Modal Properties of Asymmetric Planar Chiro-Anysotropic Optical Waveguides

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The number-analytical method of solution of modal problem is presented with interpretations of some novel physical phenomena. It is shown that fields of the modes are elliptically polarized inhomogeneously in the cross-section of the waveguide. The integral polarization parameters with obvious graphical representation are used for the qualitative and quantitative discription of modes polarization and its transformations. The particular attention is paid to the fine structure of dispersion curves near the spectral points of quasi-degeneracy. In those areas, elliptical polarization of the modes converts: the right handed elliptical polarization changes into left handed and on the contrary.

INTRODUCTION

The asymmetric planar dielectric waveguide (Fig. 1) with chiro-anysotropic guiding layer is considered in this paper. The optical axis is codirectional with ox. The substrate and upper medium are isotropic dielectrical media. The wave propagation takes place along oz axis.

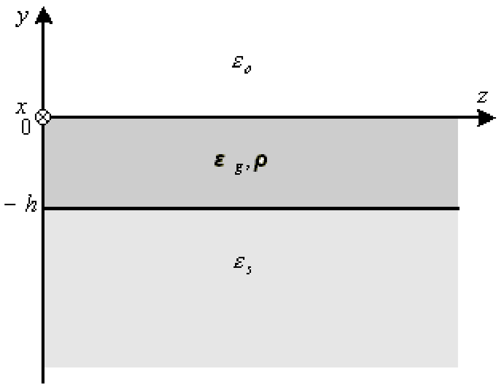

Fig. 1

THEORY

The permittivity and chirality tensors of the guiding layer are

$$\varepsilon_g = \varepsilon_g \begin{pmatrix} 1+e & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_\perp & 0 & 0 \\ 0 & \rho_\parallel & 0 \\ 0 & 0 & \rho_\parallel \end{pmatrix}, \tag{1}$$

By the generalization of the basic theory of chiral isotropic media [1] and results of [2-4] the system of dispersion equations for modes of this waveguide can be derived as

$$W^+(b,V)W^-(b,V) - \Delta_o^2 W^o(b,V) = 0\,, \tag{2}$$

$$W^\pm(b,V) = \left[\sqrt{a^2+b^2} + (1-\Delta_o)b\right]c_\pm + \left[b\sqrt{a^2+b^2} - (1-\Delta_o)c_\pm^2\right]\tan(c_\pm V), \tag{3}$$

$$W^o(b,V) = c_+ c_- \left[c_+ \tan(c_+ V) - b\right]\left[c_- \tan(c_- V) - b\right],$$

$$V = kh\sqrt{2\varepsilon_g \Delta_s}\ ,\ \gamma \cong kn_s\left(1+\Delta_s b^2\right),\ c_{\pm} = \sqrt{1+\sigma \mp \sqrt{\kappa^2 V^2 + \sigma^2} - b^2}\ , \tag{4}$$

$$k = \frac{\omega}{c}\ ,\ \Delta_{o,s} = \frac{\varepsilon_g - \varepsilon_{o,s}}{2\varepsilon_g}\ ,\ a^2 = \frac{\Delta_o}{\Delta_s} - 1,\ \sigma = \frac{e}{4\Delta_s},\ \kappa = \frac{\rho_\perp + \rho_\parallel}{h\sqrt{2\Delta_s^3}}\ ,$$

ω is frequency, c is the light velocity, γ is the propagation constant.

As it was shown in [3] the analysis of dispersion equation (2) can be carried out as a sequential analysis. The value of the term with Δ_o^2 in (2) is small for the optical waveguides with $\Delta_s << 1$. And it should be taken into consideration only for the fine structure of the dispertion curves near the points of spectral quasi-degeneracy [5]. So in an initial stage it can be ignored. Therefore (2) is turned into the following system of mode dispersion equations: $W^{\pm}(b,V) = 0$, which can be transformed to

$$c_{\pm} V = \arctan\left[\frac{b}{c_{\pm}}\right] - \arctan\left[\frac{(1-\Delta_o)c_{\pm}}{\sqrt{a^2 + b^2}}\right] + (2m+1)\frac{\pi}{2}, \tag{5}$$

