PARAMETRIC INSTABILITIES IN IONOSPHERIC HEATING EXPERIMENTS

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ABSTRACT

The instability processes excited directly by HF heating waves include parametric decay instabilities and oscillating two stream instabilities. These instabilities provide effective channels to convert heating waves to electrostatic plasma waves in the F region of the ionosphere. The high frequency sidebands include Langmuir waves and upper hybrid waves. Ion acoustic waves, lower hybrid waves, and purely growing field-aligned density irregularities are among the low frequency decay modes. The instability thresholds, growth rates, angular distribution, and regions of excitation are determined.

INTRODUCTION

Parametric instabilities provide the most effective channels to convert HF heating waves into electrostatic plasma waves of high and low frequencies in ionosphere heating experiments. In the midlatitude region parametric instabilities were excited in the region slightly below the reflection height of the O-mode HF heating wave [1], where the heating wave became linearly polarized in the geomagnetic field direction. The sidebands excited by both parametric decay instability (PDI) and oscillating two-stream instability (OTSI) were Langmuir waves [2,3]. In the high latitude heating experiments, PDI and OTSI by the O-mode HF heating wave had to compete with thermal parametric instabilities excited in the upper hybrid resonance region located at a lower height, where the O-mode heating wave was still dominated by the field component of right-hand circular polarization. Sidebands of instabilities were upper hybrid waves propagating nearly perpendicular to the geomagnetic field [4,5]. Upper hybrid waves were found to play a key role in the generation of “stimulated electromagnetic emissions (SEEs)”, observed in Tromso heating experiments [6,7]. Due to the field-aligned nature of upper hybrid waves, these waves could not be detected directly by EISCAT’s UHF and VHF radar [8,9] and did not contribute to Tromso’s HFPLs.

In the present work, PDI and OTSI are analyzed systematically to explore underlying mechanisms responsible for the relevant phenomena observed in heating experiments.

COUPLED MODE EQUATIONS FOR PARAMETRIC INSTABILITIES

A large amplitude high frequency wave $\mathbf{E}_p(\omega_0, \mathbf{k}_p)$ in plasma can act as a pump wave to excite plasma modes through parametric couplings. For example, this pump wave electric field can drive a nonlinear current in the electron density perturbation $n_s(\omega_s, \mathbf{k}_s)$ of a low frequency plasma mode to produce beat waves $\mathbf{E}_1(\omega_1, \mathbf{k}_1)$ and $\mathbf{E}_1'(\omega_1', \mathbf{k}_1')$, where the frequency and wavevector matching conditions: $\omega_0 = \omega_1 + \omega_s^* = \omega_1' - \omega_s$ and $\mathbf{k}_p = \mathbf{k}_1 + \mathbf{k}_s = \mathbf{k}_1' - \mathbf{k}_s$ are imposed. The coupling is strong when the beat wave is a plasma mode. Beat waves, in turn, also couple with the pump wave to introduce a low frequency nonlinear force on electrons, which produces plasma density perturbation having the same frequency and wavevector as $n_s(\omega_s, \mathbf{k}_s)$. Hence, this coupling produces a feedback to the original density perturbation $n_s(\omega_s, \mathbf{k}_s)$. If the feedback is positive and large enough to overcome linear losses of coupled waves, the coupling becomes unstable and coupled waves grow exponentially in the expense of pump wave energy. This is called “parametric instability”, by which the pump wave $\mathbf{E}_p(\omega_0, \mathbf{k}_p)$ decays to two sidebands $\mathbf{E}_1(\omega_1, \mathbf{k}_1)$ and $\mathbf{E}_1'(\omega_1', \mathbf{k}_1')$ through a low frequency decay mode $n_s(\omega_s, \mathbf{k}_s)$. The parametric coupling is imposed by the frequency and wavevector matching conditions as well as a threshold condition on the pump wave field intensity. This process can be reduced to a three-wave coupling process when the decay mode $n_s(\omega_s, \mathbf{k}_s)$ has a finite oscillating frequency. In this situation, two sidebands cannot satisfy the same dispersion relation simultaneously. Thus the frequency-upshifted sideband $\mathbf{E}_1'(\omega_1', \mathbf{k}_1')$ is off resonant with plasma and can be neglected in the coupling.

