CIRCUIT AND WAVEGUIDE THEORIES OF ANTENNAS IN THE TIME AND FREQUENCY DOMAINS

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ABSTRACT

Recently there has been a concerted movement to improve physical understanding of radiation from antennas. Much of the newer work has used time domain simulations tracing the evolution of a current pulse on the antenna. But it is just as valid to consider an antenna as a waveguide that transduces between a guided wave at its terminals and free space. This shifts emphasis to the fields enclosed by the conductors, when a frequency domain description in terms of modes becomes more insightful. In this paper we will look at the interrelation of these two standpoints and some related questions.

INTRODUCTION

Antennas are transducers between a guided wave in the waveguide attached to their terminals and an unguided wave in free space beyond their physical boundaries. Transduction may occur in either direction but experience has shown that analysis is generally easier if the antenna transmits. In this presentation we will follow this well-worn path.

Antenna theories can be classified into two types [1]. These are circuit theories, where emphasis is on the currents and charges on the conductors, and waveguide theories, where the emphasis is on the fields within and adjacent to the antenna, with the conductors appearing only as the boundaries. In the first, the antenna is a radiating circuit. In a waveguide it is more convenient to concentrate first on finding the fields and only later and if needed the currents and charges on the guiding walls. Both theories represent the same facts from different points of view and so are complementary. What may be opaque in terms of the one is often clear in terms of the other. An example is the question of why the current wave moves at or near the velocity of light. From a waveguide point of view, the current on the conductor is not what is causing the radiation but is a “banks of the channel” response to a temporarily confined wave. It is like the rustling of the detritus on the banks of a canal as the bow wave of a boat passes by. The response must travel with the wave that produces it.

As well as two antenna theories, we can work with them either in the time or frequency domains. The latter is essentially a steady state point of view in which we build up the response to arbitrary excitation as a superposition of harmonic responses. Its advantage is simplicity but it moderates the physics through the lens of a mathematical process that sometimes obscures at least part of it. The counter is to work directly in the time domain but the offset is the appearance of often difficult to deal with convolution integrals. However, from the point of view of understanding antenna physics, most of what it is useful to know is revealed by simple forms of excitation which minimise the difficulties.

The result is to leave us with two antenna theories and two ways of manipulating them. All four possible pairings potentially add something to advancing insight. The recent movement toward a better understanding of antenna physics has focussed very much on the circuit theory, time domain approach [2], [3]. This is perhaps no more than a natural outgrowth of the traditional, circuit theory based, engineering approach to antenna performance analysis which uses the notion that the currents and charges flowing on the antenna as the sources of its radiation field. This is not entirely fortunate as it leaves out of consideration a number of fundamental antenna properties that are better approached from a waveguide theory, time harmonic based direction. Later it will be interesting to look at both and seek ways of reconciling them.

A BRIEF LOOK AT FIELD THEORY

Maxwell arrived at his celebrated equations by giving mathematical form to the more qualitative and geometrically based ideas of Faraday. Faraday’s contribution was to move the focus away from the material objects on which the currents and charges appear toward the surrounding space and to think of the phenomena in terms of a supposed tension in this space.

Both Maxwell and Faraday had a clear physical picture of what they were about [4]. They couched it in terms of displacements along lines of force in an ether, in response to impressed electric and magnetic forces. A waveguide theory of antennas is therefore a more directly Maxwellian approach. Subsequent advances in physics, though leaving the equations, have dismantled much of the scaffolding, so that in an extreme view they remain only as a set of rules for manipulating a segment of the physical world, largely divorced from any underlying reality. As Schelkunoff remarks, "it is not necessary to endow these concepts with their literal meanings: the picture remains just as useful if they are thought of as expressions of what will happen when appropriate experiments are performed" [1]. In doing this, we have perhaps arrived at a position with which many post modernists would not feel uncomfortable.

This may be true but it is not a situation with which we should rest content. Particularly in a pedagogical context, the field equations take on something of the appearance of a *deus ex machina*, the essentially mathematical manipulation of which leads to a result which ultimately and at some remove is interpretable in the physical world without any clear physical picture being carried along the way. It is no wonder that many students find electromagnetic theory excessively abstract and are turned away by its presumed difficulty. The movement in recent years led by Miller, Burke, Smith and others has been valuable in attempting to correct this by reinserting a physical picture of how radiation occurs and seeking to give meaning to such questions as from where on the antenna current distribution does it come [2], [3].

It is possible to arrive at many useful results in electromagnetic theory without following the route taken by Maxwell. Levi-Civita is reported as having observed as early as 1897 that: "we can find the essentials of Maxwell's theory even while starting from the classical laws. It is sufficient to complete them by the hypothesis that actions at a distance are propagated with finite velocity" [5]. Soon after, relativity demonstrated that Levi-Civita's insight was more than a convenient hypothesis. However, prior to this, J.J. Thompson used the idea in 1904 to obtain the field radiated by an accelerated point charge starting form Coulomb's law [6]. It is useful to retrace Thompson's footsteps in outline for the physical insight which results.

