

# GENERATION OF DENSITY IRREGULARITIES AND WHISTLER WAVES BY POWERFUL HF WAVES IN THE POLAR IONOSPHERE

**S. P. Kuo and E. Koretzky**

<sup>(1)</sup>*Department of Electrical Engineering, Polytechnic University, 901 Route 110, Farmingdale, NY 11735, USA  
E-mail: spkuo@rama.poly.edu*

<sup>(2)</sup>*Antenna Systems Department, TRW Space and Electronics Group, Redondo Beach, CA 90278, USA*

## ABSTRACT

Electron recombination rates with E region dominant ion species  $\text{NO}^+$  and  $\text{O}_2^+$  decrease with the electron temperature; thus plasma density and electrojet current are perturbed spatially in similar fashion as the electron temperature irregularities, generated along the geomagnetic field by powerful HF wave via thermal instability in the polar ionosphere. The spatial period of irregularities varies from 460 m to 1.3 Km. When modulation frequency between 2 and 30 kHz is applied, density irregularities coupling to modulated electrojet current produces spatially distributed mode current of whistler waves. This direct excitation process enhances the generation efficiency of whistler waves and reduces harmonic components.

## INTRODUCTION

Ionospheric density irregularities with a broad range of scale lengths have been observed in natural events and in HF heating experiments. These irregularities have spatial variations in the direction perpendicular to the background magnetic field, thus being called field-aligned density irregularities (FAIs). Short to medium scale (i.e., less than 100 meters) FAIs give rise to intense bistatic scattering and backscattering of ground-based HF/VHF/UHF radar signals [1]. Large scale FAIs (i.e., larger than a few hundred meters) cause spread echoes on ionograms, known as spread-F, and scintillation of beacon satellite signals [2].

In HF heating experiments, FAIs of different scale sizes can be generated artificially through various processes. Those in short to medium scale sizes are generated via secondary parametric instabilities [3-5]. The pump waves of these secondary parametric instabilities are the high frequency sidebands of the primary parametric instabilities excited directly by the HF heating waves. On the other hand, large-scale sheet-like FAIs [6] are usually generated by the HF heating wave directly via thermal filamentation instabilities [7-9].

In this work, we show that heating of E-region electrons by HF waves can excite purely growing thermal instability to produce large-scale temperature perturbations, which then evolve through the electron thermal diffusion to steady state periodic temperature irregularities along the magnetic field. Because in E region the recombination rates [10] of electrons with the dominant ion species  $\text{NO}^+$  and  $\text{O}_2^+$  decrease as the electron temperature increases, large-scale periodic density irregularities together with an average plasma density enhancement are also generated [11,12]. The favorable spatial variation direction (i.e., along the magnetic field) and scale lengths (i.e., in the kilometer range) make these new type irregularities to couple effectively with the HF wave-modulated electrojet. The coupling can induce mode currents of whistler waves, which produces whistler waves directly [12, 13].

## GENERATION OF PERIODIC IRREGULARITIES BY THE HF HEATING

In the presence of HF heating, the electron thermal energy equation [10] is given by

$$\partial T_e / \partial t + (2T_e/3)\nabla \cdot \mathbf{v}_e + \delta(T_e)v_{en}(T_e)(T_e - T_n) = (2/3n)(Q + \nabla \cdot \mathbf{K}_e \cdot \nabla T_e) \quad (1)$$

where  $n$  is the plasma density,  $\mathbf{v}_e$  is the electron fluid velocity, and the second term on the left hand side (LHS) of (1) represents the isothermal response of electrons to the thermal perturbation;  $\delta(T_e)$  is the average relative energy fraction lost in each collision,  $v_{en}(T_e)$  is the effective collision frequency of electrons with neutral particles,  $T_n$  is the temperature of the background neutral particles,  $\mathbf{K}_e = \hat{z} \hat{z} 3.16nT_e/mv_{en}$  is the along-the-magnetic field thermal diffusion tensor, and

$m$  is the electron mass;  $Q = J_e^2/\sigma \cong v_{en}n_0m[u_e^2 + \langle |v_{pe\pm}|^2 \rangle]$  is the total Ohmic heating power density in the background plasma and contributed by the electrojet current and the HF heater wave.

