CRITICAL HF HEATING POWER FOR MAJOR ENHANCEMENT OF ELF WAVE GENERATION BY MODULATED ELECTROJET

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ABSTRACT

The amplitude-modulated HF heating wave excites a stimulated thermal instability enhancing electrojet modulation. Inelastic collisions damp nonlinearly this instability, which is normally saturated at low levels. As the modulated HF heating power exceeds a critical level, it is shown that significant electron heating enhanced by the instability can cause a steep drop of electron's inelastic collision loss rate in the energy regime from 3.5 to 6 eV. Consequently, this instability saturates at a much higher level, resulting to a near step increase (of about 10 to 13 dB depending on the modulation waveform) in the spectral intensity of ELF radiation.

INTRODUCTION

In the polar region, electrojet current appears frequently in the lower ionosphere. This current is driven by a dc space charge field and can be perturbed through perturbations on the background electric conductivity. An amplitude-modulated powerful HF wave modulated at ELF/VLF frequency can be introduced to heat background plasma. The HF wave modulates the electron temperature, which results to the modulation of the electron conductivity in a similar fashion as the power modulation of the heating wave. Consequently, electrojet current become oscillating in time to act virtually as an antenna. The ac part of the current becomes the source current of ELF/VLF radiation [1-4]. The frequencies of radiation are easily controlled by the modulation frequencies of amplitude-modulated HF heating waves. However, the generation efficiency and signal quality are critical to practical applications. A beam-painting approach [5] that enlarges the modulated region of the electrojet to enhance the ionospheric antenna gain has been suggested. Kuo and Lee [6] and Kuo [7] showed that an active process by exciting stimulated thermal instability could provide more effective modulation on the electrojet. The increasing dependence of the elastic electron-neutral collision frequency ν_{en} on the electron temperature T_e, i.e., $\nu_{en} \propto T_e^{5/6}$, provides a positive feedback channel for the instability. The results of the numerical and analytical analyses [8,9] further indicate that the signal quality and the generation efficiency depend on the mode type (O or X) and frequency of the HF heating wave, and the modulation scheme and frequency, and that stimulated thermal instability is, in fact, the dominant process in electron temperature modulation before being stabilized by the nonlinear damping of inelastic collisions.

The inelastic collision frequency of electrons (e.g., vibration excitation of N_2 and O_2) has a strong dependence on the electron temperature. It starts with a rapidly increasing dependence in the low electron temperature regime. This increasing dependence on the electron temperature slows down as the electron temperature further increases. The data curve presented in Fig. 5, on page 57 of Gurevich's book [10] shows that the inelastic collision cross section of the electron, after reaching a peak at about 2.5 eV of the electron temperature (*i.e.*, electron energy at about 3.5 eV), decreases rapidly with a further increase of the electron temperature. It stays at low values for $2.5 < T_e < 4.5$ eV before the optic excitation and ionization processes become significant. It suggests that the saturation level of stimulated thermal instability be enhanced drastically by increasing electron heating to exceed a critical rate [11]. This will be verified by the numerical results presented in the following.

MODAL EQUATIONS

The electron thermal energy equation [10,12] is given by

$$\partial T_e/\partial t + (2T_e/3)\nabla\cdot\mathbf{v}_e + \delta(T_e)\nu_e(T_e)(T_e - T_n) + \text{ion ization loss}$$

$$= (2/3n_0)(Q + \nabla\cdot\,\mathbf{K}_e\cdot\nabla T_e) + \text{solar heat input} \qquad (1)$$

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where $\mathbf{v}_e$ is the electron fluid velocity, $\delta(T_e)$ is the average relative energy fraction lost in each collision, $\nu_e(T_e)$ is the effective collision frequency of electrons with neutral particles, T_n is the temperature of the background neutral particles; the ionization loss becomes significant as electrons are heated up to high temperature; Q is the total Ohmic heating power density in the background plasma and contributed by the electrojet current and the HF heater wave, $\mathbf{K}_e$ is the thermal conduction tensor, and m is the electron mass; the explicit expression of the ionization loss term on the left hand side (LHS) of (1) will be given later. Since only temporal modulation is considered, the two terms involving spatial derivatives on each side of (1) are set to zero.

