ON THE FREQUENCY BEHAVIORS OF DISCRETE SYSTEMS CHARACTERISTICS

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Abstract- Infinite Impulse Response (recursive filter) meet a given set of specifications with a much lower order than a corresponding Finite Impulse Response (non recursive filter). These filters are employed in a number of practical applications because of their simplicity, which makes them amenable for inexpensive hardware implementation. This paper discusses computer-based approach on three simple low order IIR filters to investigate frequency variations of linear discrete-time system characteristics in the Z-transform domain. The effectiveness of the filters is demonstrated through sinusoids. Clearly, the location of poles and zeros are vital in the design of low pass digital filter.

I. INTRODUCTION

The design of digital low pass filter has received much attention in recent years. All the linear digital filters fall into two categories: non-recursive and recursive (also called feedback) filters [1]. The output of a non-recursive filter is a weighed average of present inputs, some past inputs and, possibly, some future inputs. The choice of the quantity and values of the weights determines the characteristics of the filter (e.g. high pass, low pass, band pass, rejection, etc.). For a one-dimensional linear non-recursive filter, the weights are the discrete Fourier transform of the desired frequency transfer function of the filter. This property allows by applying the fast Fourier transform algorithm to the digitized transfer function of the filter. For time sequences, only present and past inputs can meaningfully be used, whereas for space sequences future inputs are always used in order to ensure zero phase distortion of the output.

It is interesting to note that IIR filter plays a central role in parametric models of random processes [2]. A model of random process that make use of an IIR filter is called an autoregressive-moving average (ARMA) which, by nature, a rather special version of time-delayed feedback control system which is successful in many control system application [3]. In this work recursive filters are used.

II. COMPUTATIONAL METHODS

A Linear Time-Invariant (LTI) system can be described by a difference equation according to

$$y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{l=0}^{M} b_l x(n-l), \tag{1}$$

where $\{a_k\}$ and $\{b_l\}$ are difference equation coefficients. The initial conditions are assumed to be zeros.
The transfer function representation in factored form can be written as

$$H(z) = b_o z^{N-M} \frac{\prod_{l=1}^{N}(z - z_l)}{\prod_{k=1}^{N}(z - z_p)}, \tag{2}$$

where z_l's are the system finite zeros and p_k's are the system finite poles, b_o is gain factor. Three discrete-time systems are considered. The gain factor is unity for each system.

2.1 Frequency Response

The regions of convergence (ROC) of our systems include a unit circle ($z=e^{j\omega}$). The frequency response function is a complex function of ω. It is computed using the expression given by

$$H(e^{j\omega}) = b_o e^{j(N-M)\omega} \frac{\prod_{1}^{M}(e^{j\omega} - z_l)}{\prod_{1}^{N}(e^{j\omega} - p_k)} \quad . \tag{3}$$

2.2 Group Delay Function

It is a measure of the linearity of the phase function [4]. It is calculated from the expression

$$\tau(\omega) = -\frac{d}{d\omega}[\angle H(e^{j\omega})]. \tag{4}$$

Fig. 1 and Fig. 2 show frequency response plots and group delay plots for the systems respectively.

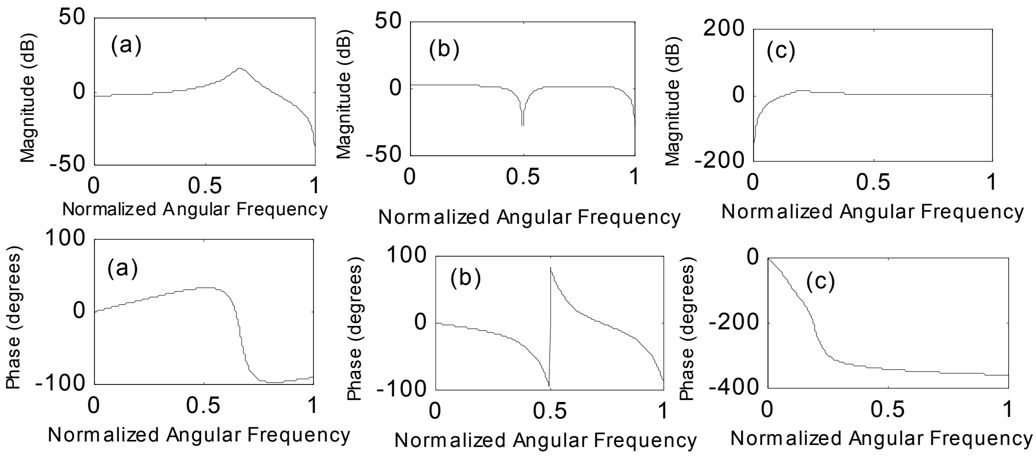

Fig. 1. Frequency response plots:(a) second order (b) third order (c) fourth order.

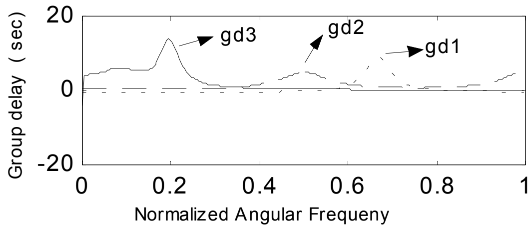

Fig. 2. Group delay plots for the three systems.

