CASCADE OF THE PARAMETRIC DECAY INSTABILITY IN IONOSPHERIC HEATING EXPERIMENTS

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ABSTRACT

Cascade of Langmuir waves excited by parametric decay instability (PDI) in ionospheric heating experiments is studied. In resonance cascade the mother Langmuir wave has to propagate downward to the resonant location of the daughter line for the resonant decay. To balance the propagation loss of the mother Langmuir wave, a large power ratio (~10 dB) between two consecutive cascade lines in observed HFPLs is required. The nonresonant cascade proceeds at the same location of PDI. The threshold power has a N^2 dependence for $\nu_e >> 2N\nu_i$ and a $\sqrt{N}$ dependence for $\nu_e << 2N\nu_i$, where N is the cascade step number.

INTRODUCTION

Heating of the ionosphere by using powerful high-frequency (HF) waves parametric instabilities provide effective channels to convert the ground-transmitted electromagnetic (em) heating waves into electrostatic plasma waves of high and low frequencies. The sidebands excited by both of the parametric decay instability (PDI) and oscillating two stream instability (OTSI) are Langmuir waves, which have been detected by the EISCAT's UHF and VHF radars during the Tromso heating experiments [1-3] and are called "HF wave enhanced plasma lines (HFPLs)". Moreover, the cascade lines of the Langmuir sidebands excited by the PDI have also been observed in the HFPLs. However, the results of early experiments conducted by progressively increasing the heating power within the range of available powers (~ 240 MW ERP) indicated the number of cascade lines in the HFPLs recorded by the EISCAT's UHF and VHF radar was limited to a value of two, independent of input power [3,4]. This was also the case in the later experiment as the heating power was increased to 270 MW ERP [5]. As the heating power was further increased to 1200 MW ERP, for the first time, up to five cascade lines in Tromso's HFPLs were observed [6]. It is known that the O-mode HF heating wave electric field is enhanced considerably near its reflection height by swelling effect, due to the gradual increase of the plasma density that decreases the wave group velocity continuously. However, the most intense cascade lines in Tromso's HFPLs are originated from the matching height of the PDI line in the HFPLs [5], which locates lower for the UHF radar than that for the VHF radar [4].

A systematic analysis of the PDI and the subsequent cascade has been performed [7]. The underlying mechanism that limits the number of cascade lines in the HFPLs of Tromso's heating experiments is presented.

PARAMETRIC DECAY INSTABILITY (PDI)

A RH circularly polarized HF heating wave propagates upward to the region near its reflection height, its wave number decreases gradually and approaches zero, causing a swell in the wave electric field intensity and converting the wave to one having an O-mode polarization in that region. Consider the decay of this dipole pump $\mathbf{E}_p(\omega_0, \mathbf{k}_p=0)$ into a Langmuir sideband $\phi(\omega, \mathbf{k})$ and an ion acoustic decay mode $n_s(\omega_s, \mathbf{k}_s)$, where $\mathbf{E}_p$ (= $\hat{z}$ E_p), ϕ, and n_s denote pump wave field, sideband's electrostatic potentials, and ion acoustic mode's density perturbation, respectively. The frequency and wavevector matching conditions for this process are $\omega_0 = \omega + \omega_s^*$ and $\mathbf{k}_p = 0 = \mathbf{k} + \mathbf{k}_s$, where $\mathbf{k} = \hat{z}\, k_z + \hat{x}\, k_\perp$ and the background magnetic field $\mathbf{B}_0 = \hat{z}\, B_0$. The coupled mode equation for the Langmuir sideband is derived to be [8]

$$\{[(\partial_t+\nu_e)^2 + \Omega_e^2](\partial_t^2 + \nu_e\partial_t + \omega_p^2 - 3v_{te}^2\nabla^2)\nabla^2 - \Omega_e^2(\omega_p^2 - 3v_{te}^2\nabla^2)\nabla_\perp^2\}\phi$$
$$= \omega_p^2[(\partial_t+\nu_e)^2 + \Omega_e^2]\partial_z\langle E_p n_s^*/n_0\rangle \quad (1)$$

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where ν_e, ω_p, and Ω_e are, respectively, the electron collision frequency, electron plasma frequency, and electron cyclotron frequency; $v_{te} = (T_e/m)^{1/2}$ is the electron thermal speed and n_0 is the unperturbed plasma density.