Consider the electrojet modulation by X-mode heating wave modulated periodically by a rectangular wave with 50% duty cycle, the left-hand circularly polarized wave electric field is expressed as

$$\mathbf{E}_p = (\hat{x} - i\hat{y})(\varepsilon_p/2)\exp[i(k_0 z - \omega_0 t)] + \text{c.c.} \tag{2}$$

where $\varepsilon_p^2 = \varepsilon_{p0}^2 M(t)$ and $M(t) = [1 + 2\ {}_{k=1}\ \mathrm{sinc}\,(k\pi/2)\cos k(\omega_1 t + \pi/2)]$ is the power modulation function; $\omega_1/2\pi = f_1$ is the modulation frequency. Thus

$$\mathbf{v}_{pe} = -i(\hat{x} - i\hat{y})[e\varepsilon_p/2m(\omega_0 - \Omega_e + i\nu_{en})]\exp[i(k_0 z - \omega_0 t)] + \text{c.c.} \tag{3}$$

The total Ohmic heating power density is given by

$$Q = J_{et}^2/\sigma \cong \nu_{en} n_0 m[u_e^2 + \langle|v_{pe}|^2\ ] \tag{4}$$

where $\mathbf{J}_{et} = -en(\mathbf{u}_e + \mathbf{v}_{pe})$ is the total electron current density in the background plasma carrying an electrojet and interacting with the HF wave fields; angle brackets indicate the time average over the HF wave period, and $\sigma = n_0 e^2/m\nu_{en}$ is the conductivity of the plasma responsible for the Ohmic loss; $u_e = eE_0/m(\nu_{en}^2 + \Omega_e^2)^{1/2}$, $\langle|v_{pe}|^2 = v_q^2 M(t)$, and $v_q^2 = (e\varepsilon_{p0}/m)^2/[(\omega_0 - \Omega_e)^2 + \nu_{en}^2]$. Radiation field $\mathbf{E}(z, t)$ at the receiver location z (z =0 at ground) generated by heating induced modulation current distributed in a layer at z = h and thickness of Δh is given by

$$\mathbf{E}(z, t) = -(\hat{x} - i\hat{y})(5A_0 n_0 e^2 E_0/6mc)\exp[k_{i0}(z-h)][(\Omega_e - i\nu_{en})/(\nu_{en}^2 + \Omega_e^2)]^2(\nu_{en}/T_e)\partial_t T_e(t + k_{r0}z/\omega_{10}) \tag{5}$$

where ω_{10} and k_{r0} are the real frequency and wavenumber of radiation, and k_{i0} is its spatial damping rate in plasma; $A_0 = (i\pi\Delta h/c)\,[\exp(ik_{r0}h)/(k_{r0} + ik_{i0})]$. (1) indicates that the field intensity of radiation depends on the layer thickness Δh of the current source, plasma density n_0, current modulation rate, and attenuation in plasma.

NUMERICAL ANALYSIS AND RESULTS

The numerical analysis is carried out for electrojet modulation in the region near 100 Km altitudes. The heating wave frequency of $\omega_0/2\pi$ = 4 MHz is used and adopted E region parameters are $T_n \cong T_i \cong 300$ K, $T_{e0} \cong 1500$ K, $\nu_{en0} = 5\times10^4\ s^{-1}$, E_0 = 50 mV/m, $M_n/m = 5.52\times10^4$, $\Omega_e/2\pi$ = 1.35 MHz, and $v_{t0} = (T_{e0}/m)^{1/2} = 1.5\times10^5$ m/s. In the strong heating power regime as considered, the numerical results are insensitive to the initial value of the electron temperature.