THOMPSON'S KINK MODEL

Thompson begins by considering a point charge moving along a straight line, initially at constant velocity. The charge carries along with it a radial electric field that moves parallel to itself. Then the charge is accelerated for a brief period after which it is allowed to continue at a now increased constant velocity. Near the charge, the field lines continue to move with it but, short of assuming that the effect of accelerating the charge can be felt instantaneously everywhere, further out the field lines will continue to have the same form as would have been the case without the acceleration. The result is a discontinuity or kink in the lines of force that propagates radially outwards through the field carrying energy away from the charge. This gives a simple and quantitative picture of the radiation field in terms of lines of force, it being like the disturbance propagated along a rope when one end is given a sharp flick.

Thompson himself traces the genesis of the idea to a letter written by Faraday to Maxwell in which he offered "tremors in the lines of force" as a possible explanation for the propagation of light. By considering the current in a filamentary wire as a continuous flow of point charges, it is easy to generalise Thompson's kink model to consider what happens when there is charge acceleration. If in turn the acceleration is made time harmonic, Hertz' result for the radiation field of a current element, the fundamental Green's function of the circuit theory approach [7], follows quickly. Several things emerge from this picture, but the most fundamental is to associate radiation with charge acceleration. Radiation from an antenna will occur not where the current is greatest but from those regions where the charge acceleration is a maximum.

A CIRCUIT THEORY APPROACH TO DIPOLE RADIATION

We can imagine a conducting filament that is excited by passing round its middle a magnetically conducting loop to which it is normal. When the loop is pulsed with a magnetic current, the resulting impulsive electromotive force will draw charge carriers from one side, where a local depletion is created, and use them to produce a local excess on the other. It is the propagation of these local excesses and depletions that constitute the impulsive macroscopic charge and current waves. The result is to have impulsive currents, positive on one arm and negative on the other, which move out away from their point of generation as travelling waves at or near the speed of light. This is not, of course, to imply that a slug of charge moves along the antenna at this speed. The mobilities of the charge carriers in any physical conductor are much too small to permit this. What we are dealing with is a local disequilibrium of these charge carriers and it is the disequilibrium that is propagating, not the charge carriers themselves.

One interesting way to look at the unfolding of this scenario is in terms of Thompson's kink model. The time derivative of the impulsive current, the acceleration of the charge, is an impulse doublet. Assuming for the moment that nothing acts to change or diminish the impulse as it travels outwardly along the filament, almost everywhere the result must be no radiation as the line of force is kinked first one way and then immediately equally and oppositely back the other, with no net effect. The exceptions to this are in the regions where there is net acceleration of charge, i.e. at the centre of the filament where the disturbance is launched and from its ends where the outgoing travelling waves are reflected [2], [7]. This would indicate that radiation is to be sourced to three spherical waves emanating from the ends and middle of the radiator. It is interesting that when one looks at the time harmonic solution of this problem in which the filament supports a sinusoidally distributed standing wave of current, itself the result of interference of equal, oppositely directed travelling waves, the phase terms corroborate the time domain picture.

An interesting addendum to this problem appears when attempts are made to solve it by time domain, computer simulations [8]. There one must use a conducting rod of finite diameter and a pulse of finite, albeit short, duration. The results appear to show that, in addition to centre and end radiation, there is further radiation from along the length of the conductor. It is important to note that the previous conclusions resulted from an assumed, not modified as it progresses, current wave and not through the solution of any boundary value problem that might have established its exact form. This can only be the case if the impedance presented to the wave is uniform along the whole of the rod. That would be so for biconical conductors, of which a filament is the zero cone angle limit, but not for a conductor of uniform section. The result will be continuing deceleration of portion of the charge wave as it progresses and, like as previously only at the ends, this acts to produce the necessary net acceleration needed to account for the simulation results. The truth of this is told in a reflected charge wave that starts to appear immediately the impulse is launched on its way.

Another insight comes from looking more closely at the lines of force; in the foregoing more use was made of the notion of radiation being associated with charge acceleration than with their exact shape. Lines of force will join the positive and negative charge disequilibria and, from a circuit theory perspective, be towed along with them as they move along the conductor, becoming increasingly bowed out as they approach the ends. On reflection, in the neighbourhood of the charge pulses, the lines of force will return with them but nearer the plane normal to the conductor the inertia of the field will prevent its immediate return. The result will ultimately be to produce the closed field lines necessary in a radiation field [7].

