ACCELERATION OF ELECTRONS IN FIELD-ALIGNED DENSITY IRREGULARITIES BY TRAPPED CYLINDRICAL UPPER HYBRID OSCILLATIONS

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ABSTRACT

A high frequency pump wave of ordinary mode polarization in the ionosphere creates small scale filamentary density depletions stretched along the geomagnetic field. By scattering off these irregularities the electromagnetic wave generates cylindrical UH waves with azimuthal wave numbers, which can be trapped in the density depletions. The electric field of such UH oscillations rotates in the direction of the gyro motion of electrons, which leads to electron acceleration transverse to the geomagnetic field. These accelerated electrons can excite atmospheric atoms and molecules which produce the enhanced airglow observed in pump–ionosphere interaction experiments at high latitudes.

INTRODUCTION

An electromagnetic wave propagating in the ionosphere polarizes the plasma. The resulting polarization $\mathbf{P}$ is proportional to the electric field $\mathbf{E}$ of the wave. For a high frequency ($\omega \simeq \omega_{pe}$) ordinary mode wave $\mathbf{P}$ is determined by the motion of the magnetized electrons

$$\mathbf{P}_\perp = -\frac{1}{4\pi}\frac{\omega_{pe}^2}{\omega(\omega+\omega_{ce})}\mathbf{E}_\perp . \tag{1}$$

The index '$\perp$' denotes the component orthogonal to the geomagnetic field. A nonuniform $\mathbf{P}$ excites an electron charge density ϱ,

$$\varrho = -\nabla\cdot\mathbf{P} = \frac{\omega_{pe0}^2}{4\pi\omega(\omega+\omega_{ce})}\nabla\cdot\left[(1+\eta)\mathbf{E}_\perp\right] . \tag{2}$$

Here η is the relative electron concentration, $1+\eta = n_e/n_0$ and $\omega_{pe0}^2 = 4\pi n_0 e^2/m_e$. We see that any short scale inhomogeneity of the plasma density, or of the electromagnetic wave amplitude are sources of the oscillating charge density. When ω coincides with the eigen plasma mode frequencies, the electromagnetic wave efficiently excites the eigen plasma waves. Therefore, in the region of the upper hybrid (UH) resonance, where $\omega \simeq \omega_{UH} = (\omega_{pe}^2+\omega_{ce}^2)^{1/2}$, small scale plasma inhomogeneities, strongly elongated along the magnetic field due to the large parallel transport coefficients, generates UH waves. This effect is amplified by the trapping of UH oscillations in the density cavities, which minimizes the convective losses. As seen in (2), the source of UH wave generation is the scattering of the uniform electromagnetic waves by the density irregularities, $\varrho \propto \nabla\eta\cdot\mathbf{E}_\perp$. In addition, from (2) it is also seen that potential waves can be generated by small scale inhomogeneities of the electromagnetic field itself, $\varrho \simeq (1+\eta)\nabla\cdot\mathbf{E}_\perp$. Such inhomogeneities can arise by the diffraction of the transverse wave on the field-aligned cylindrically symmetric plasma density depletions near the UH resonance. This problem was discussed by Istomin and Leyser [1], who showed that the initially uniform wave beam, scattered by the field-aligned striations, decays into bunches of size L, where

$$L = \bar{l}\left[\frac{c^2}{\omega\omega_{ce}}\overline{\left|\int \eta d\mathbf{r}_\perp\right|^{-1}}\right]^{1/2} . \tag{3}$$

Here $\bar{l}$ is the mean distance between the small scale striations and $\int \eta d\mathbf{r}_\perp$ is a dimensionless length density of a single striation. The overline denotes averaging over different depletions. For $\eta \simeq -0.1$, $\bar{l} \simeq 1$ m and the wave frequency $\omega \simeq 4\cdot 10^7$ s^{-1}, we have $L \simeq 30\bar{l}$. This means that inside one bunch there are on the order of 10^3 striations. Scattering of the electromagnetic wave by the striations results not only in the average field being divided into bunches, but also in small scale electromagnetic wave inhomogeneities.