In the numerical analysis, dimensionless variables and parameters are introduced: $\chi = T_e/T_{e0}$ as that defined early, $\nu_{en}/\nu_{en0} = \chi^{5/6}$, $\tau = \nu_{en0}t/100$, $Z = (\nu_{en0}k_{r0}/100\omega_0)z$, $H = (\nu_{en0}/100c)h$, $\xi = \tau + Z$, $K_{i0} = k_{i0}(100\omega_0/\nu_{en0}k_{r0})$, $\varepsilon(\xi) = |\mathbf{E}(z, t)/E_0|$, $\omega_{10} = 100\omega_1/\nu_{en0}$, T_n/T_{e0} =0.2, $\eta = (eE_0/m\Omega_e v_{t0})^2$, $b = (\nu_{en0}/\Omega_e)^2$, $\beta = (5/6)10^{-4}(\omega_{pe}^2\nu_{en0}^3 A_0/4\pi c^2\Omega_e^2)$, and $q = \alpha M(t)$, where $\alpha = (v_q/v_{t0})^2$, $v_q = 1.04\times10^4\varepsilon_{p0}$ m/s, ε_{p0} is in V/m, and c is the speed of light in vacuum. It is noted that Z = 0 at ground. Chosen $\varepsilon_{p0} = \varepsilon_0$ = 2.5 V/m as a reference field amplitude, leads to $v_{q0} = 2.6\times10^4$ m/s, $\alpha_0 = \alpha(v_q = v_{q0}) = 0.03$, and $\alpha = 0.03p$, where $p = (\varepsilon_{p0}/\varepsilon_0)^2$ is the heating wave power normalized to the reference power.

Thus (1) and (5) are normalized to the dimensionless forms

$$\begin{aligned} &d\chi/d\tau + 200(m/M_n)[\chi^{5/6} + 8.38\chi^{-1/2} + \mu_I(\chi)]\,(\chi - 0.2) \\ &+ \chi^{1/2}[0.86D_1(93 + \chi)e^{-94.3/\chi} + 3.14D_2(120 + \chi)e^{-121.7/\chi}] \\ &= (200/3)\chi^{5/6}[\eta/(1 + b\chi^{5/3}) + q] + 200[19.3(m/M_n) - \eta/3] \end{aligned} \tag{6}$$

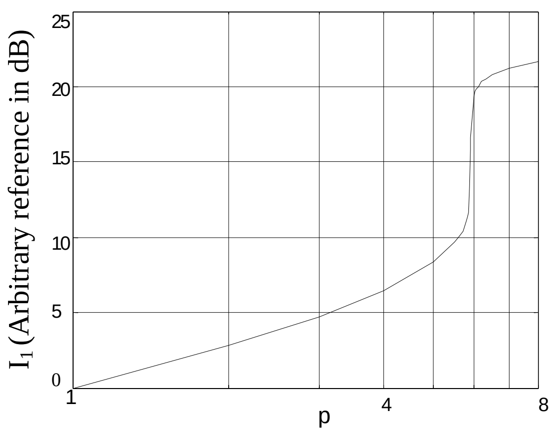

FIG. 1. The dependence of the spectral intensity I_1 of the fundamental line of the radiation on the normalized heating power p, which shows a (near) step increase in the spectral intensity as the heating power exceeds a threshold $p_c = 6$; the modulation frequency $f_1 = 100$ Hz.

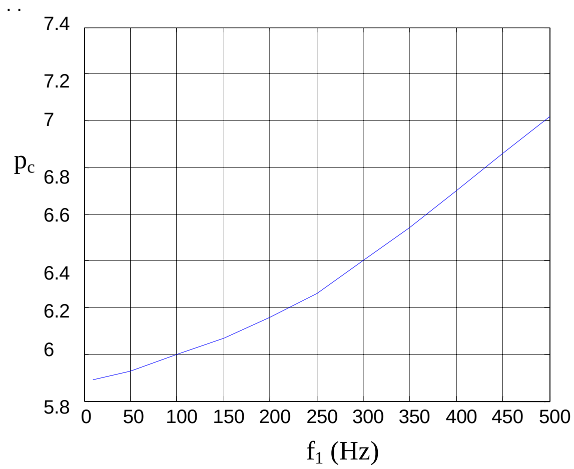

FIG. 2. The dependence of the threshold power p_c on the modulation frequency f_1.

and

$$\boldsymbol{\varepsilon}(\xi) = \sqrt{2}\,\beta \exp[K_{i0}(Z - H)]\,\chi^{-1/6}(\xi)(d/d\tau)\{\chi(\xi)/[1 + b\chi^{5/3}(\xi)]\} \quad \text{for } Z \geq Z_0$$
$$= \sqrt{2}\,\beta \exp[K_{i0}(Z_0 - H)]\,\chi^{-1/6}(\xi)(d/d\tau)\{\chi(\xi)/[1 + b\chi^{5/3}(\xi)]\} \quad \text{for } Z \leq Z_0 \qquad (7)$$

where the spatially dependent terms (i.e., the divergent terms) in (1) have been neglected; the third term on the LH side of (6) is the normalized ionization loss term and the two terms in the bracket are contributed from the ionization of O_2 and N_2, respectively; Z_0 is the normalized lowest height of plasma.