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Parametric excitation of Langmuir/upper hybrid waves $\phi(\omega, \mathbf{k})$ and low-frequency (including ion acoustic, purely growing, and lower hybrid) waves $n_s(\omega_s, \mathbf{k}_s)$ by electromagnetic pump waves $\mathbf{E}_p(\omega_0, \mathbf{k}_p)$ is considered. Langmuir waves can have large oblique propagation angles (with respect to the magnetic field $\mathbf{B}_0 = \hat{z} B_0$) and upper hybrid waves propagate near perpendicularly. The coupled mode equation for the Langmuir/upper hybrid sideband is derived from the electron continuity and momentum equations, and Poisson's equation to be [10]

$$\{[(\partial_t + \nu_e)^2 + \Omega_e^2](\partial_t^2 + \nu_e\partial_t + \omega_p^2 - 3v_{te}^2\nabla^2)\nabla^2 - \Omega_e^2(\omega_p^2 - 3v_{te}^2\nabla^2)\nabla_\perp^2\}\phi$$

$$= \omega_p^2\{[(\partial_t + \nu_e)^2\boldsymbol{\nabla} + \Omega_e^2\boldsymbol{\nabla}_z] \cdot \ \mathbf{E}_p n_s^*/n_0 \ - \Omega_e(\partial_t + \nu_e) \cdot \langle\boldsymbol{\nabla}(n_s^*/n_0) \times \mathbf{E}_P \ \} \tag{1}$$

The formulation of the coupled mode equation for low frequency waves needs to include both electron and ion fluid equations. Since electrons and ions tend to move together, the formulation can be simplified by introducing the quasineutral condition: $n_{si} \cong n_{se} = n_s$. Also included in the formulation is the electron thermal energy equation [11]

$$\partial_t T_e + (2T_{e0}/3)\boldsymbol{\nabla} \cdot \mathbf{v}_e = (2/3n_e)\boldsymbol{\nabla} \cdot (\kappa_z\nabla_z + \kappa_\perp\nabla_\perp)T_e - 2\nu_e(m/M)(T_e - T_{e0}) + 2\nu_e m\langle v_e^2 \ /3 \tag{2}$$

where $\kappa_z = 3n_0T_{e0}/2m\nu_e$, $\kappa_\perp = (\nu_e/\Omega_e)^2\kappa_z$, and T_{e0} is the unperturbed electron temperature. Using the same procedure as that outlined in [12], the coupled mode equation for the low frequency mode in the collisional case is derived to be

$$\{\partial_t^3\{(\partial_t + \nu_e)[\partial_t(\partial_t + \nu_i) - C_s^2\nabla^2] + \Omega_e\Omega_i\partial_t\}\nabla_\perp^2 + \Omega_e^2\{(\partial_t^2 + \Omega_i^2)[\partial_t(\partial_t + \nu_i) - C_s^2\nabla^2] + \Omega_i^2C_s^2\nabla_\perp^2\}\nabla_z^2 \quad (n_s/n_0)$$

$$= (m/M)[(\partial_t^2 + \Omega_i^2)\nabla_z^2 + \partial_t^2\nabla_\perp^2][\partial_t(\partial_t + \nu_e)\boldsymbol{\nabla}_\perp \cdot (\mathbf{a}_{p\perp} + \boldsymbol{\nabla}_\perp\delta T_e/m) + \Omega_e^2(\partial_z a_{pz} - \partial_t\boldsymbol{\nabla} \cdot \mathbf{J}_B/n_0) - \Omega_e\partial_t\boldsymbol{\nabla} \cdot \mathbf{a}_p \times \hat{z}\ ] \tag{3}$$