III. RESULTS AND DISCUSSION

The results of frequency behavior of Linear-Time Invariant (LTI) discrete-time systems are presented in this section. The poles of second-order system occur in complex – conjugate pair. The locations of the poles indicate that, the system is a high pass filter. In the case of three-poles system (or a third-order filter) two poles occur in complex conjugate pair at the marginal stability, the third pole is at high frequency region. While the fourth poles of the fourth-order filter occur as complex-conjugate pairs. Their locations indicate a low pass filter. The systems are all stable since all the poles are within the unit circle.

In the complex frequency responses we observe that the second-order system exhibit asymmetric frequency-response behavior. The second system, on the other hand, demonstrates notch filter frequency-response characteristics. The third system, however, the frequency response behavior is that of low pass filter.

In comparison of group delay for the three filters, the label 'gd1' indicates group delay of the second-order filter,'gd2' is group delay of the third-order system, and 'gd3' represents the group delay of the fourth-order filter. We note that each system has a peak delay at different value of frequency. The peaks occur at around ω = 0.65, 0.50 and 0.20 for the first, second and third system respectively, with the third order having the least delay. The greater the value of the delay the more the phase linearity of the system, this behavior that has been noted before [5].

We then examine the effects of sinusoids on frequency response of the systems. To this end, 200 samples of the input sequence given by

$$x(n) = \sin(\prod n/3) + 5\cos(\prod n), \tag{5}$$

is processed through the filters. The characteristics complex frequency-response behaviors of the systems are reported in Figs. 3 (a-c). Several comments are in order here. First, appearance of lobes in magnitude response in all the systems with mainlobe occurring at normalized angular frequency $\omega = 0.33$, corresponding to the frequency of the sinusoidal part of the input. Second, the phase frequency responses of the three systems become piecewise linear-phase. The amplitudes and phases of the input sinusoids have affected the complex frequency response behavior of the filters. Fig. 4 shows 200 samples of the steady-state output y1(n), y2(n), y3(n) and input x(n) of the filters. The first system pass the low- frequency component of the input sequence and heavily attenuated the high frequency part. The steady state is reached at n = 24. The second system allows the low frequency part of the sinusoids to pass and stop the high-frequency component. The steady state is reached at n = 18. In both cases, the amplitude of the input signal that passed is preserved. In the fourth-order system, the filter does not influence the input signals.

The frequency behaviors explanation of the responses is in two folds. First, it is mainly due to the time delay in the systems [6]. Second, it can be attributed to the pole positions in the unit circle.

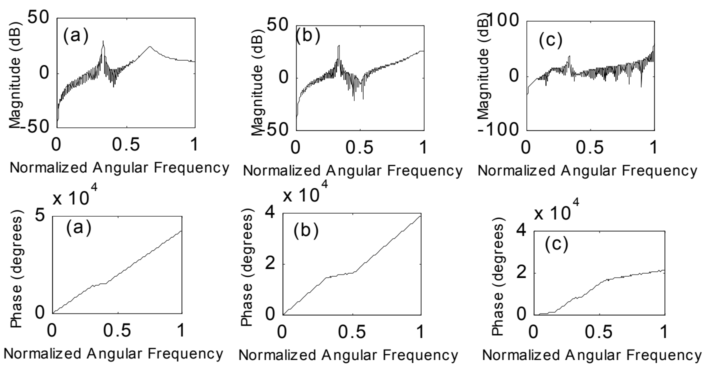

Fig. 3. Effect of sinusoids on frequency response of the filters

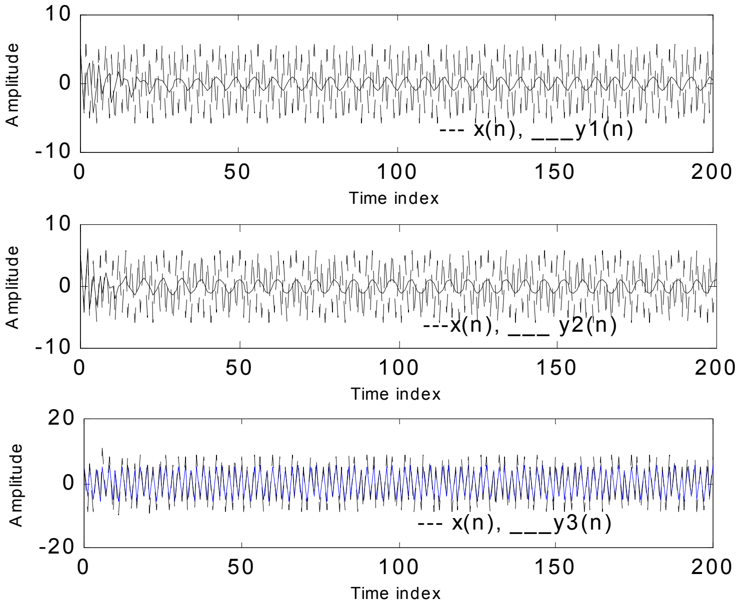

Fig. 4. Output y1(n), y2(n), y3(n) (solid line), input x(n) (dash line) signal to the filters

IV. CONCLUSIONS

This paper presents two main results: 1) the design of low order low pass recursive digital filters. Their effectiveness has been demonstrated through the applications of those filters to sinusoids. The filters significantly modified the characteristic of the input signals. These filters (the first and second) are capable of separating any effects that may affect the signals. 2) It has also been shown that the position of poles and zeros in the z-plane pole-zero diagram plays a significant role in the frequency behaviors of discrete systems.

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