SCATTERED FIELD

The diffraction of an ordinary mode wave is described by the parabolic equation [1]

$$-L^{-2}E_0 + \triangle_\perp E_0 - \frac{2\omega\omega_{ce}}{c^2}\left(\sum_i \eta_i\right) E_0 = 0. \tag{4}$$

In average the third term in (4) cancels the first term, and the wave field is uniform on the scale L. However, there are also the field irregularities due to scattering of the wave field by the density cavities. This is seen from the integral form of (4)

$$E_0(\mathbf{r}_\perp) = \frac{2\omega\omega_{ce}}{c^2}\sum_i \int G(\mathbf{r}_\perp, \mathbf{r}'_\perp)\eta_i(\mathbf{r}'_\perp)E_0(\mathbf{r}'_\perp)d\mathbf{r}'_\perp. \tag{5}$$

The function $G(\mathbf{r}_\perp, \mathbf{r}'_\perp)$ is the Green function of the equation $(\triangle_\perp - L^{-2}) = 0$,

$$G(r_\perp, \phi, r'_\perp, \phi') = -\frac{1}{2\pi}\sum_{m=-\infty}^{\infty} \exp[im(\phi - \phi')]I_m(r_{\perp<}/L)\cdot K_m(r_{\perp>}/L), \tag{6}$$

where I_m and K_m are the modified Bessel functions of the mth order, $r_{\perp<} = \min(r_\perp, r'_\perp)$, and $r_{\perp>} = \max(r_\perp, r'_\perp)$. The term for $m = 0$ in (6) gives the average distribution of the field $\bar{E}_0$ in the region $r_\perp \simeq L$. Terms for $m \neq 0$ describe the field irregularities $\tilde{E}_0$, which are less than $\bar{E}_0$, as we will see below. Iterating (5), we can write

$$\tilde{E}_{0j}(\mathbf{r}_\perp) = \frac{\omega\omega_{ce}}{\pi c^2}\bar{E}_0(\mathbf{r}_\perp)\sum_{m\neq 0} I_m(r_\perp/L)e^{im\phi}\sum_{i\neq j} K_m(r_{\perp i}/L)e^{-im\phi_i}\int |\eta_i| d\mathbf{r}_\perp. \tag{7}$$

Expression (7) is the wave field near the jth density irregularity with the coordinate origin in its center and $r_\perp < \bar{l}$. We neglect in the sum (7) the contribution of the field scattered by its own density fluctuation due to $|\eta_j| l_j^2 \omega\omega_{ce}/c^2 << 1$. The perturbed field $\tilde{E}_0(\mathbf{r}_\perp)$ is the sum of the waves scattered by all density fluctuations. Because of $r_\perp < \bar{l} << L$ we can expand the Bessel functions over their arguments

$$\tilde{E}_{0j}(\mathbf{r}_\perp) = \bar{E}_0 \sum_{m\neq 0}\frac{1}{2m^2}\left(\frac{r_\perp}{\bar{l}}\right)^{|m|}\exp(im\phi)a_{j,m}, \tag{8}$$

where $a_{j,m}$ is the amplitude of the mth harmonic of the wave field given by

$$a_{j,m} = \frac{\sum_{i\neq j} 2m^2(\bar{l}/2L)^{|m|}K_m(r_{\perp i}/L)e^{-im\phi_i}\int |\eta_i| d\mathbf{r}_\perp}{\sum_{i\neq j}\int |\eta_i| d\mathbf{r}_\perp}. \tag{9}$$

The value of $a_{j,m}$ depends on the position of the point 'j' among the scattering centers. We have $a_{j,m} = 0$ in the center of symmetry and $a_{j,m}$ approaches unity on the periphery. Further, we see that the amplitude of the mth harmonic of the perturbed field is maximum at $r_\perp \simeq \bar{l}/2$ and is at least $2^{|m|+1}m^2$ times less than the amplitude of the uniform field $\bar{E}_0$. This field creates the charge density $\varrho_m \propto \nabla\cdot\tilde{\mathbf{E}}_m$,

$$\nabla\cdot\tilde{\mathbf{E}} = \sum_{m>0}\frac{1}{2^{1/2}m\bar{l}}\left(\frac{r_\perp}{\bar{l}}\right)^{m-1} e^{i(m-1)\phi}|\bar{E}_0|a_{j,m}. \tag{10}$$

The characteristic scale of these density variations is $\bar{l}$, which is the distance between striations. It is important to note that the charge density ϱ_m exists only for $m > 0$. For $m < 0$, the derivatives with respect to $r_\perp$ and ϕ cancel each other. This is a very important property, namely, that $\tilde{E}$, which results from the scattering of the ordinary mode wave by the field-aligned density inhomogeneities, rotates in the ϕ-direction in the same sense as the gyrating electrons $\Omega_m = \omega/(m-1)$, $m > 1$, and can therefore resonantly accelerate the electrons.

