ON THE OSCILLATING TWO STREAM INSTABILITY IN IONOSPHERIC HEATING EXPERIMENTS

S. P. Kuo

Department of Electrical Engineering, Polytechnic University
901 Route 110, Farmingdale, NY 11735, USA
E-mail: spkuo@rama.poly.edu

ABSTRACT

It is shown that oscillating two-stream instability (OTSI) can be excited in sizable spatial region below the reflection height of the O-mode HF heating wave in mid-latitude ionospheric heating experiments. Near the reflection height, the threshold field of OTSI increases drastically with the oblique propagation angles of its Langmuir sidebands whose angular distribution is thus confined in narrow cones around the geomagnetic field. Therefore, obliquely propagating Langmuir sidebands of OTSI prefer to be excited in their matching height regions, where sidebands satisfy local dispersion relation. Their matching heights move downward from the reflection height as oblique propagation angles increase.

INTRODUCTION

In ionospheric heating experiments [1-4], parametric decay instability (PDI) and oscillating two stream instability (OTSI) are excited most effectively in regions between the reflection height of the O-mode HF heating wave and matching heights of instabilities, where the heating wave becomes linearly polarized in the geomagnetic field direction and its electric field amplitude increases considerably owing to the slowdown of wave propagation. The sidebands of PDI and OTSI are Langmuir (plasma) waves [5,6], which then cascade through secondary parametric instabilities to new Langmuir (plasma) waves [7-9]. Backscatter radar is used to monitor the enhancement of plasma waves in heating experiments. It can effectively detect those plasma waves, each one having a wavenumber twice of the wavenumber of the probing radar signal and propagating in a direction either parallel or antiparallel to the pointing direction of radar. Those plasma waves are termed "HFPLs", which have the frequency spectrum recorded by backscatter radar.
The results of radar detections indicated that the originating height of HFPLs was much closer to the matching height of the sidebands of OTSI than to the reflection height of the O-mode HF heating wave. This is in contrast to the unmagnetized case that OTSI can only be excited in a narrow region near the cutoff layer of the pump wave [10]. Thus it is necessary to analyze OTSI in magnetized plasma to explore its spatial distribution.

THEORY

The OTSI process involves the decay of a dipole pump $\mathbf{E}_p(\omega_0, \mathbf{k}_p=0)$ to two Langmuir sidebands $\phi_1(\omega_1, \mathbf{k}_1)$ and $\phi_1'(\omega_1', \mathbf{k}_1')$ and a purely growing mode $n_s(\omega_s=i\gamma_s, \mathbf{k}_s)$, where $\mathbf{E}_p = \hat{z}\, E_p$, ϕ_1 and ϕ_1', and n_s are pump wave field, sidebands' electrostatic potentials, and purely growing mode's density perturbation, respectively, and γ_s is the growth rate of the instability. The frequency and wavevector matching conditions imposed by this parametric coupling process are given by $\omega_0 = \omega_1 + \omega_s^* = \omega_1' - \omega_s$ and $\mathbf{k}_p = 0 = \mathbf{k}_1 + \mathbf{k}_s = \mathbf{k}_1' - \mathbf{k}_s$, where $\mathbf{k}_1 = \hat{z}\, k_0 + \hat{x}\, k_\perp$ and the background magnetic field $\mathbf{B}_0 = \hat{z}\, B_0$; the matching conditions lead to $\omega_1 = \omega_1' = \omega_0 + i\gamma_s$ and $\mathbf{k}_1' = \mathbf{k}_s = -\mathbf{k}_1$.
The coupled mode equations for Langmuir sidebands are derived from the electron continuity equation and electron momentum equation as [11]

$$\{[(\partial_t + \nu_e)^2 + \Omega_e^2](\partial_t^2 + \nu_e\partial_t + \omega_p^2 - 3v_{te}^2\nabla^2)\nabla^2 - \Omega_e^2(\omega_p^2 - 3v_{te}^2\nabla^2)\nabla_\perp^2\}\phi_{1\pm}$$
$$= \omega_p^2[(\partial_t + \nu_e)^2 + \Omega_e^2]\partial_z\, E_p n_{s\pm}/n_0 \qquad (1)$$

