THE PONDEROMOTIVE ACTION OF POWERFUL ELECTROMAGNETIC WAVE ON IONOSPHERIC PLASMA

Ya. N. Istomin[1]

[1] ***P.N. Lebedev Physical Institute, Leninsky Prospect 53, 119991, Moscow, Russia.***
E-mail: istomin@td.lpi.ac.ru

ABSTRACT

The ponderomotive force acting on the electrons in a cold magnetized ionospheric plasma is discussed. This force is the result not only of the change in the wave energy density but also its energy flux, momentum density and angular momentum density. The response of the plasma to this ponderomotive force is also discussed. The calculated transverse electric field and plasma motion are compared with measurements during HF pumping. Also it is suggested to use for ionospheric experiments an electromagnetic beam with a spiral wave front, which increases the electric currents in the region of the wave reflection.

INTRODUCTION

There are many nonlinear effects which arise when a powerful high frequency ordinary mode electromagnetic wave propagates in the F region of the ionosphere. These include anomalous absorption of electromagnetic waves, anomalous heating of the plasma and stimulation of electromagnetic emission. They are connected with excitation of plasma waves near the reflection level and origin of the different plasma density irregularities stretching along the geomagnetic field. Among these nonlinear effects there is also the polarization of the ionospheric plasma and its motion under the action of a powerful wave. When the electromagnetic wave propagates in the media it acts on it via a force known as the ponderomotive force [1]. The wave possesses energy density $\mathcal{E}$, energy flux $\mathbf{S}$, momentum density $\mathbf{P}$ and angular momentum density $\mathbf{M}$. All are proportional to the square of the wave amplitude. But in the propagation media which is not a vacuum it is reasonable only to consider conservation of electromagnetic energy only, because the wave and the medium can exchange momentum and angular momentum through the pondromotive force.

PONDEROMOTIVE FORCE

Further we will use Euler equations for electrons and ions motion. Thus, we calculate the mean force $\mathbf{f}_e$ acting on an electron

$$\mathbf{f}_e = \langle -m_e(\mathbf{v}\cdot\nabla)\mathbf{v} - e\mathbf{v}\times\tilde{\mathbf{B}}\rangle. \tag{1}$$

In the general case the ponderomotive force (1) consists of the three different terms:

1) a gradient of some scalar

$$f^1_{e\alpha} = \frac{\epsilon_0}{4n_e}\nabla_\alpha\left(\delta\epsilon_{\beta\gamma}E_{0\gamma}E^*_{0\beta}\right); \quad \delta\epsilon_{\beta\gamma} = \epsilon_{\beta\gamma} - \delta_{\beta\gamma}, \tag{2}$$

which is close to part of the wave energy density $-\omega_p^2\mathcal{E}/2\omega^2$, but does not equal it;

2) an anisotropy force, which is proportional to the space derivatives of the vector $\mathbf{E}_0$

$$f^2_{e\alpha} = -\frac{\epsilon_0}{4n_e}\left\{\left(\frac{\omega^2}{\omega_p^2}\delta\epsilon_{\beta\alpha} + \delta_{\beta\alpha}\right)\left[\delta\epsilon_{k\gamma}\frac{\partial}{\partial r_k}(E_{0\gamma}E^*_{0\beta}) + E^*_{0\beta}\nabla\cdot\mathbf{E}_0\right]\right\} + c.c.; \tag{3}$$

3) a force, which is proportional to the wave vector $\mathbf{k}$ and does not depend on the derivatives of the amplitude of the wave field $\mathbf{E}_0$

$$f^3_{e\alpha} = -\frac{i\epsilon_0}{4n_e}\left(\frac{\omega^2}{\omega_p^2}\delta\epsilon_{\beta\alpha} + \delta_{\beta\alpha}\right)E^*_{0\beta}\quad \mathbf{k}\cdot\mathbf{E}_0 + c.c. \tag{4}$$

The third term exists only in a gyrotropic medium. This force is directed perpendicular to the magnetic field $\mathbf{B}$ and to the wave vector $\mathbf{k}$, $\mathbf{f}_e^3 \propto \mathbf{k} \times \mathbf{B}$. This term originates from two factors: gyrorotation of the electrons and rotation of the wave polarization, $E_y \propto iE_x$. The spin–spin interaction of the medium particles and the wave field results in a force acting on the medium.

