DIFFUSIVE AND PROPAGATIVE WAVE FIELD INVERSION WITH EMPHASIS ON NONDESTRUCTIVE EVALUATION OF MAN-MADE & NATURAL OBJECTS

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ABSTRACT

Inverse scattering problems in the field of electromagnetics and novel, mostly non-linearized solutions thereof are considered. Emphasis is on the retrieval of cylindrical or three-dimensionally bounded obstacles that are buried in a known, planarly layered environment. Numerical illustrations come from three challenging applications, the evaluation of voids in a damaged metal layer at eddy-current frequencies, the characterization of mineral bodies deeply embedded in a less conductive Earth, these two being considered within the diffusive regime, and the imaging of objects shallowly buried in a half space and interrogated from above in the propagative regime at near-resonance.

KEY ELEMENTS OF THE INVERSION

Inverse scattering problems in the field of electromagnetics and novel (mostly non-linearized) solutions thereof are considered in this contribution. More precisely, one is assuming that an obstacle is found fully within a given test domain. The obstacle can either be the two-dimensional cross-section of an infinitely long cylinder, or a three-dimensionally bounded structure, which is made of a linear, isotropic and non-magnetic lossy dielectric penetrable material —only its conductive properties will matter in the so-called diffusive regime at low enough frequency— or even an impenetrable one, which is corresponding to a perfectly conducting body. The test domain is some closed box, which, for example, has been made known to us by preliminary experimentation. It is itself located within a given layer of a planarly stratified environment which is often reduced to two homogeneous half spaces, or to a homogeneous slab set between two such half spaces. This environment has known electrical and geometrical properties; its constitutive material is linear, isotropic and non-magnetic, usually lossy dielectric, or essentially conductive, or simply made of air.

One further assumes that one is able to prescribe in the exterior of the test domain a (generally small) number of time-harmonic sources. Those are typically modeled as line sources with linear electric or magnetic polarization in the axially symmetric geometry, and as electrical current coils in the three-dimensional geometry. One places as well some receivers, point-wise and sensitive to the local electric or magnetic field in the most ideal cases, but possibly more complicated also, e.g., coil probes whose variations of impedance are collected. Monochromatic as well as frequency-diverse fields, at a few discrete frequencies usually, are considered.

The obstacle itself is (partially) unknown and has to be characterized from the data collected by the receivers for the various source locations envisaged, being emphasized that most recent approaches by the authors —and this is the focus of this contribution— have attempted to address a rather specific —yet still of good applicability— set of problems, the so-called binary case. That is, one is considering that one knows beforehand, or at least that one has a good guess of, the properties of the constitutive material (even an upper bound of the contrast with respect to its immediate environment might suffice) and that the said obstacle is homogeneous, being noticed that it might be singly connected or multiply connected, this topological information being made available to us, or not.

Now, in this configurational case, one is de facto searching for the shape of the obstacle, or of its several parts. This search can be carried out in implicit fashion; the boundary is modeled as a zero level of a level set which is evolved in some optimal fashion from an initial domain [1-2] or the boundary is found as a by-product of the iterative construction of a distribution of cells with prescribed contrast [3-4]. This search can also be carried out in more explicit fashion, then by using a closed-form analytic representation of the contour whose suitable characteristic coefficients are adjusted step-by-step [5], and, especially with scarce data in complex environments, by identification of a simple yet versatile ellipsoidal obstacle [6] and even of equivalent electric and/or magnetic localized sources only [7].

The above being said, and though one has already much restricted the scope of the inverse problems one is dealing with herein, two major hurdles remain. Indeed, the collected signals contain an encrypted information on the interrogated obstacle, and the inversion methods mentioned in the above, be they aiming at a direct identification of topology —the question is to draw contour lines or surfaces where a known boundary condition is holding— or at identification of a parameter distribution —the question is to reproduce the variations of a pertinent electric feature throughout space from which the boundary proceeds— may be quite involved both mathematically and numerically, This is true notably in view of non-linearity vis-à-vis the contour or parameter sought and ill-posedness, theoretical or practical as well, of the inversion problems, referring for discussion to [8].

Evidently, to build up meaningful inverse solutions, one must tackle the corresponding direct radiation and scattering problems. This has to be done to appraise how the probing signals generated by known sources interact with the probed obstacle in a certain environment, to exhibit key electrical and geometrical parameters involved in this interaction, to develop exact models or approximated ones (e.g., extended-Born), with conflicting issues of computational burden and accuracy, and to acquire proper data sets to comprehensively test the proposed inversion method. And it has to be reminded that what finally matters is how such a method can be put in practice, which means successful processing of laboratory-controlled data first, and proof that data of industrial origin can be handled too, later on at least.