We now study the dependence of the radiation intensity on the heating power. A modulation frequency $f_1 = 100$ Hz is first considered. Thus $q = 0.03pM(\tau)$, $\omega_{10} = 0.4\pi$, and p is the only variable parameter left in (6). For a given p, (6) is solved numerically subject to the initial condition $\chi(0) = 1$. The result is then substituted into (7) to obtain the time dependent radiation field $\boldsymbol{\varepsilon}(\tau; p)$ at Z = 0 with p as a variable parameter. Numerical results show that when the heating power exceeds a threshold level, electron temperature and radiation intensity increase abruptly. The stimulated thermal instability becomes the dominant electrojet heating and modulation process. Its saturation level becomes weakly dependent on the heating power. The dependence of the spectral intensity I_1 of the fundamental line of radiation, for $f_1 = 100$ Hz case, on the normalized heating power p, varied from 1 to 8, is presented in Fig. 1. It is shown that this function has a narrow transition region of width Δp. As p passes this transition region to exceed a threshold value $p_c = 6$, the spectral intensity of the signal is suddenly increased by near 10 dB. The threshold value p_c varies with the modulation frequency f_1. Similar calculations have been carried out for different modulation frequencies in the range from 10 Hz to 500 Hz. This dependence, showing monotonic increase of the threshold value p_c with the modulation frequency f_1, is presented in Fig. 2.

DISCUSSION AND CONCLUSION

Amplitude-modulated HF heating wave in the electrojet excites stimulated thermal instability because the electron-heating rate increases with electron temperature [as shown by the first term on the RHS of (6)]. Instability leads the transient temperature response of electrojet plasma to grow exponentially at the expense of the free energy of the heating wave [the second term in the first bracket on the RHS of (6)] as well as the background electrojet current [the first term in the first bracket on the RHS of (6)]. Inelastic collisions of electrons with neutral particles [mainly due to vibration excitation of N_2] introduce nonlinear damping to stabilize the instability. The nonlinear damping rate μ_I on the LHS of (6) turns out to become a rapidly decreasing function of χ after reaching a peak at $\chi \cong 10$. Therefore, as the HF heating power exceeds a threshold level, significant electron heating causes a steep drop in the electron inelastic collision rate and stimulated thermal instability saturates at a much higher level, resulting to a (near) step enhancement (about 8 dB for f_1 = 100 Hz) in the generation efficiency of ELF radiation. The threshold power is rather high. For example, the threshold field, for using 4 MHz X-mode heating wave modulating at 100 Hz, is about 6.1 V/m. But this threshold field is modulation-scheme dependent. It reduces to about 4.4 V/m by using a half-wave rectified-wave modulation scheme, having $M(\tau) = 1 - (16/\pi)\sum_{k=1} [(\sin k\pi/2)/k(k^2 - 4)] \cos k(\omega_{10}\tau - \pi/2)$. This modulation scheme also advantageously increases the (near) step jump in the radiation intensity to 11 dB. Without including the absorption of the HF power by the D region ionosphere, this threshold power evaluated for the E region parameters (~ 100 km altitude) requires the effective radiated power ERP ≥ 90 dBw. D region absorption during daytime or under disturbed conditions could severely reduce the HF power transmitted to altitudes near 100 km. This further increases the ERP requirement for taking advantage of thermal instability to enhance electrojet modulation. Nevertheless, this extraordinary physical phenomenon could well be explored in future heating experiments by the European Incoherent Scatter (EISCAT) facility's superheater [13], in Tromso, Norway, under favorable ionospheric conditions, or by the High Frequency Active Auroral Research Program (HAARP) heating facility [14], in Gakona, Alaska, when its effective radiated power reaches its planned level.

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