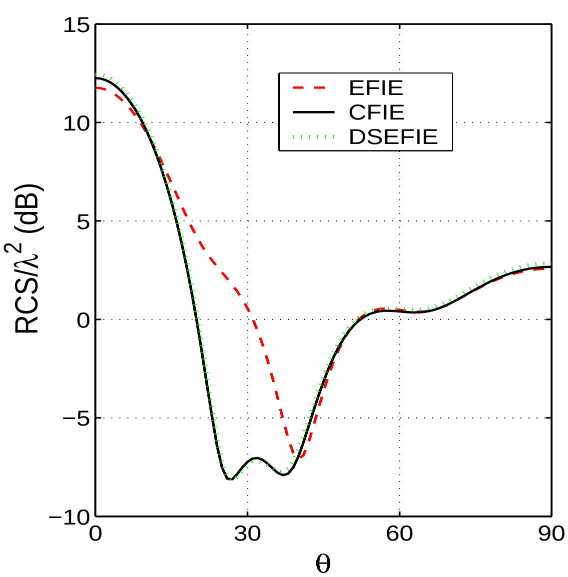

Figure 2: Monostatic RCS for a PEC cylinder.