where Ω_i the ion cyclotron frequency, $C_s = [(T_e + 3T_i)/M]^{1/2}$ the ion acoustic speed, $v_{ti} = (T_i/M)^{1/2}$ the electron thermal speed, and M the ion (O^+) mass; $\nu_e = \nu_{ei}$ and $\nu_i \cong (\pi/2)^{1/2}(\omega_s^2/k_z v_{ti})(T_e/T_i)\exp(-\omega_s^2/2k_z^2v_{ti}^2)$ the ion Landau damping rate; the coupling terms $\mathbf{a}_p = \langle\mathbf{v}_e\cdot\boldsymbol{\nabla}\mathbf{v}_e\rangle$ and $\mathbf{J}_B = \langle n_e\mathbf{v}_e\rangle$ arise from plasma nonlinearities and can be expressed explicitly by using only the linear part of electron velocity and density responses to high frequency wave fields; and $\delta T_e = T_e - T_{e0}$ evaluated from (2) is the result of the differential Ohmic heating, important only for the field-aligned purely-growing modes. The spatial and temporal variation of physical functions in the form of $p = p \exp[i(\boldsymbol{\kappa}\cdot\mathbf{r} - \varpi t)]$ will be assumed to reduce the coupled differential equations (1) and (3) to coupled algebraic equations leading to the dispersion relation for each parametric instability, where $\boldsymbol{\kappa}$ and ϖ are the appropriate wavevector and frequency of each physical quantity.

PARAMETRIC DECAY INSTABILITIES (PDI):

Consider the decay of a dipole pump $\mathbf{E}_p(\omega_0, \mathbf{k}_p=0)$ into a Langmuir/upper hybrid sideband $\phi(\omega, \mathbf{k})$ and an ion acoustic/lower hybrid decay mode $n_s(\omega_s, \mathbf{k}_s)$, where $\mathbf{E}_p = \hat{z}\ E_p$ near the reflection height and = $(\hat{x} - i\hat{y}\ )E_p$ in high-latitude upper hybrid resonance region; $\mathbf{k} = \hat{z}\ k_z + \hat{x}\ k_\perp$; matching conditions lead to $\omega = \omega_0 - \omega_s^*$ and $\mathbf{k}_s = -\mathbf{k}$.

A. Decay to Langmuir and ion acoustic waves:

The coupled mode equations (1) and (3) are combined to obtain the dispersion relation

$$[\omega(\omega + i\nu_e) - \omega_{k\theta}^2][\omega_s^*(\omega_s^* - i\nu_i) - k^2C_s^2] = (k_z^2\omega_p^4/4\pi n_0M\omega_0\omega)|E_p|^2 \tag{4}$$

We now set $\omega = \omega_r + i\gamma_k$ and $\omega_s = \omega_{sr} + i\gamma_k$ in (4) to evaluate the threshold field $E_{pth}(k, \theta)$ and growth rate $\gamma_k(\theta)$ of the instability excited at an arbitrary height h_1, where $\omega_p^2(h_1) = \omega_r^2 - 3k_1^2v_{te}^2 - \Omega_e^2\sin^2\theta_1$ is the matching height of the (k_1, θ_1) Langmuir wave. Thus in the general case that when the sideband and decay wave of the instability are driven waves, rather than eigen modes of plasma, the threshold field and growth rate of the instability are obtained, respectively, to be

$$|E_{pth}(k,\theta; k_1,\theta_1)| = (1 + \Delta\omega_1^4/\omega_0^2\nu_e^2)^{1/2}(mM/e^2)^{1/2}\,(\nu_e\nu_i\ \omega_{sr}\ \omega_0^3)^{1/2}/k\cos\theta\omega_p \tag{5}$$

$$\gamma_k \cong [(\nu_e\nu_i/4)(E_p/E_{pth})^2 + (\nu_e - \nu_i)^2/16]^{1/2} - (\nu_e + \nu_i)/4 \tag{6}$$

where $\Delta\omega_1^2 = \omega_{k\theta}^2 - \omega_r^2 = 3(k^2-k_1^2)v_{te}^2 + \Omega_e^2(\sin^2\theta - \sin^2\theta_1)$; $\omega_{sr}^2 = k^2C_s^2 - \omega_{sr}\nu_i\Delta\omega_1^2/\omega_r\nu_e$. As shown by (5) that the threshold field varies with the propagation angle θ and wavelength λ_1 of the Langmuir sideband as well as the location of excitation. When the instability is excited at the matching height h of its Langmuir sideband (k, θ), i.e., $\Delta\omega_1 = 0$, the threshold field is the minimum given by $|E_{pth}(k,\theta)|_m = (mM/e^2)^{1/2} (\nu_e\nu_i\, \omega_{sr}\, \omega_0^3)^{1/2}/k\cos\theta\omega_p$.

