COMPARISON BETWEEN THE INTEGRAL EQUATION APPROACH AND THE TRANSMISSION LINE THEORY FOR ANALYSIS OF A LOOP ANTENNA IN A MAGNETOPLASMA

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ABSTRACT
A study is made of the electrodynamic characteristics of a circular thin strip-loop antenna immersed in a cold uniform magnetoplasma. The main attention is devoted to comparison of the results of calculations of the antenna input impedance obtained using an integral equation method with the results of existing approximate theories such as that of the transmission line theory. The ranges of parameters are determined in which previously developed approximate theories should be refined for correct predictions of the input impedance of a loop antenna in a resonant magnetoplasma.

ANALYSIS AND RESULTS
We consider a loop antenna which is an infinitely thin, perfectly conducting strip of half-width \( b \) coiled into the ring of radius \( a \). The antenna is immersed in a magnetoplasma so that its axis is parallel to the ambient magnetic field \( B_0 = B_0 \hat{z} \). The antenna is excited by a given, time-harmonic voltage which creates the only azimuthal component of the electric field distinct from zero in a narrow angular interval at \( \rho = a \). The plasma is described by the dielectric tensor \( \hat{\varepsilon} = \varepsilon_0(\varepsilon \hat{\rho} \rho - ig \hat{\phi} \phi + ig \hat{\phi} \phi + \varepsilon \hat{z} \hat{z}) \). The emphasis is placed on frequencies lying in the resonant interval of the whistler band where \( \varepsilon > 0 \) and \( \eta < 0 \).

The antenna input impedance yielded by the approximate transmission-line theory [1] is as follows:

\[
Z = R + iX = iZ_0 k_0 \pi^{-1} h^{-1} \ln(4a/b) \tan(\pi ha),
\]

where \( k_0 = \omega (\varepsilon_0 \mu_0)^{1/2} \), \( Z_0 = (\mu_0/\varepsilon_0)^{1/2} \), and \( h = k_0 |\varepsilon \eta|^{1/4}(1 - i)/2^{-1/2} \). Recently, the problem of finding the current distribution and the input impedance of the loop antenna in a magnetoplasma has been solved by using an accurate integral equation method [2]. We have found that the simple expression (1) follows from the results of the rigorous analysis [2] only in the limit \( \ln(4a/b) \to \infty \) and ceases to be accurate for the general case.

Fig. 1 shows \( R \) and \( X \) as functions of radius \( a \) for plasma parameters typical of the earth’s ionosphere (plasma density \( N = 10^6 \) cm\(^{-3}\) and \( B_0 = 0.5 \) G) in the resonant interval of the whistler band (\( \omega = 1.88 \times 10^3 \) s\(^{-1}\)). Curves of \( X \) yielded by the accurate and approximate theories almost coincide for \( a < 10 \) m, in which case the condition \(|h|/a \ll 1\) is satisfied with a sufficient margin. As \( a \) increases, there appears some difference in the reactances given by the accurate and the approximate formulas. Nevertheless, for a moderately large antenna radius, expression (1) gives a result accurate to about 20%. Similar behavior is observed for the resistive part of the impedance for a comparatively large radius \( a \). But for a small loop \(|h|/a \ll 1\), formula (1) ceases to be accurate for determining \( R \), as is evident from Fig. 1. Note that \( R \) is much greater than the resistive part \( R_0 \) of the uniform-current loop impedance, which is widely used for small loop antennas. Thus, the accurate approach based on the integral equation method is required for correctly determining the input impedance of such an antenna in a resonant magnetoplasma.

REFERENCES