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where the notations $\phi_{1+} = \phi_1$, $\phi_{1-} = \phi_1'$, and $n_{s+}^* = n_s = n_{s-}$ are used; ν_e, ω_p, and Ω_e are the electron collision frequency, electron plasma frequency, and electron cyclotron frequency; $v_{te} = (T_e/m)^{1/2}$ is the electron thermal speed and n_0 is the unperturbed plasma density; stands for a filter which keeps only terms having the same phase functions as $\phi_{1\pm}$, the corresponding physical functions on the left hand side of (1).

The parallel (to the magnetic field) component of the wave vector of the short scale purely growing mode is not negligibly small, thus the short scale purely growing mode is mainly driven by the parallel component of the ponderomotive force induced by the high frequency wave fields. From the fluid equations of electron and ion, the coupled mode equation for the purely growing mode is derived to be [12]

$$\{(\partial_t^2 + \Omega_i^2)[\partial_t(\partial_t + \nu_i) - C_s^2\nabla^2] + \Omega_i^2 C_s^2 \nabla_\perp^2\}\nabla_z^2(n_s/n_0)$$

$$= (m/M)\,[(\partial_t^2 + \Omega_i^2)\nabla_z^2 + \partial_t^2\nabla_\perp^2]\partial_z a_{pz} \tag{2}$$

where ν_i and Ω_i are the ion-neutral particle collision frequency and ion cyclotron frequency, respectively; $C_s = [(T_e + 3T_i)/M]^{1/2}$ is the ion acoustic speed, and M is the ion mass; $a_{pz} = \langle \mathbf{v}_e\cdot\nabla v_{ez} = \partial_z \langle v_e^2/2$.

Equations. (1) and (2) lead to the dispersion relation

$$\{(\gamma_s^2 + \Omega_i^2)[\gamma_s(\gamma_s + \nu_i) + k_1^2C_s^2] - \Omega_i^2\, k_\perp^2 C_s^2\}$$

$$= 2(e^2/mM)\, k_1^2\cos^2\theta\, (\gamma_s^2 + \Omega_i^2\cos^2\theta)\{\Delta\omega^2/[\Delta\omega^4 + \omega_0^2(2\gamma_s + \nu_e)^2]\}|E_p|^2 \tag{3}$$

where $\Delta\omega^2 = \omega_p^2 + 3k_1^2v_{te}^2 + \Omega_e^2\sin^2\theta - \omega_0^2$ and $\theta = \sin^{-1}(k_\perp/k_1)$ is the oblique propagation angle of Langmuir sidebands. The threshold field of the instability is determined, by setting $\gamma_s = 0$ in (3), to be

$$|E_p(\theta)|_{th} = (mM/2e^2)^{1/2}C_s[(\Delta\omega^4 + \omega_0^2\nu_e^2)/\Delta\omega^2\,]^{1/2}/\cos\theta \tag{4}$$

For each propagation angle θ and wavelength λ_1, the instability has the minimum threshold field

$$|E_p(k_1,\theta)|_m = (mM/e^2)^{1/2}C_s(\omega_0\nu_e)^{1/2}/\cos\theta \tag{5}$$

as it is excited in a preferential height layer at $h = h_1$ with $\Delta\omega^2(k_1,\theta) = \omega_0\nu_e$, *i.e.*, $\omega_p^2(h_1) = \omega_p^2(k_1,\theta) = \omega_0(\omega_0 + \nu_e) - 3k_1^2v_{te}^2 - \Omega_e^2\sin^2\theta$, where the altitude h_1 of the preferential layer is very close to the reflection height of the O-mode heating wave only in the unmagnetized case or in the case of small oblique propagation angle, *i.e.*, $\sin^2\theta << 1$. In other words, the spectral lines of the Langmuir sidebands excited by OTSI have an angular (θ) and a spectral (k_1) distribution, as well as a spatial (h) distribution in a finite altitude region. This minimum threshold field (5) increases with the increase of the oblique propagation angle θ of (k_1, θ) lines. The altitude h of the minimum threshold region for the excitation of (k_1, θ) lines moves downward as the oblique propagation angle θ of these lines increases.