The electrojet current is driven by a dc electric field  $\mathbf{E}_0 = \hat{x} E_0$  in a collision plasma which is embedded in a background magnetic field  $\mathbf{B}_0 = -\hat{z} B_0$ . The circularly polarized HF wave fields are given by  $\mathbf{E}_{p\pm} = (\hat{x} \pm i\hat{y})(\epsilon_p/2)\exp[i(k_0z - \omega_0t)] + \text{c.c.}$ , where  $\epsilon_p^2 = \epsilon_{p0}^2 M(t)$ , i.e., the HF heating power is modulated by a periodic modulation function  $M(t) = \sum_{p=0} M_p \cos(\omega_0 t + \phi)$ , and ‘ $\pm$ ’ refer to ‘O’ mode and ‘X’ mode, respectively. Hence,  $u_e = eE_0/m(v_{en}^2 + \Omega_e^2)^{1/2}$  and  $\langle |v_{pe\pm}|^2 \rangle = v_{q\pm}^2 M(t)$ , where  $v_{q\pm}^2 = (e\epsilon_{p0}/m)^2/[(\omega_0 \pm \Omega_e)^2 + v_{en}^2]$ . It will be shown that only the dc part of  $M(t)$ , i.e., the term proportional to  $M_0$ , contributes to the generation of density irregularities.

Both elastic and inelastic collisions contribute to the heat loss. The main processes involved in the inelastic collisions are the rotational and vibration excitation of  $N_2$  and  $O_2$ . The total collision frequency can be expressed in terms of the unperturbed elastic collision frequency  $v_{en}(T_{e0}) = v_{e0}$  as [10,14]

$$\begin{aligned} \delta(T_e)v_{en}(T_e) \cong & 2(m/M_n)v_{e0} (T_e/T_{e0})^{5/6} + 8.6(T_e/T_{e0})^{-1/2} + (T_e/T_{e0})^{-5/2}\{0.25\exp[15.7(1 - T_{e0}/T_e)] \\ & + 27\exp[3(1 - T_{e0}/T_e)]\} \end{aligned} \quad (2)$$

where  $M_n$  is the mass of neutral particles ( $N_2$ ); the first term on the right hand side (RHS) of (2) is attributed to the elastic collision; the second term on the RHS of (2) accounts for the total rotational excitation loss rate; the two exponential functions in the last term of the RHS of (2) account for vibration excitation of  $N_2$  and  $O_2$ , respectively.

The dependency of  $v_{en} \propto T_e^{5/6}$  and the heating rate  $\propto v_{en}$  give a positive feedback to excite thermal instability. As temperature perturbations grow, the background plasma density also varies accordingly, through the dependence of the electron-ion ( $NO^+$  and  $O_2^+$ ) recombination rates on the electron temperature, with the relation [10]

$$n(T_e) / n(T_{e0}) \cong [0.6 (T_e/T_{e0})^{0.7} + 0.4 (T_e/T_{e0})^{1.2}]^{1/2} \quad (3)$$

