LOCALIZED DISTORTED BORN APPROXIMATION –

ON-ROUTE TOWARDS LARGE SCALE INVERSE SCATTERING

Timor Melamed

Ben-Gurion University of the Negev, Dept. of Electrical and Computer Engineering, Beer Sheva 84105, Israel.
Email: timormel@bgumail.bgu.ac.il

ABSTRACT

A novel Localized Distorted Born Approximation and its application to the nonlinear inverse scattering is presented. This approximation is carried out in a local transform domain thereby utilizing a priori localization of wave-object interactions. Further localization is achieved by using collimated pulsed-beams (PB), which provide highly resolved wavepackets for high resolution probing. The propagation over the inhomogeneous background is done using the wavepacket equation. In the Localized Distorted Born Approximation, the evaluation of the 3D Green's function in the local transform domain may be approximated analytically, thereby giving raise to a large scale inverse procedures.

INTRODUCTION

The present work extends a recent study of linearized scattering and imaging [1, 2]. It was shown there that the local spectrum of the data, obtained by processing the space-time scattering data, localizes the spectral regions wherein the "physical" signal resides, thereby isolating pulsed beam (PB) contribution to the measured field. The highly space-time resolved PB incident and scattered wavepackets excise along their trajectories through the object domain highly space-time resolved scattering cells perpendicular to their respective axes. Where these trajectories intersect, their combined effect is localized around a single scattering cell, thereby tying the properties of these PBs to the local (one-dimensional) medium stratification along the bisector axis. We now extend these results for the nonlinear inverse problem using a novel Localized Distorted Born Approximation. This approximation is carried out in the local (phase space) domain thereby utilizing a priori localization.
We are concern with the reconstruction of an object characterized by a wave velocity of $v(\mathbf{r})$, embedded in a homogeneous background of v_o (see Fig. 1). The total field $u(\mathbf{r}, t)$ satisfies the time domain (TD) wave equation

$$[\ \nabla^2 - \frac{1}{v^2(\mathbf{r})}\partial_t^2\ ]\ u(\mathbf{r}, t) = 0 \quad , \quad \partial_t \equiv \partial/\partial t. \tag{1}$$

DISTORTED BORN APPROXIMATION

The field $u(\mathbf{r}, t)$ scattered by an object having a wave velocity $v(\mathbf{r})$, may be approximated by the Distorted Wave Born Approximation where, for convenience, the medium is described by the *object function*

$$O_n(\mathbf{r}) = v_o^2[\frac{1}{v^2(\mathbf{r})} - \frac{1}{v_n^2(\mathbf{r})}], \tag{2}$$

The scattered field is measured over a plane (see Fig. 1). for simplicity, we shall assume here that the scattering field is measured over the $z = 0$ plane. The processed data $u_n^d(\mathbf{x}, t) \equiv u(\mathbf{r}, t) - u_n^i(\mathbf{r})|_{z=0}$ is given by

$$u_n^d(\mathbf{x}, t) = -v_o^{-2}\partial_t^2 \int d^3r' \int dt' O_n(\mathbf{r}')u_n^i(\mathbf{r}', t')G_n(\mathbf{r}, t; \mathbf{r}', t'))|_{z=0} \tag{3}$$

