CASCADE OF LANGMUIR WAVES VIA PARAMETRIC COUPLING TO LOWER HYBRID WAVES

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ABSTRACT

Cascade of obliquely propagating Langmuir waves, through parametric decay to obliquely propagating Langmuir sidebands and lower hybrid decay modes, is analyzed. Wave vectors of parametric coupling have to be matched in three-dimensional space, rather than on a plane for the conventional simplification. It generates a broad downshifted spectrum of obliquely propagating Langmuir waves, including those 40^0 plasma lines (HFPLs) detected by UHF radar in Arecibo heating experiments. It is also shown that HFPLs prefer to be excited at the matching height of the mother Langmuir wave excited by OTSI or PDI, rather than near the HF reflection height.

INTRODUCTION

In Arecibo heating experiments, parametric decay instability (PDI) and oscillating two-stream instability (OTSI) excite Langmuir waves as instability sidebands in sizable region below the reflection height of the O-mode HF heating wave [1,2], where the heating wave converts to nearly linear polarization in the geomagnetic field direction. Theory [3] also shows that those Langmuir sidebands (with a 40^0 oblique propagation angle) contributing to HFPLs favor to be excited close to their matching height region, where the 40^0 Langmuir sidebands can closely satisfy the local dispersion relation. With improved spatial resolution of radar detection, HFPLs were found to originate from a narrow altitude region located slightly (~300–600 m) above the height of below-threshold plasma lines [4] (which was defined the matching height of the plasma lines and was about 1.9 km below the HF reflection height for an inhomogeneous scale length of 50 km).

Likewise, in the high latitude heating experiments such as at Tromso, O-mode HF heating wave also excites PDI and OTSI in the region below its reflection height; however, both Langmuir waves and upper hybrid waves, excited in slightly different height regions [5-7], are the high frequency sidebands of the instabilities. Experimental results also showed that HFPLs were detected from the matching height [8] of the PDI line in HFPLs, rather than from the HF reflection height.

Usually, frequency downshifted HFPLs have narrow spectra, attributed to cascade of Langmuir waves through ion acoustic waves [9,10]. However, HFPLs, showing broad cascade spectra with bandwidths as large as 50 kHz, were also observed in Arecibo heating experiments [4]. Both discrete cascade and continuum features appeared in the spectra. These HFPLs were characteristically different from those broad HFPLs observed in Tromso heating experiments, which were frequency up-shifted and much wider, and originated from regions near and above the HF reflection height.

Although Langmuir waves excited by OTSI and PDI in the region below the HF reflection height can become pump waves to excite secondary parametric instabilities, which generate frequency-downshifted Langmuir waves to be their sidebands, successive cascade of Langmuir waves through secondary parametric instabilities is required to produce a broad downshifted frequency spectrum. However, the permissible number of cascade and required pump threshold field depend on the secondary parametric instability process. In this work, a secondary parametric instability process as a cascade channel to broaden the downshifted frequency spectrum of Langmuir waves is studied. It is a parametric decay of an obliquely propagating Langmuir pump to an obliquely propagating Langmuir sideband and a lower hybrid decay wave, in which the lower hybrid wave is generalized from the lower hybrid resonance wave, which was considered in the previous work [11], by including a finite parallel component in its wave vector. Thus the frequency spectrum of lower hybrid decay waves is broad and continuum, and the cascade spectrum of Langmuir waves can carry both discrete and continuum features.

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FORMULATION AND ANALYSIS

Consider the decay of a Langmuir pump $\phi_1(\omega_1, \mathbf{k}_1)$ to a Langmuir sideband $\phi_2(\omega_2, \mathbf{k}_2)$ and a nearly field-aligned lower hybrid resonance mode $n_3(\omega_3, \mathbf{k}_3)$ in each cascade step. The first (mother) Langmuir pump $\phi_{10}(\omega_0, \mathbf{k}_{10})$ is generated either by OTSI or by PDI in a layer region located at a height $h = h_{10}$, where $\omega_p^2(h_{10}) = \omega_0(\omega_0 + \nu_e) - 3k_{10}^2 v_{te}^2 - \Omega_e^2 \sin^2\theta_0$ (i.e., h_{10} is the matching height of a Langmuir wave having propagation angle θ_0 and $k_0 = k_{10z} = k_{10}\cos\theta_0$ as its parallel wave number). The frequency and wave vector matching conditions are $\omega_1 = \omega_2 + \omega_3^*$ and $\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$. The coupled mode equation for the Langmuir sideband is given by [12]

$$\{[(\partial_t+\nu_e)^2 + \Omega_e^2](\partial_t^2 + \nu_e\partial_t + \omega_p^2 - 3v_{te}^2\nabla^2)\nabla^2 - \Omega_e^2(\omega_p^2 - 3v_{te}^2\nabla^2)\nabla_\perp^2\}\phi_2$$

$$= -\omega_p^2\{[(\partial_t+\nu_e)^2\nabla + \Omega_e^2\nabla_z]\cdot\langle\nabla\phi_1(n_3/n_0)\rangle - \Omega_e(\partial_t+\nu_e)\,\hat{z}\cdot\nabla(n_3^*/n_0)\times\nabla\phi_1\} \qquad (1)$$