Therefore, in the talk, one will outline, how mathematically sound and numerically effective tools —which enable the decryption of field data and involve well understood direct models— have been developed this last triennal. Then, from pros and cons of the above methods at the present (recognized as still early) stage of development, and limitations as well as strengths of the underlying direct models, one will attempt to see how demanding, even feasible, are the steps ahead, notably those involved by the processing of magnetic materials and the mapping of cracks in eddy current problems, or the consideration of subsurface cavities or other inclusions in Earth at higher frequencies.

NUMERICAL ILLUSTRATIONS

Numerical illustrations of corresponding advances will come from three challenging domains. The first two are in the diffusive regime: the electromagnetic non-destructive evaluation of metal structures (as in the nuclear industry and in aeronautics) at eddy-current frequencies [9-10]; the electromagnetic characterization of deep bodies in Earth (mostly but not only for evaluation of mineral ores) again at low frequencies of operation [11]. The third illustration is in the propagative regime: the imaging of dielectric or conducting objects shallowly embedded in a given half space and interrogated at UHF and microwave frequencies (with good success overall but a lesser degree of application than the two previous ones at the present time), laboratory data treated so far in this applicational field being in free space only [12]. Next, lack of place precluding us to give more examples, one considers three typical results.

In Fig. 1 one is considering the retrieval of two voids opening in air (they are parallelepipeds with cross-sectional areas of $1.1\times1.1\ \mathrm{mm}^2$ and depths 1 mm and 0.5 mm, respectively, their vertical axes being distant from 3.11 mm). They are affecting a 2 mm thick metal plate with conductivity 1 MS/m. The plate is interrogated by an air-cored pancake coil (inner and outer radii 0.6 and 1.6 mm, height 0.8 mm) operated at 150 kHz frequency at fixed location above (0.9 mm lift-off). The search box is divided into $48\times48\times18$ small cubic voxels of 0.113 mm side and the vertical component of the magnetic field is sampled above the plate each 0.1 mm in a square area of $5.2\times5.2\ \mathrm{mm}^2$ at 1.55 mm lift-off. The inversion algorithm is a binary specialization (in the spirit of [2-3]) of a contrast-source solution method (in essence, the one introduced in [13]). The involved machinery is out of the scope of this paper —refer to [14]. What mostly matters is that one successively seeks equivalent electrical currents in the box (they radiate the anomalous magnetic field) and the conductivity contrast of the voxels (they support these currents), this contrast being progressively shifted to be close to – 1 (void) or 0 (metal). Notice that data are interpolated from fields calculated on a different mesh in the observation plane, using a different voxel distribution to describe the voids.

In Fig. 2 one is considering the retrieval of a deep orebody in Earth from vector magnetic fields collected by a 3-component sensor which is displaced along a single borehole passing near a sought metallic ore, a large electrical current loop being placed at the surface of the Earth. Data here are courtesy of B. Bourgeois, BRGM, and have been acquired during a Geo-Nickel measurement campaign on a mining test site. One aims at building up either an equivalent ellipsoidal body or an equivalent spherical body, assumed of infinite conductivity (impenetrable), placed somewhere in between the borehole and the source, such that the induced magnetic field suitably fits the data, and whose geometrical parameters (coordinates of the center, semi-axis lengths and angles of orientation, or radius) are retrieved to that end by a quasi-Newton search algorithm. The calculation of this field involves a low-frequency model of the scattering phenomenon, here which has been limited to a computationally light but still mathematically involved static model (only the in-phase or real-valued parts of the three field components are estimated) —refer to [6] from which the results shown here have been extracted. An ellipsoidal body, whose orientation, size and location are very realistic in view of our knowledge of the test site, and are in particular fairly independent of the first guesses, can be shown to yield an excellent fit to all three components at the operation frequency (1120 Hz) whilst a spherical one fails to retrieve the data (the best results are displayed).

In Fig. 3 one is considering the retrieval of two identical circular cylinders of diameter 3 cm, which are made of a dielectric material with relative permittivity about 3, and which have been set vertically within an anechoic chamber at a distance axis-to-axis of about 9 cm. These obstacles are illuminated and seen by horn antennas located in the far field, these sources and sensors being in the TM polarization mode and operating in the microwave regime (here, the frequency of operation is 8 GHz), the data field being sampled at high rate in a 240° observation aperture facing the emitting antenna for each location of this antenna. *In fine*, the configuration is such that one is facing a 2-D inverse scattering problem instead of a 3-D one as previously, with a good data coverage. But the availability of experimental data even acquired in this controlled laboratory situation is still a good challenge. Two solution methods have been employed to retrieve the obstacles (no hypothesis is made on their shapes and number). A controlled evolution of level sets aims at the identification of the contours, being assumed that the permittivity contrast is known beforehand; a binary specialization of a contrast-source method aims at the construction of a binary distribution of pixels. Again the machineries are out of the scope of this paper as well as technical details —refer to [2, 4] from which the results shown here have been extracted.