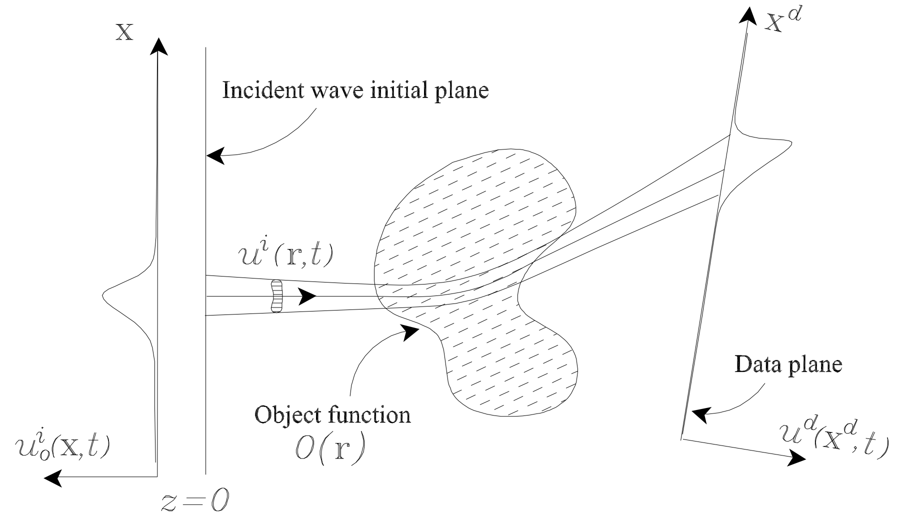

Fig. 1. Scattering scenario for the Localized Distorted Born Approximation. The incident field is a localized wavepacket (Pulsed Beam) propagating over a known inhomogeneous background. The scattered field, $u_n^d(\mathbf{x},t)$ is measured over the data plane, and processed using a local (phase-space) transform

where G_n is the v_n medium TD Green's function, and u_n^i is the incident wave propagating in v_n medium. In the inverse scattering scenario at the nth iteration, $v_n(\mathbf{r})$ is known, from which u_n^i and G_n are found, and an inverse procedure is applied to evaluate O from the measured field, yielding v_{n+1}. The effort to recalculate both u_n^i and G_n (either in the time or frequency domain) are the main limitation of the procedures based on this approximation.

In the Localized Distorted Born Approximation, the incident field is a wavepacket and may be evaluated *analytically* by use of the wavepacket equation [3]. The evaluation of Green's function is the major drawback of most procedures suggested so far, and made the use of these schemes limited to the reconstruction of limited size objects. Since wave iteration with the media have been found to be local in nature, it is suggested that the evaluation of Green's function should not be carried out in the $\mathbf{r}$ space but in a local transform (phase-space) domain, which extracts local radiation properties of the data and by that synthesize local wave-medium interactions [1, 2].

FORWARD PROPAGATION IN THE PERTURBED MEDIA

The object is illuminated by an incident wave defined by its boundary condition on $z = 0$ plane u_o^i

$$u_o^i(\mathbf{x},t) = \mathrm{Re}\ \overset{+}{f}[t + v_o^{-1}(\frac{1}{2}\mathbf{x}\,\mathbf{\Gamma}_o \cdot \mathbf{x})], \quad \mathbf{\Gamma}_o = \begin{bmatrix} \Gamma_o & 0 \\ 0 & \Gamma_o \end{bmatrix} \tag{4}$$

where $\overset{+}{\delta}$ is the analytic delta function $\overset{+}{\delta}(t) = (\pi i t)^{-1}$.

An approximated (high frequency) solution for propagating u_o^i in a media with a know wave velocity $v_n(\mathbf{r})$, is given in [3]. The wavepacket $u_n^i(\mathbf{r},t)$ is propagating along ray trajectories Σ (see Fig. 1). Denoting σ as the arc length along Σ, one finds that the field is given by

$$u_n^i(\mathbf{r},t) = \frac{1}{\Gamma_o\sqrt{v_o}}\sqrt{\frac{v_n(\sigma)}{|\mathbf{Q}_n(\sigma)|}}\ \overset{+}{f}\left(t - \int^{\sigma} d\sigma'/v_n(\sigma') - \frac{1}{2}\mathbf{x}\,\mathbf{\Gamma}_n(\sigma)\cdot\mathbf{x}\right), \quad \mathbf{\Gamma}_n = \mathbf{P}_n\mathbf{Q}_n{}^{-1} \tag{5}$$

where $\mathbf{P}_n,\ \mathbf{Q}_n$ are found from

$$\mathbf{Q}_n' = v_o \mathbf{P}_n, \quad \mathbf{P}_n' = -v_o \mathbf{V}_n^{(2)} \mathbf{Q}_n, \quad \mathbf{V}_{n_{ij}}^{(2)} = \partial_i \partial_j v_n|_\Sigma \tag{6}$$

where the prime denotes a derivative with respect to the argument. The initial conditions are $\mathbf{Q}_n(0) = \mathbf{\Gamma}_o^{-1}, \quad \mathbf{P}_n(0) = \mathbf{I}$. The wavepacket equation provide us with an analytic approximation for the propagating of the incident field in the v_n inhomogeneous medium as long as the object grid is fine enough for the solution of (6).