The coupled mode equation for the lower hybrid decay mode $n_3(\omega_3, \mathbf{k}_3)$ is derived to be [13]

$$\partial_t\{\nabla_\perp^2(\partial_t+\nu_e)[\partial_t(\partial_t+\nu_i) - C_s^2\nabla^2] + \Omega_e\Omega_i(\nabla_\perp^2 + M\nabla_z^2/m)\partial_t\}(n_3/n_0)$$

$$= (m/M)\nabla^2[(\partial_t+\nu_e)\partial_t\nabla_\perp\cdot\mathbf{a}_p + \Omega_e^2\partial_z a_{pz} - \Omega_e^2\partial_t\nabla\cdot\mathbf{J}_B/n_0 - \Omega_e\partial_t\nabla\cdot\mathbf{a}_p\times\hat{z}] \qquad (2)$$

where $\mathbf{a}_p = \mathbf{v}_e\cdot\nabla\mathbf{v}_e$ and $\mathbf{J}_B = \langle n_e\mathbf{v}_e$. Equations (1) and (2) are the set of coupled mode equations for the cascading process considered in the present work. In the following (1) and (2) will be analyzed in the spectral domain by assuming a general expression $p = p\exp[i(\boldsymbol{\kappa}\cdot\mathbf{r} - \varpi t)]$ as the spatial and temporal variation of the perturbation quantities, where $\boldsymbol{\kappa}$ and ϖ are the appropriate wave vector and frequency of each perturbation. The wave vector matching condition can be satisfied in a two-dimensional arrangement, which is usually a favorable choice for simplifying the analysis, or a general three-dimensional arrangement.

A. Two-dimensional Coupling

We first consider a two-dimensional arrangement having $\mathbf{k}_1$, $\mathbf{k}_2$, and the z-axis on the same (e.g., x-z) plane, i.e., $\mathbf{k}_1 = \hat{z}\,k_0 + \hat{x}\,k_\perp$, $\mathbf{k}_2 = \hat{z}\,(k_0 - k_z) - \hat{x}\,k_\perp$, and thus $\mathbf{k}_3 = \hat{x}\,2k_\perp + \hat{z}\,k_z$, where $k_\perp/k_0 = \tan\theta$ and $|k_z/k_\perp| << 1$. In the spectral domain, (1) and (2) can be combined to obtain the dispersion relation of the instability for the Nth cascade as

$$[i\omega_0(2\gamma_N + \nu_e) - \Delta\omega_N^2]\,[-i\omega_{30}(2\gamma_N + \nu_3) - \Delta\omega_3^2] =$$

$$4(e^2/mM)[\omega_p^2\cos^2\theta/(\omega_0^2 - \Omega_e^2)][1 - \omega_0^2\tan^2\theta/(\omega_0^2 - \Omega_e^2)]\times$$

$$[k_\perp^2(k_0^2 - k_\perp^2) + (k_z k_0\Omega_e^2/\omega_{30}\omega_0)(k_\perp^2 - k_0^2/2) - k_z^2 k_0^2\Omega_e^2/4\omega_{30}^2]\,|\phi_N|^2 \qquad (3)$$

where $\omega_3 = \omega_{30} + i\gamma_N$ is assumed and γ_N is the growth rate of the instability; $\Delta\omega_N^2 = \omega_0^2 - \omega_N^2 + \Omega_e^2(\sin^2\theta - \sin^2\theta_0) + 3k_0^2 v_{te}^2(\tan^2\theta - \tan^2\theta_0)$, and $\Delta\omega_3^2 = 4k_\perp^2 C_s^2 + \Omega_e\Omega_i\xi + \nu_e\nu_i - \omega_{30}^2$; $\nu_3 = \nu_i + \nu_e(1 - 4k_\perp^2 C_s^2/\omega_{30}^2)$; $|\omega_1| \approx \omega_0$, $\nu_e/\omega_{30} << \Omega_e/\omega_0$ and $k_\perp^2\Omega_e^2/\omega_0^2 k_1^2 << 1$ are assumed.

