

# TOMOGRAPHIC GROUND SURFACE IMAGING USING RADIO WAVES

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## ABSTRACT

A technique for the tomographic ground surface imaging using radiowaves is described. The image of the ground surface is obtained by reconstructing the normalised surface impedance distribution of the surface from the data of scattered electric fields using an error minimisation method. Numerical simulations of the following ground features: 1) a wet and dry mixed ground surface, 2) a wet surface with icy patches, and 3) a forest covered surface with a hidden object, have demonstrated that the images of the ground surfaces can be well reconstructed using the technique.

## INTRODUCTION

Over the past century, radiowave propagation over the ground surface of the Earth has been extensively studied [1, 2]. A number of propagation models have been established to obtain analytical or numerical solutions for the prediction of electric or magnetic fields at points above the ground surface of the Earth. In the formulations, the distribution of dielectric properties or surface impedance of the ground surface is always assumed to be known so that the propagation prediction is dealt with as a forward problem. As the radiated fields of an antenna over the ground surface are related to the dielectric properties of the ground, the measurements of the fields with antennas positioned at different azimuthal locations can be used to determine the distribution of ground properties [3, 4]. This forms the inverse problem. The antennas are arranged in a circle surrounding the surface area being imaged creating "multi-views" of the ground surface. The ground surface can then be mapped in terms of its reconstructed surface impedance distribution from the measured scattered electric fields of the surface. The tomographic image of the ground surface can thus be obtained. The technique will be described in the next section. It will be applied to the image reconstruction of four ground features, namely 1) a wet and dry mixed ground surface, 2) a wet surface with icy patches, and 3) a forest covered surface with a hidden object. The simulation results of image reconstruction will be presented in the following sections.

## RADIO GROUND WAVE SCATTERING BY AN ISOLATED INHOMOGENEOUS SURFACE

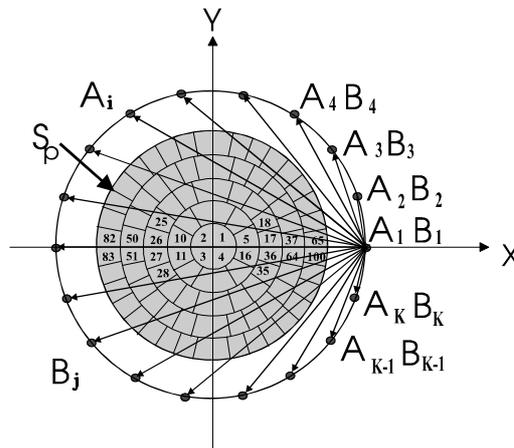


Fig.1 Measurement configuration

Consider the transmission of a radio wave over a circular isolated inhomogeneous surface  $S_p$  from a vertical antenna  $I_1 dl$  at  $A_i$  to a receiving antenna  $I_2 dl$  at  $B_j$ , both on the ground surface, as shown in Fig.1. Following the work given in [5], the vertical electric field  $E_{z1}(B)$  at  $B_j$  can be expressed as the sum of the unperturbed electric field  $E_{z1}^0(B_j)$  over a ground surface with the same normalised surface impedance  $\Delta_0$  as that of the surrounding area, and the scattered electric field  $E_{z1}^s(B)$ , i.e.

$$E_{z1}(B_j) = E_{z1}^0(B_j) + E_{z1}^s(B_j) \quad (1)$$

The unperturbed electric field  $E_{z1}^0(B_j)$  is given by [5]

$$E_{z1}^0(B_j) = A_{PG} F(w_{1B}) \{ e^{-jk_0 \rho_{1B}} / \rho_{1B} \} \quad (2)$$

where  $A_{PG}$  is a constant depending on the transmitted power and antenna gain,  $k_0$  is the wave number in free space,  $\rho_{1B}$  is the distance between A and B,  $F(w_{1B})$  is the Sommerfeld attenuation function with the numerical distance  $w_{1B}$ , i.e.

$$F(w_{1B}) = [1 - j\sqrt{\pi w} e^{-w} \operatorname{erfc}(j\sqrt{w})] \Big|_{w=w_{1B} = -jk_0 \rho_{1B} \Delta_0^2 / 2} \quad (3)$$