The ion acoustic decay mode is mainly driven by the parallel (with respect to the magnetic field) ponderomotive force induced by the high frequency wave fields. Using the fluid equations of electron and ion, the coupled mode equation for the ion acoustic mode is derived to be [9]

$$\{(\partial_t^2 + \Omega_i^2)[\partial_t(\partial_t+\nu_i) - C_s^2\nabla^2] + \Omega_i^2 C_s^2 \nabla_\perp^2\}\nabla_z^2(n_s/n_0)$$

$$= (m/M)\,[(\partial_t^2 + \Omega_i^2)\nabla_z^2 + \partial_t^2\nabla_\perp^2]\partial_z a_{pz} \quad (2)$$

where $\nu_i = (\pi/2)^{1/2}k_s C_s (T_e/T_i)^{3/2}\exp[-(3 + T_e/T_i)/2]$ and Ω_i are the ion Landau damping rate and ion cyclotron frequency, respectively; $C_s = [(T_e + 3T_i)/M]^{1/2}$ is the ion acoustic speed, and M is the ion (O^+) mass; $a_{pz} = < \mathbf{v}_e\cdot\nabla v_{ez} > = \partial_z < v_e^2/2 >$. Let $E_p = E_p\exp(-i\omega_0 t)$ + c.c. and express the spatial and temporal variation of perturbations in the form of p = p $\exp[i(\boldsymbol{\kappa}\cdot\mathbf{r} - \varpi t)]$, where $\boldsymbol{\kappa}$ and ϖ are the appropriate wavevector and frequency of each perturbation, (1) and (2) are combined into the dispersion relation for the PDI

$$[\omega(\omega + i\nu_e) - \omega_{k\theta}^2][\omega_s^*(\omega_s^* - i\nu_i) - k^2C_s^2] = (k_z^2\omega_p^4/4\pi n_0 M\omega_0\omega)|E_p|^2 \quad (3)$$

We now set $\omega = \omega_r + i\gamma_k$ and $\omega_s = \omega_{sr} + i\gamma_k$ in (3) and evaluate the threshold field $E_{pth}(k, \theta)$ and growth rate $\gamma_k(\theta)$ of the instability excited in the resonance region of the Langmuir sideband, i.e., in the height region that $\omega_r \cong \omega_{k\theta}$, $\omega_{sr} \cong kC_s$, and $\omega_r + \omega_{sr} = \omega_0$. The results are

$$|E_{pth}(\theta)| = (mM/e^2)^{1/2}\,(\nu_e\nu_i\,\omega_{sr}\,\omega_0^3)^{1/2}/k\cos\theta\,\omega_p \quad (4)$$

and

$$\gamma_k = [(\nu_e\nu_i/4)(E_p/E_{pth})^2 + (\nu_e - \nu_i)^2/16]^{1/2} - (\nu_e + \nu_i)/4 \quad (5)$$

It is shown that the threshold field, $\propto 1/\cos\theta$, has a weak dependence on the propagation angle θ (with respect to the magnetic field) of the Langmuir sideband. The growth rate in the strong pump field case, i.e., $(E_p/E_{pth})^2 >> 1$, is proportional to $\sqrt{k\cos\theta}$. Therefore, the excited Langmuir waves are expected to have a spectral and angular distribution as well as a spatial distribution over an altitude region. Those lines shown in recorded HFPLs do not have the lowest threshold field and highest growth rate. They are most favorably excited in a region called their "matching height", which is below the reflection height of the O-mode HF heating wave by a distance $d \cong L(12k_R^2v_{te}^2 + \Omega_e^2\sin^2\theta_0)/\omega_0^2$, where L is the linear scale length of the background plasma, k_R is the wavenumber of the backscatter radar signal, and θ_0 is the conjugate angle of the magnetic dip angle.

CASCADE OF PDI-EXCITED LANGMUIR WAVES

As these Langmuir waves grow to large amplitudes, they become pump waves to excite secondary parametric instabilities. One instability process to be considered in the following is the decay of a Langmuir pump into an ion acoustic decay mode and a Langmuir sideband. As will be shown through this process Langmuir waves excited by the PDI can cascade in frequency to broaden the downshifted frequency spectrum of Langmuir waves.

The secondary parametric decay instability can occur at the same location as that of the primary parametric decay instability, which generates the mother line of the first cascade line in the HFPLs. However, the cascading process is a nonresonant decay, namely, the sideband of the secondary parametric decay instability can not satisfy the local dispersion relation of the Langmuir wave. This is because the HFPLs have the same wavevector but different frequencies. In order to achieve a resonant decay, the mother line as the pump has to propagate downward to a location where the sideband of the secondary parametric decay instability is also a local plasma mode. Thus, cascades occur at different altitudes. Thus it is necessary to include the propagation loss of Langmuir waves in determining the threshold field of each resonant cascade step.