It turns out that there is no instability (no cascade) for large oblique propagation angles. This is because the coupling coefficient on the right hand side (RHS) of (3) becomes very small and turns to a negative value.

B. Three-dimensional Coupling

Therefore, the matching of wave vectors in the coupling is extended to a three-dimensional arrangement with $\mathbf{k}_1 = \hat{z}\,k_0 + \hat{x}\,k_\perp$, $\mathbf{k}_2 = \hat{z}\,(k_0 - k_z) + \hat{y}\,k_\perp$, and thus $\mathbf{k}_3 = (\hat{x} - \hat{y})\,k_\perp + \hat{z}\,k_z$. Again, $k_\perp/k_0 = \tan\theta$, θ is the oblique propagation angle of the Langmuir pump, and $|k_z/k_\perp| << 1$. Then instability can be excited to lead to continuous cascade of the Langmuir sidebands of OTSI and PDI as being shown in the following. The cascading process is first described as follows. When the $\mathbf{k}_2$-line grows to a level exceeding the threshold of the subsequent cascading, the wave vectors of the sideband and decay mode will be $\mathbf{k}_1' = \hat{z}\,k_0 - \hat{x}\,k_\perp$ and $\mathbf{k}_3' = (\hat{x} + \hat{y})\,k_\perp - \hat{z}\,k_z$, which are separated from the $\mathbf{k}_1$ and $\mathbf{k}_3$-lines. This decay process continues when the $\mathbf{k}_1'$-line becomes strong enough to be a pump, which decays to a sideband with $\mathbf{k}_2' = \hat{z}\,(k_0 + k_z') - \hat{y}\,k_\perp$ and a decay mode with $\mathbf{k}_3'' = -(\hat{x} - \hat{y})\,k_\perp - \hat{z}\,k_z'$; again, $|k_z'/k_\perp| << 1$.

Since the third and fourth coupling terms on the RHS of (2) cancel each other, it leaves the first two terms on the RHS of (2), attributed to the transverse convective force and parallel ponderomotive force, to be the dominant coupling terms. (1) and (2) are combined to obtain the dispersion relation of the instability for the Nth cascade as

$$[i\omega_0(2\gamma_N + \nu_e) - \Delta\omega_N^2]\,[-i\omega_{30}(2\gamma_N + \nu_3) - \Delta\omega_3^2] =$$

$$2\xi^{-1}(e^2/mM)(\omega_p^2/\omega_0^2)\cos^2\theta(1+\xi\Omega_e^2/\omega_0^2+i\xi\Omega_e/\omega_0)[1+i\Omega_e\omega_0\tan^2\theta/(\omega_0^2-\Omega_e^2)]k_0^2k_\perp^2|\phi_N|^2 \qquad (4)$$

where again, $\omega_3 = \omega_{30} + i\gamma_N$ is assumed and γ_N is the growth rate of the instability; $\Delta\omega_N^2 = 2\omega_0(N\omega_{30} + \nu_e/2) + \Omega_e^2(\sin^2\theta - \sin^2\theta_0) + 3k_0^2v_{te}^2(\tan^2\theta - \tan^2\theta_0)$, and $\Delta\omega_3^2 = 2k_\perp^2C_s^2 + \Omega_e\Omega_i\xi + \nu_e\nu_i - \omega_{30}^2$; $\nu_3 = \nu_i + \nu_e(1 - 2k_\perp^2C_s^2/\omega_{30}^2)$; $\xi = 1 + Mk_z^2/2mk_\perp^2$. The threshold field is determined, by setting $\gamma_N = 0$ in (4), to be

$$|\phi_N|_{th} = (m/e)\xi[\Delta\omega_N^2\nu_e\Omega_e\omega_0/2\omega_{30}(1+\xi\Omega_e^2\sin^2\theta/\omega_0^2)k_0^2k_\perp^2]^{1/2} \qquad (5)$$

Next, the growth rate is found to be

$$2\gamma_N = [\nu_e^2 + (\nu_e^2 + \Delta\omega_N^2\nu_e\Omega_i/\omega_{30}^2)(\,|\phi_N|^2/|\phi_N|_{th}^2 - 1)]^{1/2} - \nu_e \qquad (6)$$

where the slight frequency shift of the decay mode owing to the pump field is neglected. Since the threshold field determined by (5) increases with $\Delta\omega_N$ and $\Delta\omega_N^2$ increases with the number of the cascade process, cascade of Langmuir waves eventually stops when the threshold field becomes too high.

