A Highly Efficient Higher-Order Hybrid Finite Element–Boundary Integral Method for Large-Scale Scattering Analysis

Jian-Ming Jin and Jian Liu

Center for Computational Electromagnetics
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801-2991

Traditional finite element (FE) analysis of electromagnetic scattering uses either an absorbing boundary condition (ABC) or a boundary integral equation (BIE) to truncate the computational domain. The FE–ABC approach yields a purely sparse matrix, which can be stored and solved very efficiently; however, its solution is approximate. To improve the accuracy, the ABC is usually placed far away from the scatterer, which results in a large computational domain and hence a large number of unknowns. The FE–BIE approach is accurate and the truncation boundary can be placed very close to the scatterer; unfortunately, its numerical system is a partly sparse and partly full matrix, which is inefficient to solve. Moreover, this matrix is found very ill conditioned, especially when higher-order elements are used, and an iterative solution often requires an excessive number of iterations. Therefore, the FE–ABC approach is efficient but approximate and the FE–BIE approach is accurate but time-consuming. In this paper, we present a new approach, which can be considered either as an extension of the FE–ABC approach using the BIE or as an extension of the FE–BIE approach using the ABC. It bridges the FE–ABC and FE–BIE approaches and yields an accurate and efficient FE solution of scattering problems. This new approach starts with a traditional ABC and then constructs an adaptive ABC using the BIE. As such, it produces a purely sparse system matrix and the truncation boundary can be placed very close to the scatterer to minimize the computational domain. More importantly, the solution converges quickly to the true solution of the problem. It can be shown that this approach is actually a special case of a more general approach that uses the FE–ABC system as the preconditioner for the FE–BIE system, based on the observation that the large eigenvalues of the FE–ABC system are similar to those of the FE–BI system. As a result, the preconditioned system has a spectrum distribution clustered around 1 in the complex plane, and when a Krylov subspace based method is employed to solve the preconditioned system, the convergence is greatly accelerated. To demonstrate its potential, the new approach is implemented using higher-order curvilinear vector elements. A mixed functional is designed to yield both electric and magnetic fields on an integration surface, without numerical differentiation. The evaluation of boundary integrals is carried out using the multilevel fast multipole algorithm (MLFMA), which greatly reduces both the memory requirement and CPU time. The preconditioner is constructed either using the multifrontal algorithm or the generalized minimum residual (GMRES) algorithm in conjunction with an ILU preconditioner. The preconditioned FE–BIE system is solved using the GMRES algorithm. The resulting simulation code has been applied to a variety of large-scale scattering problems involving complex geometries and material composition. Numerical results show that the new approach is accurate and more efficient than the traditional FE–BIE approach by approximately two orders of magnitude for large-scale problems.