As an intermediate step this secondary parametric decay instability excited by the PDI-excited Langmuir waves is first considered. The decay mode is the ion acoustic wave, which propagates in the same direction as that of the pump wave and has a wavelength about half of the pump wave's wavelength. The Langmuir sideband propagates in the direction opposite to that of the pump wave and has a wavelength about the same as that of the pump wave. This Langmuir sideband is called the first cascade line of the sideband of the PDI. It will continue to cascade through the same instability process if its saturation amplitude exceeds the instability threshold. The number of cascade of the Langmuir waves excited by the PDI is the focus of the following analysis. In the parametric decay of a Langmuir pump wave: $\phi(\omega, \mathbf{k})$ into a Langmuir sideband: $\phi_1(\omega_1, \mathbf{k}_1)$ and an ion acoustic mode: $n_{s1}(\omega_{s1}, \mathbf{k}_{s1})$, the frequency and wavevector matching conditions are $\omega = \omega_1 + \omega_{s1}^*$ and $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_{s1}$, where $\mathbf{k}_1 \cong -\mathbf{k}$ and thus, $\mathbf{k}_{s1} \cong 2\mathbf{k}$.
The dispersion relation of the instability is derived, from (1) and (2), to be

$$[\omega_1(\omega_1+i\nu_e)-\omega_{k\theta}^2][\omega_{s1}^*(\omega_{s1}^*-i\nu_i)-4k^2C_s^2] = k^2[k_z^2+k_\perp^2\omega^2/(\omega^2-\Omega_e^2)](\omega_p^4/\pi n_0M\omega^2)|\phi|^2 \quad (6)$$

A. Resonant Decay

In the resonant decay process, the Langmuir pump wave has to propagate down from its excited altitude z_0 to the decay altitude z_1. When the wave propagates in the region outside of its excitation region, its field amplitude decays exponentially due to collision and collisionless losses. Thus, the actual threshold field has to be increased by an exponential factor $e^{\alpha\Delta z}$, where α is the spatial damping rate of the Langmuir wave and $\Delta z = |z_0-z_1|$ is the distance between the two altitudes. The spatial damping rate can be derived from the dispersion relation $\omega(\omega + i\nu_e) - \omega_{k\theta}^2 = 0$, where ν_e is the total effective collision frequency including the Landau damping effect. Setting $k = k_r + i\alpha$ into the dispersion relation for a real ω, the damping rate $\alpha \cong \nu_e/2v_g$ is derived, where $v_g = 3k_rv_{te}^2/\omega$ is the group velocity of the Langmuir wave. The distance Δz can be determined in terms of the inhomogenty scale length L of the plasma density. Since the mother line and the daughter line in the HFPLs have the same wavevector at their respective originating altitudes z_0 and z_1, thus, $\omega^2 - \omega_1^2 = \omega_p^2(z_0) - \omega_p^2(z_1)$, which leads to $\Delta z = 4\omega\ k_rC_sL/\omega_p^2(z_0)$. Hence, $\alpha\Delta z = 2C_s\nu_e\omega^2L/3\omega_p^2v_{te}^2 \geq 1$ for L = 20 ~ 50 km, $T_i \cong T_e$, and $\nu_e \geq 500$ Hz.
Assuming that the daughter line is saturated at about the same level as that of the mother line in the excitation region of the daughter line, then the power ratio of the mother line and the daughter line in the HFPLs will be $e^{2\alpha\Delta z}$, which leads to about 10 dB power difference. The same analysis and results are applicable to the following cascades. In other words, the spectral intensities of the cascade lines in the HFPLs should decrease consecutively by a factor $e^{-2\alpha\Delta z}$ if the resonant cascading process generates them. However, the spectral power difference between two consecutive cascade lines in the HFPLs is, in general, much less than 10 dB as shown by both Tromso and Arecibo's experimental results. Moreover, the spectral distributions of HFPLs detected from different heights in Arecibo heating experiments were usually in similar patterns, suggested that the resonant cascade not be a dominant process. Therefore, the nonresonant decay process is studied in great length in the following.

B. Nonresonant Decay

Introducing $\omega_{s1} = \omega_{s1r} + i\gamma_1$ and $\omega_1 = \omega_{k\theta} - \omega_{s1r} + i\gamma_1$ in (6). At threshold field, $\gamma_1 = 0$, we obtain $\omega_{s1r}^2 = [\nu_e/(\nu_e+2\nu_i)]4k^2C_s^2$ and the threshold field of the instability

$$|\phi|_{th}^2 = [1 + 16k^2C_s^2/\nu_e(\nu_e+2\nu_i)]\ [\nu_e/(\nu_e+2\nu_i)]^{1/2}|\phi|_0^2 \ \cong 8C_s^2\cos^2\theta/[\nu_e(\nu_e+2\nu_i)^3]^{1/2}|E_{pth}(\theta)|^2 \quad (7)$$