The scattered electric field  $E_{z1}^s(B_j)$  in (1) is given by

$$E_{z1}^s(B_j) = -\frac{jk_0}{2\pi} \iint_{S_p} [\Delta_s(P) - \Delta_0] E_{z1}(P) F(w_{2P}) \frac{e^{-jk_0 \rho_{2P}}}{\rho_{2P}} dS \quad (4)$$

where  $E_{z1}(P)$  is the vertical electric field at P on the surface  $S_p$  from the radiation of the same antenna,  $\Delta_s(P)$  is the normalised surface impedance of the surface  $S_p$ ,  $\rho_{2P}$  is the distance between B and P, and  $w_{2P} = -jk_0 \rho_{2P} \Delta_0^2 / 2$ . When the surface  $S_p$  is discretised into 5 rings and  $4(2n_r+1)$  segments in each ring for  $n_r=1$  to 5 giving a total of  $N=100$  cells of equal area as shown in Fig.1, the scattered electric field can be obtained to be [3, 4],

$$E_{z1}^{s,est(i,j)} = \sum_{n=1}^N E_n \frac{e^{-jk_0 \rho_{Bn}}}{jk_0 \rho_{Bn}} (\Delta_{sn}^{est0} - \Delta_0) F(w_{2n}) (1 - \cos(k_0 a_n)) \quad (5)$$

where  $E_n$  is the electric field on the n-th cell, with n starting from 1 to 100 anticlockwise from the inner ring to the outer rings. The values of  $E_n$  are the solutions of the matrix equation

$$[E_n] = [C_{mn}]^{-1} [E_m^0] \quad (6)$$

where  $E_m^0$  is the unperturbed electric field at the centre of the m-th cell obtained using (2) but with field point at the centre. The elements of the matrix  $[C_{mn}]$  in (6) are given by [3, 4]

$$C_{mn} = \frac{e^{-jk_0 \rho_{mn}}}{-jk_0 \rho_{mn}} (\Delta_{sn} - \Delta_0) F(w_{mn}) (1 - \cos(k_0 a_n)) \quad \text{for } m \neq n \quad \text{or} \quad C_{mn} = 1 + (\Delta_{sn}^{est0} - \Delta_0) (1 - e^{-jk_0 a_n}) \quad \text{for } m = n \quad (7)$$

where  $\rho_{mn}$  is the distance from the m-th cell to the n-th cell and  $w_{mn} = -jk_0 \rho_{mn} \Delta_0^2 / 2$ .

## IMAGE RECONSTRUCTION ALGORITHM

For image reconstruction, a total of K antennas capable of transmitting and receiving are positioned in a circle with a diameter D enclosing the isolated surface with equal spacing at  $A_i$  for  $i=1$  to K or  $B_j$  for  $j=1$  to K as shown in Fig.1 so as to create 'multi-views' of the surface. The measured scattered field at  $B_j$  are denoted as  $E_{z1}^{s,meas(i,j)}$ , when the i-th antenna radiates. Since the number of measurements taken is not generally equal to the number of unknowns  $\Delta_{sn}$ , the solution of  $\Delta_s(P)$  is not unique. An approximated solution is then found by searching a set of  $\Delta_{sn}$  which minimise the difference between the measured scattered field  $E_{z1}^{s,meas(i,j)}$  and the corresponding estimated scattered field, denoted as  $E_{z1}^{s,est(i,j)}$  produced by the estimated distribution of  $\Delta_{sn}$  for the same transmitting and receiving positions i and j, or the cost function defined as

$$F_{cost}(\Delta_s^{est}) = \sum_{i=1}^K \sum_{j=1}^K |\Psi_{ij}|^2 = \sum_{i=1}^K \sum_{j=1}^K |E_{z1}^{s,meas(i,j)} - E_{z1}^{s,est(i,j)}|^2 \quad (8)$$

Using the steepest descent (SD) method, the search for the approximated solution starts with a set of guessed values of  $\Delta_{sn}^{est0}$ . The complex updating direction  $d_n$ , which combines the directions for changes in real and imaginary parts, for

each  $\Delta_{sn}^{est0}$  is then determined so as to reduce the error in (8), and it is chosen as the negative direction of the complex slope when the real and imaginary parts of  $\Delta_{sn}^{est}$  are varied respectively from  $\Delta_{sn}^{est0}$ . It can be derived that

$$d_n = -\sum_{i=1}^K \sum_{j=1}^K 2\Psi_{ij} \frac{\partial \Psi_{ij}^*}{\partial \text{Re}(\Delta_{sn})} \quad (9)$$