where the 3 to 2 ratio of the daytime densities of  $O_2^+$  and  $NO^+$  in the region near 120 Km height is assumed. Therefore, the spatial spectrum of the density irregularities is determined directly by the steady state temperature perturbations. Since the growth rate of the thermal instability ( $\propto v_{e0}$ ) is quite high and the nonlinearity in the inelastic damping is strong, only the steady state perturbation is of interest. (1) is solved for the steady state situation by setting the time derivative term on its LHS equal to zero. Again, the second term on the LHS of (1) is derived, from the continuity equation ( $\nabla \cdot n_e \mathbf{v}_e = 0$ ) and momentum equation ( $-\nabla p_e - mn_e v_{en} \mathbf{v}_e \cong 0$ ) of electrons and the ratio of specific heats for electrons  $\gamma_e = 3$ , to be  $(2T_e/3)\nabla \cdot \mathbf{v}_e = -(2mv_{en})^{-1}(dT_e/dz)^2$ . This term and the thermal diffusion term on the RHS of (1) can be combined into a single diffusion term. For the convenience of numerical analysis, the normalized function  $f = T_e/T_{e0}$  and normalized variable  $\eta = (m/1.05M_n)^{1/2}(v_{e0}/v_{te})z$  are used, where the normalization factor  $(m/1.05M_n)^{1/2}(v_{e0}/v_{te}) = 8.28 \times 10^{-4} \text{ m}^{-1}$  is the same as that used for  $\lambda$ . Thus (1) is converted to the equation used in the numerical analysis [13]

$$\begin{aligned} & (0.6 f^{0.7} + 0.4 f^{1.2})^{-1/2} f^{-1/4} d_\eta (0.6 f^{0.7} + 0.4 f^{1.2})^{1/2} f^{5/12} d_\eta f \\ & = -a_\pm f^{5/6} - 36.85(1 - t_n) + \{f^{5/6} + 8.6 f^{-1/2} + f^{-5/2}[0.25e^{15.7(1-1/f)} + 27e^{3(1-1/f)}]\}(f - t_n) \end{aligned} \quad (4)$$

In the numerical analysis, (4) is found to have a periodic solution only if the values of  $a_\pm$  are in the range of  $150 \leq a_\pm \leq 1819$ , i.e.,  $0.52 < [\epsilon_{p0}/(F_0 \pm 1.35)]\sqrt{M_0} < 1.69$ . In this range of  $a_\pm$ , the RHS of (4) has three zeros: at  $f = f_{\min}$ ,  $f_a$ , and  $f_{\max}$ . The periodic solution  $f(\eta)$  of (4) oscillates between the minimum  $f_{\min}$  and the maximum  $f_{\max}$  around the steady state stable value  $f_a$ .  $f(\eta)$  varies in space periodically along the geomagnetic field (i.e., the  $z$  axis). The spatial period  $\lambda_s$  of the generated periodic density irregularities is between 0.46 Km and 1.3 Km. When the values of  $a_\pm$  are outside the range of [150, 1819], the solution of (4) becomes a constant  $f = f_a$  and thus, only an enhanced density layer, rather than irregularities, is formed. The functional dependence of the normalized  $\lambda_s$  on  $a_\pm$  is plotted in Fig. 1.

## EXCITATION OF WHISTLER (VLF) WAVES

The electron drift speed, driven by a dc field in the electrojet, is modulated by the amplitude-modulated heating wave in the same fashion as the conductivity of the electron plasma. As the periodic density irregularities distributed along the geomagnetic field with a large spatial period are also generated by the heating wave in the same region, their coupling

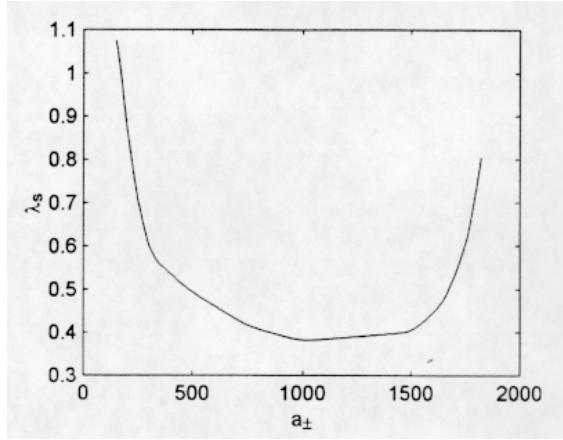


Fig.1. Normalized spatial period  $\lambda_s$  of the density irregularities, by a factor of  $8.28 \times 10^{-4} \text{ m}^{-1}$ , as a function of  $a_{\pm}$ .