ANTENNAS AS WAVEGUIDES

As we have already alluded, the other main paradigm is to place antennas in the domain of waveguides, where in the time harmonic case the emphasis is in terms of modes [1]. There is a view that modes are mathematical creations rather than physical entities, that they are not physical as are charges, currents, lines of force, etc. So far as electromagnetic fields are concerned, it can, of course, be remarked that none of the above have any direct sensory appeal and to that extent all are equally imaginary. However it is a distinction that we do not make in other domains of physics. In the study of sound, for example, we have no difficulty with the idea that we can hear harmonics and non-harmonically related overtones. Moreover Chladni's figures which make visible the modal patterns of vibrating plates by sprinkling them with a light powder are surely just as real as is making magnetic lines of force visible with iron powder or the use of fragments of paper to lend visual appeal to electrostatic fields [9]. Here we propose to recognise modes as as much physical entities as any other of the paraphernalia that routinely we bring with us to the study of electromagnetic fields.

It is convenient to associate with any antenna its circumscribing sphere, the smallest sphere that is just able to contain it. We regard the region internal to the sphere as the antenna's waveguide region and that external to it as free space into which it radiates. Those parts of the boundary not taken up with any of the antenna conductors constitute its aperture from which the radiation field is launched. The function of the antenna is to convey energy from the guided wave present at its input port to the aperture, generally conditioning it on the way to produce what for the application at hand is an appropriate radiation field. Beyond the input port but still within the circumscribing sphere, there may also be the currents and charges that we might still like to regard as the ultimate sources of the input guided wave, but now they have no importance to us.

Within the antenna the field exists as a set of modes appropriate to the boundary conditions imposed by the antenna conductors. Harrington and others have shown that, even though they no longer have material support, there also exists a set of free space modes in terms of which the field produced by any antenna beyond its aperture can be expressed [10]. At a sufficient additional distance this metamorphoses into its radiation field. What occurs at the aperture surface is a process of mode matching in which (generally some of – the rest is internally reflected) the energy contained in the incident guided mode is transferred into the free space modes. These free space modes are typified by having a cutoff radius in the neighbourhood of which their wave impedance moves from largely reactive within, to largely real without The available mode set for the antenna is comprised of those modes that have made this transition at a radius less than or equal to that of the aperture. While in general modes not contained in the available mode set cannot be excited, it will be the antenna that determines with what relative weights those that can be, are.

This is an important observation that exposes performance limits which are not revealed by the circuit theory approach. It explains, for example, why a highly directive antenna must necessarily be large in terms of the wavelength, why an isotropic antenna cannot be constructed and why very small antennas have highly reactive radiation impedances, low gain and simple patterns. It is also a viewpoint that renders largely meaningless the question prompted by the circuit theory approach of from where on the antenna does its radiation come. On this view, the currents and charges on the conductors are the responses to the wave flowing in the guide of which they are the boundaries, rather the causes of the radiation field.

RECONCILIATION OF CIRCUIT AND WAVEGUIDE THEORIES

Huygens' Principle or, as it is generalised in electromagnetic theory, the equivalence principle can be called upon as a vehicle to unite the circuit and waveguide theories [10]. It is interesting to observe that should the fields on the antenna aperture be transformed into equivalent distributions of electric and magnetic currents, exactly the same integrals as are used in the circuit theory approach can be used to calculate the radiation fields. This suggests that these integrals are essentially expressions of analytic continuation. Inasmuch as knowledge of the currents on the conductors must imply a knowledge of the fields in their immediate neighbourhood, in the end we might like to view which theory we use as simply a choice of the surface over which we integrate, the conductors or the aperture.

CONCLUSION

In this paper we have examined the physics of radiation from antennas in the time and frequency domains using both the circuit theory and waveguide standpoints. What emerges are complementary views, all of which are needed to form a complete picture. A reconciliation of these standpoints has been suggested through viewing the equivalence principle as a tool for analytic continuation.

REFERENCES

[1] S.A. Schelkunoff, *Advanced Antenna Theory*, Wiley, New York, 1952.

[2] G.S. Smith, "On the interpretation for radiation from simple current distributions", *IEEE A&P Magazine*, vol. 40, pp. 9-14, June 1998.

[3] E.K. Miller, "Why is there electromagnetic radiation and where does it come from?", *Course Notes, 16th Annual review of Progress in Applied Computational Electromagnetics*, US Naval Postgraduate School, Monterey, March 2000.

[4] J.C. Maxwell, "On Faraday's lines of force", *The Scientific Papers of James Clerk Maxwell*, Dover, New York, pp. 155-229, 1965.

[5] A.O'Rahilly, *Electromagnetic Theory*, Dover, New York, pp. 190, 1965.

[6] J.J. Thompson, *Electricity and Matter*, Constable, Westminster, 1904.

[7] H.E. Green, "Fundamentals of radiation from antennas", *IE Aust Monitor*, vol. 26, pp. 16-25, June-August 2001.

[8] E.K. Miller and G.J. Burke, "A time domain exploration of how equivalent charge and radiation from thin wires might be related", *IEEE International Symposium on Antennas & Propagation*, Salt Lake City, pp. 2042-2045, July 2000.

[9] J.W.S. Rayleigh, *The Theory of Sound*, vol. 1, pp. 352-394, Dover, New York, 1945.

[10] R.F. Harrington, *Time Harmonic Electromagnetic Waves*, McGraw-Hill, New York, 1961.