Equation (7) determines the threshold field for exciting the first cascade line of the Langmuir sideband of the PDI through a nonresonant decay process. We now extend the analysis to determine the threshold field for the excitation of the Nth cascade line $\phi_N(\omega_N, -\mathbf{k})$. In this case the pump is the (N−1)th cascade line $\phi_{N-1}(\omega_{N-1}, \mathbf{k})$ and the decay mode is again an ion acoustic mode $n_{sN}(\omega_s, 2\mathbf{k})$. (6) is generalized to the dispersion relation for the Nth cascade

$$[i\omega_0(2\gamma_N+\nu_e)-2\omega_0 \sum_{q=1}^{N} \omega_{sqr}][-i\omega_{sNr}(2\gamma_N+\nu_i)+\omega_{sNr}^2-4k^2C_s^2-\gamma_N(\gamma_N+\nu_i)]$$
$$= k^2[k_z^2+k_\perp^2\omega_0^2/(\omega_0^2-\Omega_e^2)](\omega_p^4/\pi n_0M\omega_0^2)|\phi_{N-1}|^2 \quad (8)$$

where $\omega_{sN} \cong \omega_{sNr} + i\gamma_N$ and $\omega_N \cong \omega_{k\theta} - \sum_{q=1}^{N} \omega_{sqr} + i\gamma_N$ are introduced and $\omega_{k\theta} \cong \omega_0 \cong \omega_{N-1}$ are assumed.
Equation (8) is solved to determine the threshold field $|\phi_{N-1}|_{th}$ by setting $\gamma_N = 0$. The result is found to be

$$|\phi_{N-1}|_{th}^2 = [1 + 16N^2k^2C_s^2/\nu_e(\nu_e+2N\nu_i)]\,[\nu_e/(\nu_e+2N\nu_i)]^{1/2}|\phi|_0^2 \cong N^2[(\nu_e+2\nu_i)/(\nu_e+2N\nu_i)]^{3/2}|\phi|_{th}^2 \quad (9)$$

where $_{q=1}^{N}\,\omega_{sqr} \cong N\omega_{sNr}$ and $16N^2k^2C_s^2/\nu_e(\nu_e+2N\nu_i) >> 1$ are assumed.
Considering two extreme cases that $\nu_e >> 2N\nu_i$ and $\nu_e << 2N\nu_i$ for comparison, (9) is reduced in these two cases to

$$|\phi_{N-1}|_{th}^2 \cong \begin{cases} N^2\,|\phi|_{th}^2 & \text{for } \nu_e >> 2N\nu_i \\ N^{1/2}\,|\phi|_{th}^2 & \text{for } \nu_e << 2N\nu_i \end{cases} \quad (10)$$

Equation (10) suggests that the feature of the PDI cascade lines depend strongly on ν_e/ν_i, i.e., the latitude of the heating site, the time of the experiment, and the frequency of the heating wave.

DISCUSSION

The cascade of the PDI excited in the ionospheric heating experiments is studied. Both resonant and nonresonant cascading processes are analyzed. In the resonant decay case, the propagation loss of Langmuir waves imposes a large power ratio $e^{2\alpha\Delta z}$ (~ 10 dB) to the two consecutive cascade lines in the HFPLs. Therefore, from the experimental results, the cascade of the PDI is not likely to be through the resonant cascading process.
Through the nonresonant cascading processes, it is shown by (10) that the threshold power for exciting cascade lines increases with the square of the number N of the cascade line in the parameter range that $2N\nu_i < \nu_e$ and $k^2C_s^2/\nu_e^2 >> 1$. For example, the threshold power for the excitation of the third cascade line is about 3.5 dB higher than that for the excitation of the second cascade line. The experimental result shows that the saturation intensity of the second cascade lines can be larger than that of the first cascade line, but the difference rarely exceeds 3.5 dB. This may explain why the second cascade line hardly generates a third in Tromso's early heating experiments [3]. After the EISCAT's heating power increased from 240 MW ERP to 1.2 GW ERP, up to five cascade lines in Tromso's HFPLs were observed [6]. This result is not inconsistent with the theory. From (10), the ratio of spectral intensities $|\phi_4|_{th}^2/|\phi_1|_{th}^2 \cong 6.25$. On the other hand, the heating power is also increased 5 times.
In Arecibo, $\nu_i \sim \nu_e$ and thus, the increase of the threshold field with the number of the cascade step ($\propto \sqrt{N}$) is not as fast as that ($\propto N^2$) in Tromso's heating experiments. Consequently, more cascade lines in the HFPLs of Arecibo's heating experiments were observed.

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