In Arecibo heating experiments, $\omega_0/2\pi = 5.1$ MHz, $\Omega_e/2\pi = 1.06$ MHz, $\nu_e = 500\ s^{-1}$, $v_{te} \sim 1.3\times10^5$ m/s, and $C_s \sim 1.4\times10^3$ m/s; magnetic dip angle is about 50^0 and HFPLs are detected by 430-MHz backscatter radar, thus k_0 and θ of the lines in the HFPLs are 4.39π (i.e., twice the parallel wave number of the 430-MHz radar signal) and 40^0, respectively. Substitute these values into (5), results to

$$|E_N|_{th} \cong 1.51\xi[N + 3.87\xi^{-1/2}(\tan^2\theta - \tan^2\theta_0) + 17.86\xi^{-1/2}(\sin^2\theta - \sin^2\theta_0)]^{1/2}/\sin2\theta\ \ \text{V/m} \qquad (7)$$

Therefore, the threshold field for the Nth cascade of (k_1, 40^0) lines occurring near the HF wave reflection height in the layer at $h_{10} = h_0$, where $k_1 = 5.73\pi\ m^{-1}$, twice the wave number of the radar signal, and $\omega_p^2(h_0) = \omega_0(\omega_0 + \nu_e) - 3k_0^2v_{te}^2$, i.e., $\theta_0 = 0$, is given by

$$|E_N(h_0)|_{th} = 1.53\xi(N + 10.1\xi^{-1/2})^{1/2}\ \ \text{V/m} \qquad (8)$$

and occurring in the matching height layer of the mother Langmuir pump at $h_{10} = h_1$, where $\omega_p^2(h_1) = \omega_0(\omega_0 + \nu_e) - 3k_1^2v_{te}^2 - \Omega_e^2\sin^2 40^0$, i.e., $\theta_0 = 40^0$, becomes

$$|E_N(h_1)|_{th} = 1.53\xi N^{1/2}\ \ \text{V/m.} \qquad (9)$$

The threshold field (9) is smaller than that of (8). Moreover, the OTSI and PDI threshold fields [3] for exciting (k_1, 40^0) sidebands in the layer at $h_{10} = h_0$ are much larger than the corresponding ones at $h_{10} = h_1$. Thus the cascade process favors to occur in the layer at $h_{10} = h_1$, where the threshold field requirement of the process is governed by (9). It is noted that the lower hybrid frequency is proportional to $\xi^{1/2}$, thus a larger ξ will need fewer number of cascades to obtain the same bandwidth, i.e., $N \propto \xi^{-1/2}$. Include this fact the threshold field in (9) is still proportional to $\xi^{3/4}$, thus the decay instability prefers to excite the field-aligned lower hybrid resonance mode having $\xi = 1$.

We now examine the threshold conditions for producing a broad frequency downshifted HFPLs of 50 kHz bandwidth. Since the lower hybrid resonance frequency $f_{30} \cong 6.2$ kHz, thus N = 8 is needed in $\xi = 1$ case. The threshold field evaluated from (9) for N = 8 is 4.33 V/m. This field amplitude is about twice of that of the HF pump wave [14], thus the required HF field amplitude for 40^0 Langmuir sidebands excited by OTSI or PDI in their matching height layer able to cascade 8 times is about 2.17 V/m. The result of 8 cascades produces a broadband of HFPLs having a broad spectral bandwidth of about 50 kHz, which is downshifted continuously from the HF wave frequency.

In Tromso heating experiments, the parameters are: $\omega_0/2\pi = 4$ MHz, $\Omega_e/2\pi = 1.35$ MHz, $\nu_e = 1$ kHz, $v_{te} \sim 1.8\times10^5$ m/s, $C_s \sim 1.66\times10^3$ m/s, and $k_{01} = 12.17\pi$ (i.e., $\lambda_{\|1} = 0.1644$ m, corresponding to 933-MHz radar) or $k_{02} = 2.92\pi$ (i.e., $\lambda_{\|2} =$

0.685 m, corresponding to 224-MHz radar); the propagation angle of HFPLs is $\theta = 12^0$. In the case detected by 933-MHz radar, the threshold field of the instability excited in the matching height layer of the mother Langmuir pump is found to be $|E_N(h_1)|_{th} = 2.16\xi N^{1/2}$ V/m. In the same situation but detected by 224-MHz radar, the threshold field is increased to $|E_N(h_1)|_{th} = 9\xi N^{1/2}$ V/m. In both cases, the threshold fields are higher than that in Arecibo.