with $m = 0,1,2,3,...$ The modes that satisfy the solution of (5) describe two classes with right- and left-handed elliptical polarization. The dispersion curves cross one another according to the approximate analysis in the spectral points of supposed spectral degeneracy (b_q, V_q), obtained from the joint solution of (5). It was shown in [5] that those points are the points of quasi-degeneracy only. In order to examine the fine structure of dispersion curves it is enough to seach for the exact solution of the initial dispersion equations only near those points, where (2) can be approximately presented as

$$\begin{aligned} &\left[(b-b_q)A^+ + (V-V_q)B^+\right]\left[(b-b_q)A^- + (V-V_q)B^-\right] - \\ &- \Delta_o^2\left[W^o(b_q,V_q) + (b-b_q)A^o + (V-V_q)B^o\right] = 0, \end{aligned} \tag{6}$$

$$A^{\pm} = \frac{dW^{\pm}(b,V)}{db},\ B^{\pm} = \frac{dW^{\pm}(b,V)}{dV},\ A^o = \frac{dW^o(b,V)}{db},\ B^o = \frac{dW^o(b,V)}{dV} \tag{7}$$

with $b = b_q$, $V = V_q$.

MODAL DISPERSION

The dispertion curves that correspond to (5) with $\Delta_o = 0{,}25$, $a = 10$, $\sigma = 1$, $\kappa = 0{,}2$ and local curves near the points of quasi-degeneracy of modes EP_o^+ and EP_1^- and modes EP_o^+ and EP_2^- are represented in Fig. 2. It is demonstrated that curves do not cross one another, they just close up.

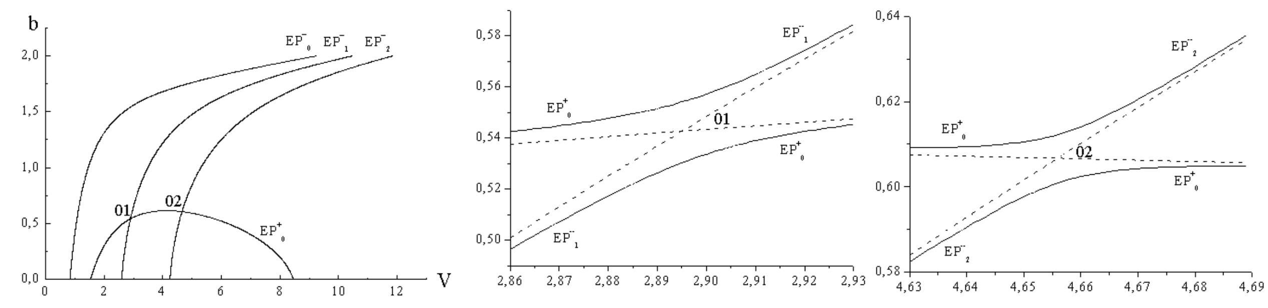

Fig. 2

POLARIZATION PROPERTIES

As it is shown in [5,6] fields of the modes are inhomogeneously polarized in the cross-section of the waveguide. The effective integral polarization parameters [7] with obvious graphical representation are used for the qualitative and quantitative description of modes polarization:

$$\overline{\Phi} = \frac{1}{h}\int_{-h}^{0}\Phi(y)dy\ ,\quad \overline{S} = \frac{1}{h}\int_{-h}^{0}S(y)dy\,, \tag{8}$$

with $\Phi(y)$, $S(y)$ - distribution functions of the polarization coefficients in the cross-section of the waveguide, here in the guiding layer. The first one is a polarization orientation coefficient: it is equal to the angle of inclination of yhe great axis of polarization ellipse to the ox axis. And the second one is a coefficient of ellipsity of polarization ellipse, its modulus is equal to the ratio of small and great ellipse axes and its sign shows the trend of $\vec{E}$ rotation in the cross-section of the waveguide: right or left ellipses. Those coefficients relate to the field components as