B. Decay to upper hybrid and lower hybrid waves in high-latitude upper hybrid resonance region:

Under the condition $k_z << k_\perp$, (1) and (3) are combined to obtain the dispersion relation

$$[-\Gamma + i\nu_e\omega(1 + \Omega_e^2/\omega_{uk}^2)][\omega_s{*}^2 - i(2 - \xi^{-1})\nu_e\omega_s{*} - \omega_{Lks}^2] \cong [k^2\omega_p^4(\omega_0 - \Omega_e)/4\pi n_0 M\omega_u^2(\omega_0 + \Omega_e)]|E_p|^2 \quad (7)$$

where $\Gamma = \omega_{uk}^2 - \omega^2$, $\omega_{uk}^2 = \omega_k^2 + \Omega_e^2 + \nu_e^2$, $\omega_k^2 = \omega_p^2 + 3k_\perp^2 v_{te}^2$, and $\omega_u^2 = \omega_p^2 + \Omega_e^2$; $\omega_{Lks}^2 = \omega_{LH}^2\xi + k^2C_s^2$, $\xi = 1 + (M/m)(k_z^2/k_\perp^2)$, and $\omega_{LH}^2 = \omega_{pi}^2/(1 + \omega_p^2/\Omega_e^2) \cong \Omega_e\Omega_i$. $k_{3x} \cong -k$ is substituted. We now set $\omega = \omega_{uk} + i\gamma_k$ and $\omega_s = \omega_{Lks} + i\gamma_k$ in (7), the threshold field E_{pth} and growth rate γ_k of the instability are determined, respectively, to be

$$E_{pth} \cong (m/e)(2\xi - 1)^{1/2}(1 + \Omega_e^2/\omega_0^2)^{1/2}(\nu_e\Omega_e/|k|)[\omega_0(\omega_0 + \Omega_e)/\omega_{Lks}(\omega_0 - \Omega_e)]^{1/2} \quad (8)$$

$$\gamma_k = (\nu_e/\sqrt{2})[(|E_p/E_{pth}|^2 + 1/8)^{1/2} - 3\sqrt{2}/4] \quad (9)$$

Using the parameters for Tromso heating experiments, the threshold field (8) can be expressed as $E_{pth} \cong 1.7\xi^{1/4}/k$ V/m.

OSCILLATING TWO-STREAM INSTABILITY (OTSI):

Due to the geomagnetic field, OTSI can be excited in a sizable region below the HF reflection height. This is in contrast to the unmagnetized case that OTSI can only be excited in a narrow region near the HF reflection height. Moreover, in the high latitude region OTSI can also occur in the upper hybrid resonance layer, which is well below the HF reflection height, to excite upper hybrid sidebands together with short-scale field-aligned density irregularities.

A. Excitation of Langmuir waves together with purely growing density striations:

This process involves the decay of a dipole pump $\mathbf{E}_p(\omega_0, \mathbf{k}_p=0)$ into two Langmuir sidebands $\phi_1(\omega_1, \mathbf{k}_1)$ and $\phi_1'(\omega_1', \mathbf{k}_1')$ and a purely growing mode $n_s(\omega_s=i\gamma_s, \mathbf{k}_s)$, where $\mathbf{E}_p = \hat{z}\, E_p$; γ_s is the growth rate of the instability and $\mathbf{k}_1 = \hat{z}\, k_0 + \hat{x}\, k_\perp$; matching conditions lead to $\omega_1 = \omega_1' = \omega_0 + i\gamma_s$ and $\mathbf{k}_1' = \mathbf{k}_s = -\mathbf{k}_1$. From (1) and (3), the dispersion relation is derived as

$$\{(\gamma_s^2 + \Omega_i^2)[\gamma_s(\gamma_s + \nu_i) + k_1^2C_s^2] - \Omega_i^2\, k_\perp^2C_s^2\} = 2(e^2/mM)k_1^2\cos^2\theta(\gamma_s^2 + \Omega_i^2\cos^2\theta)\{\Delta\omega^2/[\Delta\omega^4 + \omega_0^2(2\gamma_s + \nu_e)^2]\}\, |E_p|^2 \quad (10)$$

where $\Delta\omega^2 = \omega_p^2 + 3k_1^2v_{te}^2 + \Omega_e^2 \sin^2\theta - \omega_0^2$, and $\theta = \sin^{-1}(k_\perp/k_1)$.