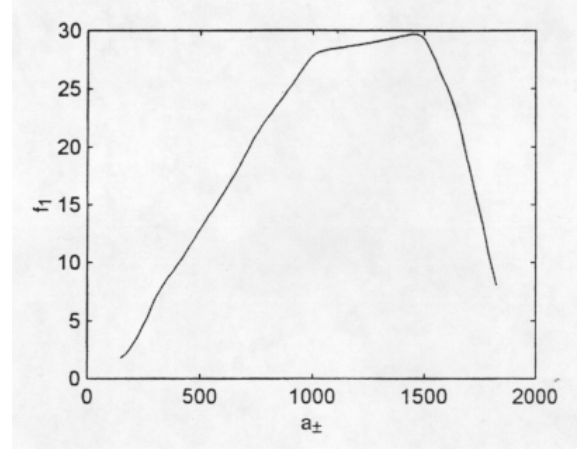


Fig 2. Modulation frequency  $f_1$  (kHz) vs  $a_{\pm}$ .

with the electron drift velocity perturbation, produces a space-time dependent current, oscillating at the modulation frequency and distributed along the geomagnetic field, as [13]

$$\Delta \mathbf{J}_e = [\hat{x} + \hat{y} 2\underline{v}_{e0}/\Omega_e] A \sum_k n_k \{ \exp[i(kz + \omega_1 t + \phi)] - \exp[i(kz - \omega_1 t - \phi)] \} + \text{c.c.} \quad (5)$$

where  $A = -i(5e^2 E_0 / 18m) (\underline{v}_{e0}^2 / \omega_1 \Omega_e^2) M_1 (v_{q\pm}^2 / v_{te}^2)$  and the density perturbation  $\Delta n$  is expressed in terms of its Fourier spectrum  $n_k$  as  $\Delta n = \sum_k n_k e^{ikz} + \text{c.c.}$ . This is a source current for plasma waves in VLF frequency range and propagating downward and upward along the geomagnetic field. If the wave number  $k_1$  of the spectral peak of  $\Delta n$  and the modulation frequency  $\omega_1$  satisfy the dispersion relation of the whistler wave, (5) becomes an effective current source for whistler wave, which is generated directly inside the current distribution with a much better directivity than the antenna radiation process. The wave equation for whistler wave is given by [13]

$$[(\partial_t + \Omega_e \hat{z} \times + \underline{v}_{e0})(c^2 \nabla^2 - \partial_t^2) - \omega_{pe}^2 \partial_t] \mathbf{E} = c^2 \mu_0 (\partial_t + \Omega_e \hat{z} \times + \underline{v}_{e0}) \partial_t \Delta \mathbf{J}_e \quad (6)$$

In (6), the wave electric field  $\mathbf{E}$  is driven by its source current density  $\Delta \mathbf{J}_e$  and can be assumed to have the form  $\mathbf{E} = \sum_k \{ \mathbf{E}_k^+ \exp[i(kz + \omega_1 t + \phi)] - \mathbf{E}_k^- \exp[i(kz - \omega_1 t - \phi)] \} + \text{c.c.}$ . It is then substituted into (6) to obtain

$$(k^2 c^2 / \omega_1) E_{k_x}^{\pm} + i(\omega_{pe}^2 / \Omega_e) (\delta_{\pm} - i \underline{v}_{e0} k^2 c^2 / \omega_1 \omega_{pe}^2) E_{k_y}^{\pm} = i c^2 \mu_0 (-\delta_{\pm} + i 2 \underline{v}_{e0} \omega_1 / \Omega_e^2) A n_k \quad (7)$$

and

$$-i(\omega_{pe}^2 / \Omega_e) (\delta_{\pm} - i \underline{v}_{e0} k^2 c^2 / \omega_1 \omega_{pe}^2) E_{k_x}^{\pm} + (k^2 c^2 / \omega_1) E_{k_y}^{\pm} = i c^2 \mu_0 (\delta_{\pm} + i \omega_1 / 3 \underline{v}_{e0}) (3 \underline{v}_{e0} / \Omega_e) A n_k$$