$$\Phi(y) = \arctan\frac{|B|^2 - 1 + |B^2+1|}{2B'},\quad S(y) = \frac{|B|^2 + 1 - |B^2+1|}{2B''}, \tag{9}$$

$$B = B' + iB'' = \frac{E_y}{E_x}\,. \tag{10}$$

For lossless planar waveguide $B' = 0$, thus according to [7]

$$\Phi = \begin{cases} 0 \quad for \quad |B''| < 1 \\ \pm\dfrac{\pi}{2} \quad for \quad |B''| > 1 \end{cases},\qquad S = \begin{cases} B'' \quad for \quad |B''| < 1 \\ \dfrac{1}{B''} \quad for \quad |B''| > 1 \end{cases}, \tag{11}$$

where values $+\pi/2$ and $-\pi/2$ are degenerated, it means that they correspond to the same angle of inclination $\pi/2$. The $\overline{S}$ values calculated along the dispersion curves near the point of quasi-degeneracy (*02*) are presented in the Fig. 3. The polarization is generally elliptical $\left(|\overline{S}| < 1\right)$ and particularly linear $\left(\overline{S} = 0\right)$. It is demonstrated that the right-handed elliptical polarization $\left(\overline{S} > 0\right)$ of the mode EP_o^+ becomes linear $\left(\overline{S} = 0\right)$ and than changes into left handed of the mode EP_2^- $\left(\overline{S} < 0\right)$. The inverse process of convertion of EP_2^- into EP_o^+ is represented in Fig. 4.

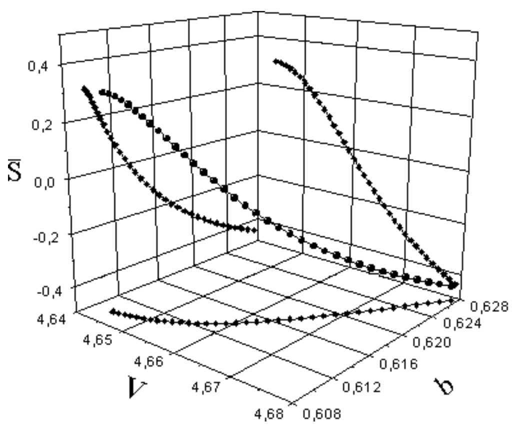

Fig..3

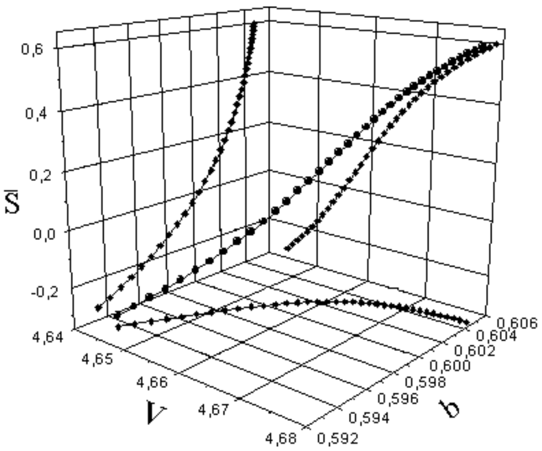

Fig.4

The analysis of the angle of inclination $\overline{\Phi}$ has some peculiarities for $B' = 0$. Expressions (9) given in [7] for the plane waves where local and integral value $\overline{\Phi}$ coincide. As far as analysis shows in case of waveguide modes there is a feature for $\overline{\Phi}(B' = 0) = \overline{\Phi}_o$, that is

$$\overline{\Phi}_o = \begin{cases} 0 & for \ \left|\overline{\Phi}\right| < \dfrac{\pi}{4} \\ \pm\dfrac{\pi}{2} & for \ \left|\overline{\Phi}\right| > \dfrac{\pi}{4} \end{cases} \qquad (12)$$

So for the process in the Fig. 3 for $\overline{S} = 0$ the linear polarization is horizontal $\overline{\Phi}_o = 0 \ (E_x \neq 0, E_y = 0)$, while in the Fig. 4 it is vertical $\overline{\Phi}_o = \pm\pi/2 \ (E_x = 0, E_y \neq 0)$.

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