LOCAL PROCESSING OF THE DATA

For the desired *local* spectral analysis of the data, we generate the time dependent local plane wave spectrum via a windowed transform of the data in *configuration* space

$$U^d(\mathbf{Y}) = \int d^2x \int dt\ u^d(\mathbf{x}, t) w[\mathbf{x} - \bar{\mathbf{x}}, t - \bar{t} - v_o^{-1} \bar{\boldsymbol{\xi}} \cdot (\mathbf{x} - \bar{\mathbf{x}})] \tag{7}$$

where $\mathbf{Y} = (\bar{\mathbf{x}}, \bar{t}, \bar{\boldsymbol{\xi}})$. Here, $w(\mathbf{x}, t)$ is a temporal spatial window function, centered at $\mathbf{x} = 0,\ t = 0$. Therefore, the space-time and spectral dependence in the phase space window kernel w of (7) implies that the window is localized about $\bar{\mathbf{x}} = (\bar{x}_1, \bar{x}_2)$ and $t = \bar{t}$ with spectral tilt $\bar{\boldsymbol{\xi}} = (\bar{\xi}_1, \bar{\xi}_2)$. The vector $\mathbf{Y}$ incorporates the configuration-spectrum *phase space coordinates* $(\bar{\mathbf{x}}, \bar{t}, \bar{\boldsymbol{\xi}})$, whence $U^d(\mathbf{Y})$ is referred to as a *phase space distribution* of the data $u^d(\mathbf{x}, t)$. The operation in (7) can be referred to as a "*local slant-stack transform*" that extracts the local spectral information from the time-dependent data.

LOCALIZED DISTORTED BORN APPROXIMATION

Due to the limited scope of this proceeding paper, we shall derive here the Data − Object relation in the Local Transform domain for the spacial case $\bar{\boldsymbol{\xi}} = 0$ (synthesizing PB reflections arriving normal to the data plane). The general case $\bar{\boldsymbol{\xi}} \neq 0$ is described in Fig. 1. By inserting (3) into (7), we obtain

$$U^d(\bar{\mathbf{x}}, \bar{t}) = -v_o^2 \int d^3r' O(\mathbf{r}') \int dt' \partial_{t'}^2 u_n^i(t') \Psi_n(\mathbf{r}', t'; \bar{\mathbf{x}}, \bar{t}) \tag{8}$$

with

$$\Psi_n(\mathbf{r}', t'; \bar{\mathbf{x}}, \bar{t}) = \int d^2x \int dt\ w(\mathbf{x} - \bar{\mathbf{x}}, t - \bar{t}) G_n(\mathbf{r}, t; \mathbf{r}', t')|_{z=0} \tag{9}$$

where $G_n(\mathbf{r}, t; \mathbf{r}', t')$ is the perturbed media time domain Green's function. Next, we rewrite (8) in the form

$$U^d(\bar{\mathbf{x}}, \bar{t}) = \int d^3r' O(\mathbf{r}') \Lambda_n(\mathbf{r}'; \bar{\mathbf{x}}, \bar{t}), \quad \Lambda_n(\mathbf{r}'; \bar{\mathbf{x}}, \bar{t}) = -v_o^2 \int dt' \partial_{t'}^2 u_n^i(t') \Psi_n(\mathbf{r}', t'; \bar{\mathbf{x}}, \bar{t}) \tag{10}$$

The above results imply that the interaction of the z-directed pulsed plane wave with the object domain, when parameterized in terms of scattered PB propagators, occurs as if each scattered PB were *specularly reflected* from the local medium inhomogeneities (Fig. 2). Each PB senses the medium as if it were locally a plane stratified medium along the bisectional axis, with the effective variations of the medium along this axis being extracted from $O(\mathbf{r})$ via the sampling operation in (10). This interpretation is a localized version of the "pseudo plane-wave reflection law" discussed in connection with the transient plane-wave analysis in [4] and with connection to the Born approximation in [1].