SUMMARY

A parametric instability decays a Langmuir pump wave to a lower hybrid wave and a Langmuir sideband is studied. This instability process cascades Langmuir sidebands of OTSI and PDI, excited by O-mode HF heating waves in heating experiments, to produce a broad spectrum of frequency downshifted Langmuir waves. It is found in the analysis that the cascade process can proceed continuously only if the wave vectors of three parametrically coupled waves in each cascade are matched three-dimensionally. If $\mathbf{k}_1$, $\mathbf{k}_2$, and the z-axis are arranged on the same plane as the conventional way to simplify the analysis, the product of the coupling terms of the two coupled mode equations will have a wrong sign and there will be no instability. In other words, the cascade events have to proceed in three-dimensional space. Moreover, the threshold fields of OTSI and PDI generating vertically propagating Langmuir waves in their matching height region is much lower than the corresponding ones in the HF reflection height region [3], the mother line of the cascade spectrum in HFPLs can be generated directly by the O-mode heating wave via OTSI or PDI excited in the matching height region of this mother line.

The results of analysis for Arecibo heating experiments show that the cascade of 40^0 Langmuir sidebands of OTSI or PDI can directly lead to a broad downshifted spectrum of HFPLs originating from the same altitude, which is the matching height of the 40^0 Langmuir waves rather than the reflection height of the O-mode HF heating wave. The required heating wave field for achieving cascade 8 times is about 2.17 V/m.

Lower hybrid waves, in general, have a broad spectrum with a spectral peak at lower hybrid resonance frequency. Thus the generated cascade spectrum is expected to contain both discrete and continuum features. For a given bandwidth of the cascade spectrum, the required number of cascades decreases as the frequency of the lower hybrid decay mode in the cascade process increases. However, the threshold field of the Langmuir pump wave in each cascade step increases with the increase of the frequency of the lower hybrid decay mode. It turns out that the threshold field of the HF heating wave for the overall process is the minimum when the decay mode is a field-aligned lower hybrid resonance mode. Thus the discrete feature is expected to prevail in the spectrum of HFPLs.

REFERENCES

[1] F. W. Perkins, C. Oberman, and E. J. Valeo, "Parametric instabilities and ionospheric modification," *J. Geophys. Res.*, vol. 79, p. 1478, 1974.

[2] J. A. Fejer, "Ionospheric modification and parametric instabilities," *Rev. Geophys.*, vol. 17, p. 135, 1979.

[3] S. P. Kuo, "Oscillating two stream instability in ionospheric heating experiments", *Phys. Plasmas*, 9, April 2002.

[4] J. A. Fejer, M. Sulzer, and F. T. Djuth, "Height dependence of the observed spectrum of radar backscatter from HF-induced ionospheric Langmuir turbulence," *J. Geophys. Res.*, vol. 96, p. 15985, 1991.

[5] P. Stubbe, H. Kopka, M. T. Rietveld, A. Frey, P. Hoeg, H. Kohl, E. Nielsen, and G. Rose, "Ionospheric modification experiments with the Tromso heating facility," *J. Atoms. Terr. Phys.*, vol. 47, p. 1151, 1985.

[6] P. Stubbe, H. Kohl, and M. T. Rietveld, "Langmuir turbulence and ionospheric modification," *J. Geophys. Res.*, vol. 97, p. 6285, 1992.

[7] P. Stubbe, "Review of ionospheric modification experiments at Tromso," *J. Atoms. Terr. Phys.*, vol.58, p.349, 1996.

[8] Rietveld, M. T., B. Isham, H. Kohl, C. LaHoz, and T. Hagfors, "Measurements of HF-enhanced plasma and ion lines at EISCAT with high-altitude resolution," *J. Geophys. Res.*, vol. 105, p. 7429, 2000.

[9] S. P. Kuo, "Cascade of the parametric decay instability in ionospheric heating experiments," *J. Geophys. Res.*, vol. 106, p. 5593, 2001.

[10] S. P. Kuo, "Cascade of parametric instabilities in ionosphere heating experiments," *J. Plasma Phys.*, vol. 66, p. 315, 2001.

[11] S. P. Kuo and M. C. Lee, "On the generation of a broad downshifted spectrum of HF wave enhanced plasma lines in the ionospheric heating experiments," *Geophys. Res. Lett.*, vol. 26, p. 3289, 1999.

[12] S. P. Kuo, B. R. Cheo, and M. C. Lee, "The role of parametric decay instabilities in generating ionospheric irregularities," *J. Geophys. Res.*, vol. 88, p. 417, 1983.

[13] S. P. Kuo, "The role of nonlinear beating currents on parametric instabilities in magnetoplasmas," *Phys. Plasmas*, vol. 3, p. 3957, 1996.

[14] W. L. Kruer, *Advances in plasma physics*, vol. 6. ed. A. Simon and W. B. Thompson, John Wiley & Son: New York, pp. 237-269, 1976.