where \* denotes the complex conjugate. Hence, an updated value of  $\Delta_{sn}^{est}$  becomes  $\Delta_{sn}^{est} = \Delta_{sn}^{est0} + \beta d_n$  where  $\beta$  is a scalar value, or the step length determined from the minimisation of the error function  $f(\beta) = F_{\cos t}(\Delta_{sn}^{est0} + \beta d_n)$  using one-dimensional line search method [6], or a closed form expression given in [3]. The iterative process will then be continued. The overall error in each iteration can be estimated using the normalised R.M.S. error function defined as

$$F_{Error} = \sqrt{\frac{\sum_{i=1}^K \sum_{j=1}^K |E_{z1}^{s,meas(i,j)} - E_{z1}^{s,est(i,j)}|^2}{\sum_{i=1}^K \sum_{j=1}^K |E_{z1}^{s,meas(i,j)}|^2}} \quad (10)$$

The iteration terminates when a pre-set error condition error is met or the required number of iterations is reached. The approximated solution can also be obtained using conjugate gradient methods [4]. For the numerical simulations described below, the required measured scattered electric fields are obtained by solving forward problems with the pre-defined surface impedance distributions, and  $\Delta_{sn}^{est0} = \Delta_{0,seawater}$ .

## NUMERICAL SIMULATION AND RESULTS

### Wet and Dry Mixed Ground Surface

The first ground feature used for simulation study consists of a dry area occupying the top half of the surface, and a wet area on the bottom half of the surface. The dry area has  $\epsilon_r = 4$  and  $\sigma = 0.001$  S/m and the wet area has  $\epsilon_r = 10$  and  $\sigma = 0.01$  S/m. The isolated surface of 300m in diameter is surrounded by seawater with  $\epsilon_r = 80$  and  $\sigma = 4$  S/m. A total of 18 antennas located around a circle of diameter  $D = 450$ m are used. At the chosen operating frequency of 1MHz, the exact and reconstructed distributions of  $\text{Re}(\Delta_s)$  using the SD method after 20 iterations are compared in Figs.2(a) and (b) respectively. The dry and wet areas can be well identified from the reconstructed image even though the dry and wet boundary is smoothed in the reconstruction. Similar observation can be made from the image of  $\text{Im}(\Delta_s)$  distribution.

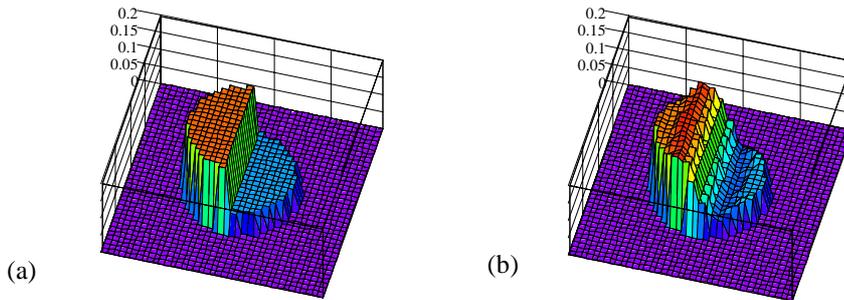


Fig.2 (a) Exact and (b) reconstructed distributions of  $\text{Re}(\Delta_s)$  of a wet and dry mixed ground surface

### Wet Ground with Icy Patches

The second ground feature is a mixed surface of wet ground and two icy patches. The isolated surface of 300m in diameter is again surrounded by seawater with  $\epsilon_r = 80$  and  $\sigma = 4$  S/m. The wet area has  $\epsilon_r = 15$  and  $\sigma = 0.05$  S/m, and the icy areas at the following cells 5, 16, 17, 18, 25, 26, 27, 28, 35 and 36 have  $\epsilon_r = 3.2$  and  $\sigma = 0.005$  S/m. A total of 18 antennas are also used in a circle of  $D = 450$ m. At the chosen operating frequency of 3MHz, the exact and reconstructed distribution of  $\text{Re}(\Delta_s)$  using the SD method obtained after 20 iterations are shown in Figs. 3(a) and (b) respectively. The distribution is well reconstructed. The icy patches can be identified from the image.