CYLINDRICAL UH WAVES

Now we discuss the excitation of the UH waves having a cylindrical structure. The electric potential for these waves is $\Psi_s = \varphi(r_\perp, t)\exp\{-i\omega t + is\phi\}$. The equation for the cylindrical UH waves is

$$\left(i\frac{\partial}{\partial t} - \Delta\omega_{UH} + \frac{3\omega_{ce}^2}{2\omega_{pe}}\rho_e^2\triangle_\perp + i\frac{\nu_2}{2} - \frac{1}{2}\omega_{pe}\eta\right)\triangle_\perp\varphi_s = -\frac{1}{2}\omega_{pe}\nabla\cdot\left[(1+\eta)\tilde{\mathbf{E}}_{s+1}\right], \tag{11}$$

where $\triangle_\perp$ is the Laplacian in the cylindrical coordinates $r_\perp, \phi$, $\Delta\omega_{UH} = \omega_{UH} - \omega$, and we took into account that near the UH resonance $\omega_{pe}^2 >> \omega_{ce}^2$. The right-hand side of (11) consists of the three terms $\nabla\eta \cdot \tilde{\mathbf{E}}_{s+1}$, $\nabla \cdot \tilde{\mathbf{E}}_{s+1}$, and $\eta\nabla \cdot \tilde{\mathbf{E}}_{s+1}$. The first term is s times less than the third because we consider the excitation of cylindrical waves by given cylindrically symmetric density cavities $\eta_i(r_\perp)$. The second term $\nabla \cdot \tilde{\mathbf{E}}_{s+1}$ describes the excitation of potential waves of the scale $\simeq \bar{l}$. Due to this we can neglect in the left-hand side of (11) the term $\omega_{pe}\eta$.

The short scale source $\eta\nabla \cdot \tilde{\mathbf{E}}_{s+1}$ in (11) results in the excitation of cylindrical UH waves, associated with the density striations. The stationary solution of the equation with this driving term is a wave trapped in the radial direction in the pre-formed density cavity. Near the bottom of the cavity of magnitude $\eta_0 < 0$, the solution is given by a Bessel function while outside the cavity the solution must be a MacDonald function. To match the different solutions we require that the first maximum of the Bessel function must be near the edge of the cavity

$$\left(|\eta_0| \frac{\omega_{pe}^2}{3\omega_{ce}^2}\right)^{1/2} \simeq s\rho_e. \tag{12}$$

Because a self-consistent cavity satisfies the condition [2]

$$|\eta_0| \left(\frac{l}{\rho_e}\right)^2 \propto (2n+1)^2, \tag{13}$$

where $(2n+1)$ is the number of zeros of the main symmetric UH wave of amplitude $\simeq |E_0|$, trapped in the density depletion, it turns out that $s \simeq 2n+1$. It is very natural that the cylindrical UH wave with index s is trapped by the cavity containing the radial UH wave with $2n+1$ radial wave number. The explicit numerical solution for a cavity of Gaussian shape shows growth of the value of $\triangle_\perp \varphi_s$ inside the cavity, and then exponential decrease for $r_\perp > l$. Analytic expressions for the fields inside the cavity have been found. The potential ϕ_s reaches its maximum value at the edge of the cavity where $r_\perp \simeq l$. In the cylindrical UH waves trapped in the density depletion the electric field rotates in the positive direction, i.e., in the same direction as the electron cyclotron rotation. Thus, the UH field can efficiently accelerate electrons inside the cavities.

ELECTRON ACCELERATION

First we consider the acceleration of electrons in the center of a cavity. Electrons see the electric fields E_ϕ, E_r on their orbits $r_\perp = \rho_c = v_\phi/\omega_{ce}$, $\phi = \omega_{ce}t + \phi_0$. The equations for the electron motion are

$$\frac{dv_r}{dt} = \frac{v_\phi^2}{r_\perp} - v_\phi\omega_{ce} - \frac{e}{m}E_r - \nu v_r, \tag{14}$$

$$\frac{dv_\phi}{dt} = -\frac{v_r v_\phi}{r_\perp} + v_r\omega_{ce} - \frac{e}{m}E_\phi - \nu v_\phi. \tag{15}$$

Here $v_\phi^2/r_\perp$ and $-v_r v_\phi/r_\perp$ are the centrifugal and Coriolis accelerations of the particle. We include also the frictional force acting on the fast particle by the thermal electrons and ions with the collision frequency $\nu \propto \mathcal{E}^{-3/2}$, where $\mathcal{E}$ is the particle energy. The radial electric field E_r gives an additional drift in the ϕ-direction, $\delta v_\phi = eE_r/m\omega_{ce}$, i.e., $v_\phi = r_\perp\omega_{ce} + eE_r/m\omega_{ce}$. Thus, the sum of the three terms for the radial motion in (14) is equal to zero and we can consider $v_r = 0$. The solution of (15) is

$$v_\phi = -\frac{ie}{m} \cdot \frac{E_\phi}{\omega - s\omega_{ce} + i\nu}. \tag{16}$$