DISCUSSIONS

Consider a group of spectral lines (k_1, θ) having $\mathbf{k_1} = \mathbf{k_0} + \mathbf{k_\perp}$, where $\mathbf{k_0} = \hat{\mathbf{z}}\, k_0$ is the common parallel component of their wave vectors and the oblique propagation angle θ increases with $k_\perp$. The threshold fields of exciting (k_1, θ) lines near the HF reflection height layer at $h = h_0$ with $\omega_p(h_0) = \omega_p(k_0,0)$ and in the matching height layer at $h = h_1$ with $\omega_p(h_1) = \omega_p(k_1,\theta)$, expressed in terms of the minimum threshold field $|E_p(k_0,0)|_m$ of exciting the (k_0, 0) line at its matching height h_0, are $|E_p(k_1,\theta)|_{th} = f(\theta,0)|E_p(k_0,0)|_m$ and $|E_p(k_1,\theta)|_m = |E_p(k_0,0)|_m/\cos\theta$, respectively, where $|E_p(k_0,0)|_m = (mM/e^2)^{1/2}C_s(\omega_0\nu_e)^{1/2}$,

$$f(\theta,0) =$$

$$(\cos\theta)^{-1}[1 + (\Delta\omega_m^4/2\omega_0\nu_e(\omega_0\nu_e + \Delta\omega_m^2)]^{1/2}, \tag{6}$$

and $\Delta\omega_m^2 = \Omega_e^2\sin^2\theta + 3k_0^2v_{te}^2\tan^2\theta$.

Use the values of the parameters in Arecibo heating experiments: $\omega_0/2\pi = 5.1$ MHz, $\Omega_e/2\pi = 1.06$ MHz, $\nu_e = 500\ s^{-1}$, $v_{te} \sim 1.3\times10^5$ m/s, $C_s \sim 1.4\times10^3$ m/s, and $k_0 = 4\pi\ m^{-1}$ (*i.e.*, $\lambda_{||} = 0.495$ m), the minimum threshold field for the $(k_0, 0)$ line evaluated from (5) is about 0.17 V/m. However, the strong increasing dependence of $f(\theta,0)$ on θ as given by (6) indicates that the threshold field of (k_1, θ) lines excited in the same height layer at $h = h_0$ increases rapidly with the obliquely propagating angle θ of (k_1, θ) lines. For example, the threshold fields for $\theta = 10^o$ and 40^0 lines are 1.22 V/m and 6.06 V/m, respectively. Thus the angular distribution of the spectrum of Langmuir sidebands excited by OTSI in the region near the HF wave reflection height, *i.e.*, at $h = h_0$, is expected to be confined in a narrow cone around the geomagnetic field. On the other hand, $f(\theta,0)\cos\theta$ also increases rapidly from 1 with the obliquely propagating angle θ, for example, $f(\theta,0)\cos\theta = 27.3$ for $\theta = 40^0$. The threshold field for $(k_1, 40^0)$ lines excited in their matching layer at $h = h_1$ with $\omega_p(h_1) = \omega_p(k_1,40^0)$ is thus reduced to 0.22 V/m, which is 27.3 times lower than the threshold field 6.06 V/m for the same lines excited in the layer at $h = h_0$. It shows that the oblique angle lines prefer to be excited in their matching height regions, rather than in the region near the HF reflection height. Moreover, large oblique angle lines can only be excited in regions close to their matching heights. Although $f(\theta,0)$ increases with θ rapidly, the minimum threshold field for the $(k_0, 0)$ line is very low, thus the excited Langmuir sidebands in the same layer at $h = h_0$ can still have a small angular distribution around the geomagnetic field. As the excitation region moves downward to the matching height h_{10} of the (k_{10}, θ_0) lines, the threshold field of (k_1, θ) lines becomes $|E_p(k_1, \theta)|_{th} = f_1(\theta,\theta_0)|E_p(k_0, 0)|_m$, where