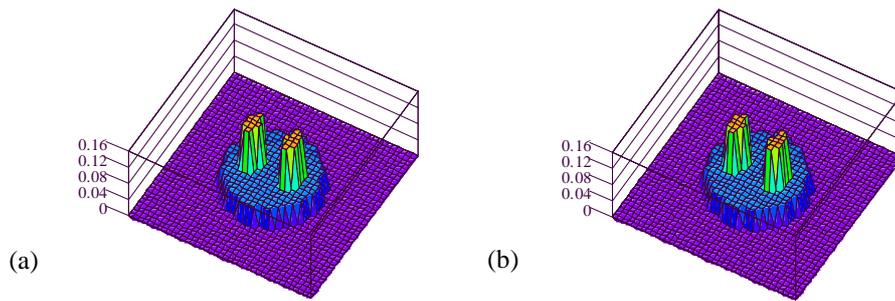


Fig.3 (a) Exact and (b) reconstructed distributions of  $\text{Re}(\Delta_s)$  of a wet ground with icy patches

### Forest Covered Surface with a Hidden Object

The third ground feature is a forest covered surface with a hidden metallic object. The forest of 4m in height is modelled with  $\epsilon_r = 10$  and  $\sigma = 0.02\text{S/m}$  and it is above a dry ground with  $\epsilon_r = 5$  and  $\sigma = 0.001\text{S/m}$ . The forest covering 50m in diameter is also surrounded by seawater. The metallic object has a diameter of 10m and a height 2.5m. The object is located at the centre of the forest area. Again 18 antennas located in a circle of  $D=75\text{m}$  are used. At the operating frequency of 6MHz, the exact and reconstructed distributions of the phase of  $\Delta_s$  after 20 iterations are shown in Figs.4(a) and (b) respectively. It can be seen that the image is well reconstructed, but with some smoothing effect. The location of the object can be identified from the image where is surface impedance is highly inductive and the phase of  $\Delta_s$  is greater than  $45^\circ$ .

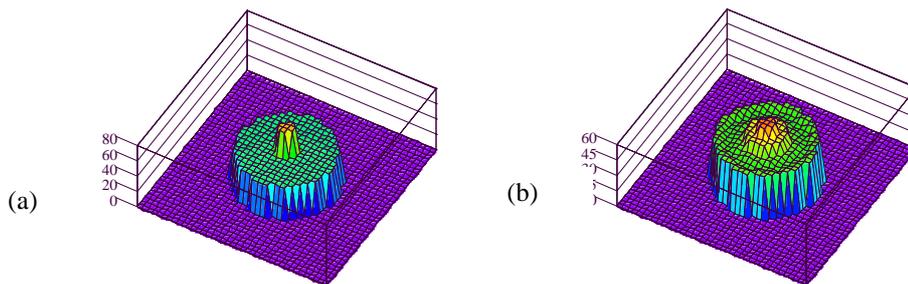


Fig.4 (a) Exact and (b) reconstructed distributions of  $\text{Phase}(\Delta_s)$  of a forest covered surface with a hidden object

### CONCLUSIONS

In this paper, a technique for the tomographic image reconstruction of ground surfaces using radiowaves is presented. The technique has been applied to the reconstruction of 1) a wet and dry mixed ground surface, 2) a wet surface with icy patches, and 3) a forest-covered surface with a hidden object. It is shown that the images of these ground features can be well reconstructed using the technique.

### REFERENCES

- [1] J. R. Wait, The ancient and modern history of EM ground wave propagation, *IEEE Antennas and Propagation Magazine*, Vol.40, No.5, p7-22, Oct. 1998.
- [2] J. R. Wait, Recent analytical investigations of electromagnetic ground wave propagation over inhomogeneous earth models, *Proc. IEEE*, Vol.62, p1061-1071, 1974.
- [3] Z. Wu, Application of Radio Ground Wave Propagation Theory to the Tomographic Imaging of Ground Surfaces, *IEEE Trans. Antennas and Propagation*, Vol.48, No.9, pp1384-1392, 2000.
- [4] Z WU, Tomographic imaging of isolated ground surfaces using radio ground waves and conjugate gradient methods, *IEE Proceedings: Radar, Sonar and Navigation*, Vol.148, No.1, 2001, pp27-34.
- [5] T. S. M. Maclean and Z. Wu, *Radiowave Propagation Over Ground*, Chapman and Hall, London, 1993.
- [6] W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, *Numerical Recipes in C*, Cambridge University Press, 2nd Edition, 1992.