where  $\delta_{\pm} = \pm 1$ ;  $\omega_1^2 \ll k^2 c^2 \ll \omega_{pe}^2$  is assumed. In the source free and collisionless case, (7) leads to the dispersion relation of the whistler wave  $\omega = k^2 c^2 \Omega_e / \omega_{pe}^2$ . Let  $k_1$  be the wave number of a whistler wave having a frequency equal to the modulation frequency  $\omega_1$ , i.e.,  $\omega_1 = k_1^2 c^2 \Omega_e / \omega_{pe}^2$ , the field amplitude of this wave is obtained from (7) to be

$$E_x^{\pm}(k_1) = -(c^2 \mu_0 \Omega_e^2 / 2 \omega_{pe}^2 \underline{v}_{e0}) (1 - i 2 \delta_{\pm} \underline{v}_{e0} \omega_1 / \Omega_e^2) A n(k_1) \text{ and } E_y^{\pm}(k_1) = i \delta_{\pm} (1 - i \delta_{\pm} \underline{v}_{e0} / \Omega_e) E_x^{\pm}(k_1) \quad (8)$$

Hence, the generated wave has a circular polarization. It is noted that the superscripts ' $\pm$ ' in (7) and (8) stand for down- and up-propagating wave, which are different from those used as subscripts to stand for O- and X-mode heater wave.

The HF heating wave can significantly enhance the daytime E-region background plasma density by reducing the recombination rates of electrons with  $\text{NO}^+$  and  $\text{O}_2^+$  ions. Let  $\alpha = \langle n(T_e) / n(T_{e0}) \rangle^{1/2} \cong [n(T_{e0}) / n(T_{e0})]^{1/2}$  stand for the average enhancement factor of the electron plasma frequency from its unperturbed value  $f_{pe0} = 2.5 \text{ MHz}$ , the dispersion relation of the whistler wave can be expressed as  $f_1 = 13.32 / \alpha^2 \lambda_1^2 \text{ kHz}$ , where  $\Omega_e / 2\pi = 1.35 \text{ MHz}$  is assumed. Setting  $\lambda_1 = \lambda_s$ , the spatial period of density irregularities presented in Fig. 1, the dependence of  $f_1$  on  $a_{\pm}$  is determined by the

dependencies of  $\alpha$  and  $\lambda_s$  on  $a_{\pm}$  and is presented in Fig. 2. By varying the power, polarization, and frequency of the HF heating wave,  $a_{\pm}$  values can be varied to change the average electron density ( $\propto \alpha$ ) and the spatial period  $\lambda_s$  of the generated density irregularities. The modulation frequency of the heating wave should be changed accordingly as the dependence of  $f_1$  on  $a_{\pm}$  shown in Fig. 2. Thus the modulation frequency satisfies the dispersion relation of whistler waves and a resonant excitation of whistler waves can be achieved. However, the choice of the modulation frequency may be optimized. As shown in Fig. 2, the dependence of  $f_1$  on  $a_{\pm}$  has a plateau around 28 kHz.