We first set $\gamma_s = 0$ in (10) to determine the threshold condition of the instability. The threshold field is obtained to be

$$|E_p(\theta)|_{th} = (mM/2e^2)^{1/2}C_s[(\Delta\omega^4 + \omega_0^2\nu_e^2)/\Delta\omega^2]^{1/2}/\cos\theta \quad (11)$$

Similar to (5) for PDI, (11) shows that the threshold field of OTSI also varies with the propagation angle θ and wavelength λ_1 of the Langmuir sidebands as well as the location of excitation. For each propagation angle θ and wavelength λ_1, the instability has the minimum threshold field $|E_p(k_1, \theta)|_m = (mM/e^2)^{1/2}C_s(\omega_0\nu_e)^{1/2}/\cos\theta$, when it is excited in a preferential height layer with $\Delta\omega^2(k_1,\theta) = \omega_0\nu_e$, i.e., $\omega_p^2(h) = \omega_p^2(k_1,\theta) = \omega_0(\omega_0 + \nu_e) - 3k_1^2v_{te}^2 - \Omega_e^2\sin^2\theta$, where h is altitude of the preferential layer. In other words, the spectral lines of the Langmuir sidebands excited by OTSI have an angular (θ) and a spectral (k_1) distribution, as well as a spatial (h) distribution in a finite altitude region. This minimum threshold field increases with the oblique propagation angle θ of (k_1, θ) lines, but it is independent of k_1. The altitude h of the preferentially excited region for (k_1, θ) lines moves downward as the oblique propagation angle θ of these lines increases. The maximum growth rate $\gamma_{sM}(k_1,\theta)$ of the instability and its excitation region $\Delta\omega'^2(k_1,\theta)$ are determined by taking partial derivative of (10) on $\Delta\omega^2$ and setting $\partial\gamma_s/\partial\Delta\omega^2 = 0$ in the resultant. It leads to $\Delta\omega'^2(k_1,\theta) = \omega_0(2\gamma_{sM} + \nu_e)$, and $\gamma_{sM}^3 + \gamma_{sM}k_1^2C_s^2 - (\nu_ek_1^2C_s^2/2)(|E_p|^2/|E_p(k_1,\theta)|_m^2) \cong 0$, where $\gamma_{sM}^2 >> \Omega_i^2$, ν_e^2 are assumed (i.e. $|E_p|^2/|E_p(k_1,\theta)|_m^2 >> 1$). This quadratic equation for γ_{sM} has a real solution $\gamma_{sM} = (G + H)^{1/3} - (G - H)^{1/3}$, where $G = [(k_1^2C_s^2/3)^3 + H^2]^{1/2}$ and $H = (\nu_ek_1^2C_s^2/4)(|E_p|^2/|E_p(k_1,\theta)|_m^2)$. In the moderate heating power regime that $|E_p|^2/|E_p(k_1,\theta)|_m^2 < 2k_1C_s/\nu_e$ (*i.e.*, $\gamma_{sM} < k_1C_s$), $\gamma_{sM} \sim (\nu_e/2)(|E_p|^2/|E_p(k_1,\theta)|_m^2)$. $\gamma_{sM} \sim (2H)^{1/3}$ in the strong power regime that $|E_p|^2/|E_p(k_1,\theta)|_m^2 > 2k_1C_s/\nu_e$ (*i.e.*, $\gamma_{sm} > k_1C_s$). The altitude h_1' of the maximum growth rate layer (*i.e.*, $\omega_p^2(h_1') = \omega_p^2(h_1) + 2\omega_0\gamma_{sM}$) is slightly higher than the altitude h_1 of the minimum threshold layer (*i.e.*, $h_1' > h_1$). The height difference is given by $\Delta h = h' - h \cong 2\gamma_{sM}L/\omega_0$, where L is the linear scale length of the plasma density distribution. The height of the preferential (maximum growth rate) layer of the instability tends to shift upward from the matching height of the instability sidebands as the heating power (*i.e.*, γ_{sM}) increases. However, in the moderate power regime this shift is negligibly small.