Next, we consider the δ-Gaussian window $w(\mathbf{x}, t) = \mathrm{Re}\left(\alpha \overset{+}{\delta}{}^{(2)}[t - \frac{i}{2}T - \frac{i}{2}|\mathbf{x}|^2/v_o\Gamma_0]\right)$ where $\overset{+}{\delta}{}^{(2)}$ is the second derivative of the analytic delta function in (4). The resulting scattering cell is found to be

$$\Lambda_n(\mathbf{r}'; \bar{\mathbf{x}}, \bar{t}) = Re\left\{\frac{-\alpha^*}{2v_o} \frac{1}{\Gamma_o v_o} \frac{v_n(\sigma)}{|\mathbf{Q}_n(\sigma)|}\ \overset{+}{f}{}^{(3)} \left[\bar{t} - i\frac{T}{2} - 2(\int^\sigma d\sigma'/v_n(\sigma') - \frac{1}{2}\mathbf{x}\, \mathbf{\Gamma}_n(\sigma) \cdot \mathbf{x}))\right]\right\} \tag{11}$$

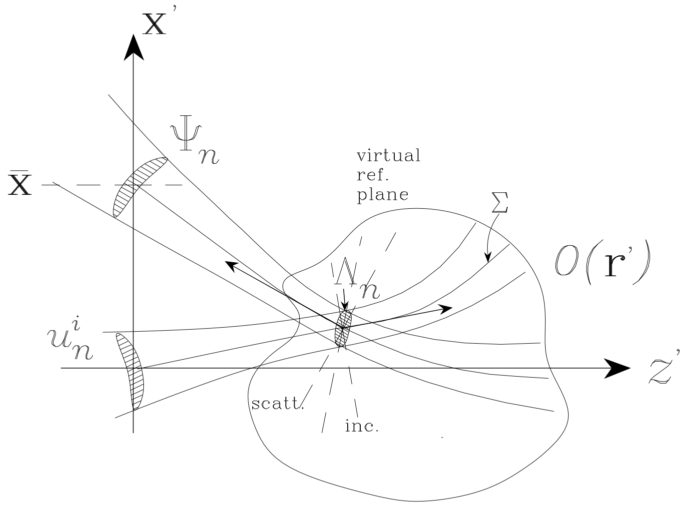

Fig. 2. Data – Object relation in the local domain. The local transform synthesizes local reflections between isolated local cells, Λ_n, dynamically oriented and located according to the phase-space processing parameters $(\bar{\mathbf{x}}, \bar{t}, \bar{\boldsymbol{\xi}})$. The reflection angle between the incident wave and the spectrally reflected PB satisfies the Snell law.

This scattering cell is localized about the ray trajectory (where the imaginary part of the analytic function is zero) and peaks for $\bar{t} = 2(\int^{\sigma} d\sigma'/v_n(\sigma')$. This results identify the location of the scattering cell for a given processing time $\bar{t}$; i.e., the location is adjusted to the propagation time of the incident field along Σ to the scattering cell plus the time required for the propagation of the spectrally reflected PB to arrive at the data plane (along the same trajectory). Equations (10) and (11) establish the building blocks for the inverse scattering procedure in which the sampling operation is inverted to evaluate $O(\mathbf{r})$ from which the next $v_n n + 1$ is found. This operation may be carried out for large scatterers since it cam be shown that under appropriate illuminating conditions, the operation in (10) mat be reduce to 1D sampling along Σ.

REFERENCES

[1] T. Melamed, E. Heyman, and L.B. Felsen, "Local spectral analysis of short-pulse-excited scattering from weakly inhomogeneous media: Part I–forward scattering," *IEEE Trans. Antennas Propagat.*, AP-47:1208–1217, July,1999.
[2] T. Melamed, E. Heyman, and L.B. Felsen, "Local spectral analysis of short-pulse-excited scattering from weakly inhomogeneous media: Part II–inverse scattering," *IEEE Trans. Antennas Propagat.*, AP-47:1218–1227, July,1999.
[3] E. Heyman, "Pulsed beam propagation in an inhomogeneous medium," *IEEE Trans. Antennas Propagat.*, AP-42:311–319, 1994.
[4] T. Melamed, Y. Ehrlich, and E. Heyman, "Short-pulse inversion of inhomogeneous media: A time-domain diffraction tomography," *Inverse Problems*, 12:977–993, 1996.