## SUMMARY

Using an amplitude-modulated HF heating wave to modulate the polar electrojet, two types of thermal instabilities are excited. The one excited by the modulation part of the heating wave oscillates only in time and causes effectively the electrojet currents to be modulated at the harmonics of the modulation frequency of the heating wave. The modulated electrojet currents, then, act as antennas to radiate electromagnetic waves at those harmonics. The other one excited by the dc part of the heating power is a purely growing thermal instability, which oscillates spatially with a broad scale length in the kilometer range. This instability has a large growth rate, but it is quickly stabilized by the inelastic (vibration) collision loss. The thermal diffusion along the geomagnetic field can, however, further evolve the steady state temperature fluctuations into a spatially periodic variation having a spatial period in the kilometer range. This temperature variation modifies the background recombination rates between electrons and  $\text{NO}^+$  and  $\text{O}_2^+$  ions, and hence, is accompanied by density variation. The generated density irregularities distribute periodically along the geomagnetic field. We further show that the combined spatial filamentation and temporal modulation of the electrojet current induces a distributed mode current for whistler waves excited at the modulation frequency of the HF wave. It is shown that this current also has a right-hand circular polarization and has its frequency and wavenumber satisfying the dispersion relation of the whistler wave. Hence, it excites whistler waves directly without going through a low efficiency antenna radiation process. This direct excitation process also works to reduce the harmonic components of the excited whistler waves. It is shown that a whistler wave with a frequency between 2 and 30 kHz can be generated by the HF wave modulated at the same frequency, via the proposed mechanism. The dependence of the required modulation frequency, in order to match the mode frequency of whistler waves, on a combined HF wave parameter  $a_{\pm}$ , is presented in Fig. 2. A plateau in the dependence of  $f_1$  on  $a_{\pm}$  appears around 28 kHz, which is, thus, the most favorable modulation frequency to be used in the future HF heating experiments.

## REFERENCES

- [1] J. Minkoff, "Radio wave scattering from a heated ionosphere," *Radio Sci.*, vol. 9, p. 997, 1974.
- [2] A. Frey, P. Stubbe, and H. Kopka, "First experimental evidence of HF produced electron density irregularities in the polar ionosphere: diagnosed by UHF radio star scintillation," *Geophys. Res. Lett.*, vol. 11, p. 523, 1984.
- [3] S. P. Kuo, B. R. Cheo, and M. C. Lee, "The role of parametric decay instabilities in generating ionospheric irregularities," *J. Geophys. Res.*, vol. 88, p. 417, 1983.
- [4] S. P. Kuo, J. Huang, and M.C. Lee, "Parametric excitation of ion Bernstein waves by parallel-propagating Langmuir waves," *J. Atmos. Terr. Phys.*, vol. 60, p. 121, 1998.
- [5] L. Stenflo, "Wave collapse in the lower part of the ionosphere," *J. Plasma Phys.*, vol. 45, p. 355, 1991.
- [6] M. C. Lee, R. J. Riddolls, W. J. Burke, S. P. Kuo, and E. M. C. Klien, "Generation of large sheet-like ionospheric plasma irregularities at Arecibo," *Geophys. Res. Lett.*, vol. 25, p. 3067, 1998.
- [7] S. P. Kuo and G. Schmidt, "Filamentation instability in magneto plasmas," *Phys. Fluids*, vol. 26, p. 2529, 1983.
- [8] S. P. Kuo and M. C. Lee, "Earth magnetic field fluctuations produced by filamentation instabilities of electromagnetic heater waves," *Geophys. Res. Lett.*, vol. 10, p. 979, 1983.
- [9] L. Stenflo and P. K. Shukla, "Filamentation instability of electron and ion cyclotron waves in the ionosphere," *J. Geophys. Res.*, vol. 93, p. 4115, 1988.
- [10] V. A. Gurevich, *Nonlinear Phenomena in the Ionosphere*, chap. 2. Springer-Verlag: New York, pp. 14-124, 1978.
- [11] S. P. Kuo, E. Koretzky, and M.C. Lee, "Plasma density enhancement and generation of spatially periodic irregularities in the ionospheric E-region by powerful HF waves," *Geophys. Res. Lett.*, vol. 26, p. 991, 1999.
- [12] S. P. Kuo, E. Koretzky, and M.C. Lee, "A new mechanism of whistler wave generation by amplitude-modulated HF waves in the polar electrojet," *Geophys. Res. Lett.*, vol. 26, p. 1677, 1999.
- [13] S. P. Kuo and E. Koretzky, "Generation of density irregularities and whistler waves by powerful radio waves in the polar ionosphere", *Phys. Plasmas*, vol. 8, p. 277, 2001.
- [14] S. P. Kuo and M. C. Lee, "Generation of ELF and VLF waves by HF heater-modulated polar electrojet via a thermal instability process," *Geophys. Res. Lett.*, vol. 20, p. 189, 1993.