The results show that the oblique angle line is preferentially excited in its matching height region, rather than in the region near the HF reflection height. The angular distribution of Langmuir sidebands excited in the matching height layer has a narrow hollow shape from θ_1 to θ_2, where $\theta_1 \sim \theta_0$, the oblique propagation angle of the resonant line (lines around $\theta = 0$ can not be excited).

B. Excitation of upper hybrid waves together with field-aligned density irregularities:

In high latitude region, right-hand circularly polarized heating wave is launched to propagate nearly along the geomagnetic field. The wave fields in the region near the upper hybrid resonance layer, which locates below the O-mode HF reflection height, are still dominated by the perpendicular components given by $\mathbf{E}_p(\omega_0, \mathbf{k}_p = 0) = (\hat{x} - i\hat{y})E_p$. This wave then decays into two upper hybrid sidebands $\phi_1(\omega_1, \mathbf{k}_1 = \hat{x}\, k_1)$ and $\phi_1'(\omega_1', \mathbf{k}_1' = -\hat{x}\, k_1)$ and a field-aligned purely growing mode $n_s(\omega_s = i\gamma_s, \mathbf{k}_s = -\hat{x}\, k_1)$, where $\omega_1 = \omega_1' = \omega_0 + i\gamma_s$. The dispersion relation is derived to be

$$[(\gamma_s + 2\nu_e m/M + \nu_e k_1^2 v_{te}^2/\Omega_e^2)(\gamma_s\Omega_e\Omega_i/\nu_e + k_1^2C_s^2) + \gamma_s k_1^2C_s^2/3](\omega_0^2\nu_e^2 + \Gamma^2)$$

$$= 2\nu_e(4e^2/3mM)k_1^2(1-2\Omega_e/\omega_0)[\Gamma - \nu_e^2 - \omega_0^2k_1^2/2k_D^2]\,|E_p|^2 \qquad (12)$$

In Tromso heating experiments, $2\pi\lambda_D(\omega_0/2\nu_e)^{1/2} \cong 5$ m. From (12), the minimum threshold field is determined to be $E_{pth}|_{min} \cong 3.2[1 + (16/\lambda_1)^2]^{1/2}$ mV/m ~ 3.2 mV/m for $\lambda_1 >> 16$ m, and $E_{pth}|_{min} \cong (16.5/\lambda_1)^2$ mV/m for $\lambda_1 < 5$ m. The growth rate of the instability is obtained to be $\gamma_s \cong 3.4\times10^{-2}\{[1 + (46/\lambda_1)^2(|E_p/E_{pth,min}|^2 - 1)]^{1/2} - 1\}$ s^{-1} for $\lambda_1 >> 31$ m and $\gamma_s \cong (5/\lambda_1)^2[(|E_p/E_{pth,min}|^2 + 0.7)^{1/2} - 1.3]$ s^{-1} for $\lambda_1 < 5$ m. These results show that this instability can generate both large-scale and small-scale field-aligned density irregularities in the high latitude ionosphere. In the large-scale case, i.e. $\lambda_1 >> 31$ m, the threshold field has a small constant value of 3.2 mV/m. The growth rate $\gamma_s \cong 3.4\times10^{-2}\{[1+(1380/\lambda_1)^2]^{1/2} - 1\}$ s^{-1} for $E_p = 0.1$ V/m, is also very small and decreases with the increase of the scale length. Thus this instability does not favor to excite kilometer scale irregularities. In the small-scale case, *i.e.* $\lambda_1 < 5$ m, the threshold field increases rapidly with the decrease of the scale length. The growth rate is generally much larger than that for the large-scale case and decreases with the decrease of the scale length due to the increase of the threshold field. For $E_p = 0.1$ V/m, the growth rate decreases to zero at $\lambda_1 = 1.65$ m, which is the lower bound of the instability scale length for this pump field.

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