$$f_1(\theta,\theta_0) =$$

$$(\cos\theta)^{-1}[1 + (\Delta\omega_m^2 - \Delta\omega_{m0}^2)^2/2\omega_0\nu_e(\omega_0\nu_e + \Delta\omega_m^2 - \Delta\omega_{m0}^2)]^{1/2} \tag{7}$$

and $\Delta\omega_{m0}^2 = \Omega_e^2\sin^2\theta_0 + 3k_0^2v_{te}^2\tan^2\theta_0$. Thus the angular distribution of the Langmuir sidebands excited in this layer becomes a hollow shape from θ_1 to θ_2, where $\theta_1 \sim \theta_0$ (*i.e.*, lines around $\theta = 0$ can not be excited because $\Delta\omega_m^2 > \Delta\omega_{m0}^2 - \omega_0\nu_e$ is required), and the angular range $\Delta\theta = \theta_2 - \theta_1$ decreases as θ_0 increases.

We have shown that OTSI prefers to be excited in the matching height region of its sidebands, where has a distance below the HF reflection height when its sidebands propagate at large oblique angles. Thus OTSI is excited over a wide altitude range for different propagation angle in heating experiments. In each narrow altitude region the excited Langmuir sidebands have a small angular distribution. The spatial distribution of the instability then leads to a large angular distribution in the excited Langmuir sidebands.

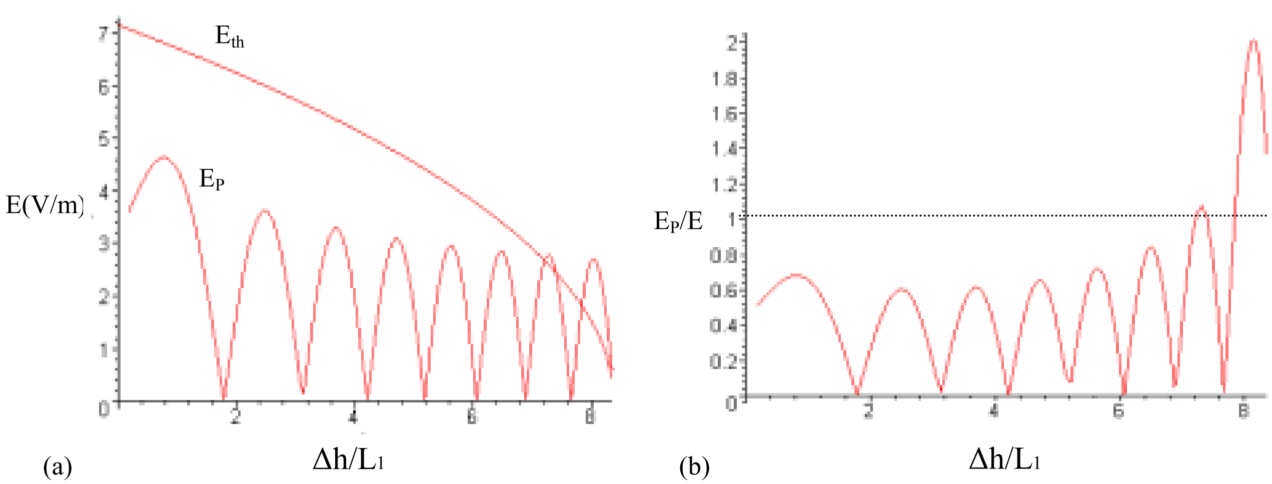

FIG. 1. (a) Altitude distributions of the heating wave electric field E_p and the threshold field E_{th} of OTSI exciting $(k_1, 40^0)$ sidebands and (b) the ratio (E_p/E_{th}) of the two fields as a function of the altitude h; $\Delta h = h_0 - h$, h_0 is the altitude of the HF reflection height, and $L_1 = 220$ m.