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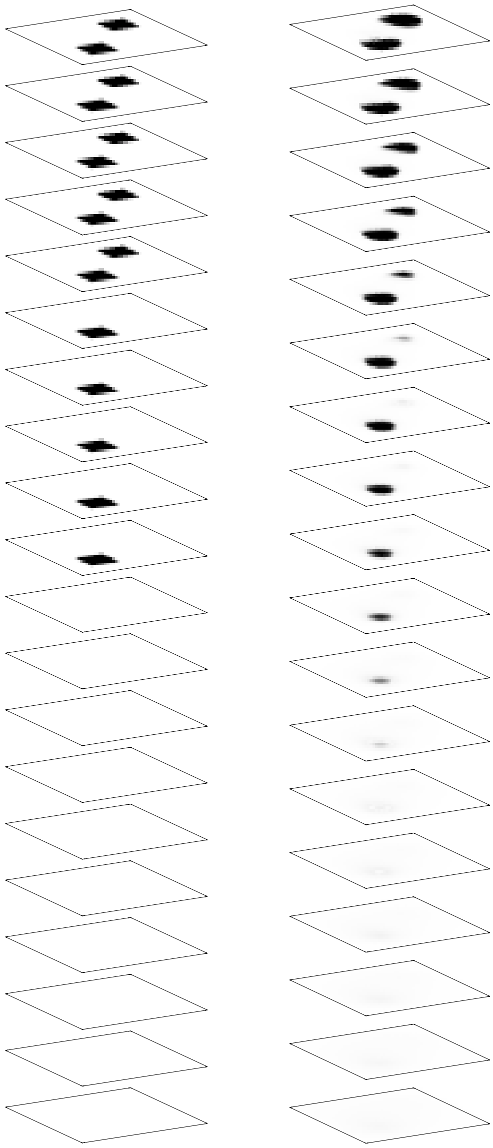

Fig. 1. Eddy-current nondestructive evaluation of a damaged metal plate using a binary-specialized contrast-source algorithm (left: the simulated voluminous voids; right: those retrieved)

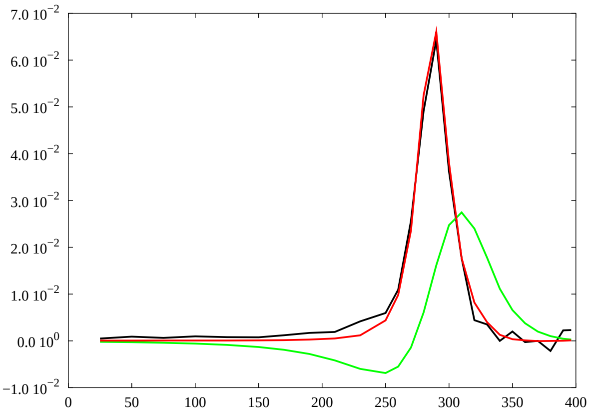

Fig. 2. Characterization of an orebody in downhole mineral exploration in the induction regime. In-phase vertical component of the secondary magnetic field (in black: the real data, in red and green: fields due to equivalent ellipsoidal and spherical bodies, respectively).

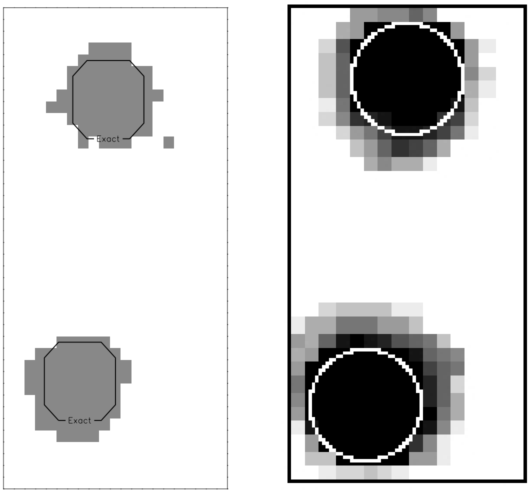

Fig. 3. Imaging of circular dielectric objects from laboratory data at microwave frequencies using a level-set algorithm (left) and a binary-specialized contrast-source one (right). (Scales are different.)