The equation for the particle acceleration then gives

$$\frac{d\mathcal{E}}{dt} = -\nu\mathcal{E} - \frac{1}{2}e\left(E_\phi v_\phi^* + c.c.\right) = -\nu\mathcal{E} + \frac{e^2}{m} \cdot \frac{\nu|E_\phi|^2}{(\omega - s\omega_{ce})^2 + \nu^2}, \tag{17}$$

where $\mathcal{E} = m(v_\phi^2 + v_z^2)/2$. The maximum particle energy is not defined by the relation $d\mathcal{E}/dt = 0$ and $\mathcal{E}_{max} = e^2|E_\phi|^2/[(\omega - s\omega_{ce})^2 + \nu^2]m$. Indeed, the accelerating electric field E_ϕ grows with the distance $r_\perp$ from the center of the cavity as $r_\perp^{s+1}$, or $|E_\phi|^2 \propto \rho_c^{2(s+1)} \propto \mathcal{E}^{s+1}$. The rate of acceleration increases with increasing particle energy. This implies that the maximum energy depends on the duration of the acceleration, i.e., on the time during which the particle is inside the depletion of a finite parallel scale L_z. We see also from (17) that the acceleration has a resonant character

and is extremal for $s = [\omega/\omega_{ce}]$, where [...] denotes the integer part. Thus, for the particle acceleration in the density cavity it is most efficient to use the pump frequency ω close to an electron cyclotron harmonic ($\omega \simeq s\omega_{ce}$). However, we know that the excitation of small scale density irregularities is suppressed when $|\omega - s\omega_{ce}| < \omega_{LH}$ due to appearance of Bernstein modes. The maximum acceleration effect therefore has to be observed when $|\omega - s\omega_{ce}| \simeq \omega_{LH}$, where $\omega_{LH} = (\omega_{ce}\omega_{ci})^{1/2}$ is the lower hybrid frequency.

We have discussed above the acceleration of electrons located in the center of the cavity. In addition we can calculate the acceleration of electrons located the distance $r_\perp >> \rho_c$ from the center of the cavity. Since we know the rate of the particle acceleration in the ϕ-direction

$$\frac{dv_\phi}{dt} = \frac{1}{mv_\phi}\frac{d\mathcal{E}}{dt},$$

we can determine the distribution function $f(v_z, v_\phi, \phi)$ of the fast particles from

$$v_z\frac{\partial f}{\partial z} + \frac{\partial}{\partial v_\phi}\left(\frac{dv_\phi}{dt}f\right) = 0. \tag{18}$$

Taking into account the dependence of the collision frequency ν on the velocity $\nu = \nu_T(v_T/v_\phi)^3$ and the amplitude of a cylindrical UH wave also on v_ϕ, $|E_\phi| \propto v_\phi^{s+1}$, we find an analytic expression for f. The function f is the maxwellian distribution function for $v_\phi < v_\phi^*$ and the power-law function for $v_\phi > v_\phi^*$,

$$f = \frac{n_e}{(2\pi)^{3/2}v_T^3}\left[\frac{v_\phi^*}{v_\phi}\right]^{2s-2}\exp\left\{-\frac{m(v_z^2 + v_{\phi 0}^2)}{2T_e}\right\}, \tag{19}$$

where

$$v_\phi^* = v_T\left[\frac{|v_z|}{z\nu_T}\right]^{\frac{1}{2s-3}}\left\{\frac{288(s+1)^4}{s^2(2s-3)}\frac{4\pi n_e T_e}{|\bar{E}_0|^2}\left[\frac{\omega_{ce}^2(\omega - s\omega_{ce})}{\omega_{pe}^3}\right]^2|a_{s-1}|^{-2}\right\}^{\frac{1}{2s-3}}\left(\frac{\bar{l}}{\rho_{cT}}\right)^{\frac{2s+2}{2s-3}}. \tag{20}$$

Finally, at small parallel velocities v_z there exists a region in the velocity space where the electron distribution function has a power-law tail, so that $f \propto v_\phi^{-(2s-2)}$.

CONCLUSIONS

Electrons accelerated by the proposed mechanism are suggested to explain the enhanced airglow observed in experiments at high latitudes in which a powerful ordinary mode pump wave is transmitted nearly vertically into the F-region ionosphere [3,4]. The occurrence of the airglow together with anomalous heating, and therefore anomalous absorption, indicates that the airglow is enhanced by UH turbulence [4]. This is also consistent with the minimum in airglow enhancement observed for pump frequencies near an harmonic of the electron cyclotron frequency [5]. Analysis of experimental results suggest that the airglow is not enhanced by thermal electrons but instead by accelerated electrons [6].

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