Owing to the swelling effect, the HF wave electric field amplitude increases with the altitude in the region near the HF reflection height. Thus the ratio of the heating wave field amplitudes at the two heights should be evaluated for making a properly conclusion on where the preferential altitude of exciting OTSI is. This is demonstrated in Fig. 1. Altitude distributions of the heating wave electric field E_p and the threshold field E_{th} of OTSI exciting (k_1, 40^0) sidebands are plotted for inhomogeneous ionosphere having a linear scale length L = 50 Km, *i.e.*, for $\omega_p^2 = \omega_0^2(1 - \Delta h/L)$, where $\Delta h = h_0 - h$, E_p is governed by the Airy function, and E_{th} is given by (4) for $k_1 = 5.22\pi\ m^{-1}$ and $\theta = 40^0$. As shown, the largest value of the ratio (E_p/E_{th}) of the two fields is located close to the matching height. Therefore, the matching height of the Langmuir sidebands, rather than the HF reflection height, is the preferential altitude of exciting OTSI in Arecibo heating experiments [13].

REFERENCES

[1] H. C. Carlson, W. E. Gordon, and R. L. Showen, "High-frequency induced enhancements of the incoherent scatter spectrum at Arecibo," *J. Geophys. Res.*, vol. 77, p. 1242, 1972.

[3] R. L. Showen and D. M. Kim, "Time variations of HF-enhanced plasma waves," *J. Geophys. Res.*, vol. 83, p. 623, 1978.

[3] P. Stubbe, H. Kohl, and M. T. Rietveld, "Langmuir turbulence and ionospheric modification," *J. Geophys. Res.*, vol. 97, p. 6285, 1992.

[4] Rietveld, M. T., B. Isham, H. Kohl, C. LaHoz, and T. Hagfors, "Measurements of HF-enhanced plasma and ion lines at EISCAT with high-altitude resolution," *J. Geophys. Res.*, vol. 105, p. 7429, 2000.

[5] F. W. Perkins, C. Oberman, and E. J. Valeo, "Parametric instabilities and ionospheric modification," *J. Geophys. Res.*, vol. 79, p. 1478, 1974.

[6] J. A. Fejer, "Ionospheric modification and parametric instabilities," *Rev. Geophys.*, vol. 17, p. 135, 1979.

[7] S. P. Kuo, "Cascade of the parametric decay instability in ionospheric heating experiments," *J. Geophys. Res.*, vol. 106, p. 5593, 2001.

[8] S. P. Kuo and M. C. Lee, "On the generation of a broad downshifted spectrum of HF wave enhanced plasma lines in the ionospheric heating experiments," *Geophys. Res. Lett.*, vol. 26, p. 3289, 1999.

[9] S. P. Kuo, "Cascade of parametric instabilities in ionosphere heating experiments," *J. Plasma Phys.*, vol. 66, p. 315, 2001.

[10] K. Nishikawa, "Parametric excitation of coupled waves, I. General formulation & II. Parametric plasma-photon interaction," *J. Phys. Soc. Japan*, vol. 24, pp. 916 & 1152, 1968.

[11] S. P. Kuo, B. R. Cheo, and M. C. Lee, "The role of parametric decay instabilities in generating ionospheric irregularities," *J. Geophys. Res.*, vol. 88, p. 417, 1983.

[12] S. P. Kuo, "The role of nonlinear beating currents on parametric instabilities in magnetoplasmas," *Phys. Plasmas*, vol. 3, p. 3957, 1996.

[13] S. P. Kuo, "Oscillating two stream instability in ionospheric heating experiments", *Phys. Plasmas*, vol. 9, April 2002.