Any change of the energy flux $\mathbf{S}$ in space results in a force acting on the medium. It occurs in an inhomogeneous medium, when the wave trajectory is not straight line. If the flux energy vector $\mathbf{S}$ turns without changing it's absolute value, then the change of $\mathbf{S}$ in neighboring points, separated by Δl along the trajectory is $\Delta\mathbf{S} = -\mathbf{n}(S/\rho)\Delta l$, where $\mathbf{n}$ is the unit vector normal to the trajectory and ρ is it's radius of curvature. The flux of the energy $\mathbf{S}$ is proportional to the energy density $\mathcal{E}$ and, by definition, this is the group velocity $\mathbf{v}_g$, $\mathbf{S} = \mathbf{v}_g\mathcal{E}$. The wave momentum density $\mathbf{P}$ is directed along the flux $\mathbf{S}$, $\mathbf{P} = \mathbf{S}/v_g^2$. Thus, $\Delta\mathbf{P} = \Delta\mathbf{S}/v_g^2$ and the ponderomotive force, $n_e\mathbf{f}_e = -\Delta\mathbf{P}/\Delta t$, is

$$\mathbf{f}_e^c = \mathbf{n}\frac{\mathcal{E}}{n_e\rho}. \tag{5}$$

For a small enough radius of curvature of the trajectory ρ this part of the ponderomotive force $\mathbf{f}_e^c$ becomes large. It has a orthogonal to the magnetic field component, as well as longitudinal one. We will see below that this fact is very important.

Apart from the ponderomotive force discussed above existing in an transparent medium there are also forces due to absorption and reflection in matter. Because the O-mode wave reflects in the ionosphere from the level where the local plasma frequency $\omega_p(\mathbf{r})$ coincides with the wave frequency ω, the electrons in the plasma are forced by the reflecting electromagnetic beam near this region

$$\mathbf{f}_e^r = \cos\alpha(1+R)\frac{\mathcal{E}}{n_e d}\mathbf{h}. \tag{6}$$

Here $\mathbf{h}$ is the unit vector normal to the surface of constant electron density, α is the angle between the flux energy vector and this direction, R is the coefficient of reflection of the wave flux and d is the effective width of a plasma layer, reflecting the electromagnetic wave. For linear dependence of the plasma density on the height h, $\omega_p^2 - \omega^2 = \omega^2 h/H$, the value of d is of $d \simeq (\lambda^2 H)^{1/3}/2$. λ is the wave length of the incident wave. For $\lambda \simeq 60m$ and $H \simeq 50km$, $d \simeq 200m$ (see [2])

The plasma ordinary mode is elliptically polarized. It's electric field in the plane perpendicular to the geomagnetic field, rotates in the direction opposite to the gyrorotation of the electrons. For longitudinal propagation $k_x = 0$, the "O" wave is purely circularly polarized, $\alpha_y = -1$. Polarized light in vacuum possesses angular momentum. The spin of each photon is equal to $\hbar$. If the density of the photons is n_{ph}, then the density of the spins is $\hbar n_{ph}$. On the other hand, the density of the photons is their density energy divided by $\hbar\omega$. Thus, the spin density is $\mathcal{E}/\omega$. But this is not the density of the angular momentum $M_\parallel$ along the propagation path of the light. If the energy density $\mathcal{E}$ is constant in the direction perpendicular to $\mathbf{k}$, then rotation in neighboring spins conceals. For a cylindrically symmetric electromagnetic beam, when the value of $\mathcal{E}$ depends only on r, the cylindrical radius, $M_\parallel = -\partial(r\mathcal{E}/\omega)/\partial r$. If the light is absorbed, then the angular momentum transfers to the medium. We denote the imaginary part of the wave vector as $\kappa = Im(k)$. Then this force acts also on the electrons

$$\mathbf{f}_e^a = -\frac{\epsilon_0\kappa}{kn_e}\frac{1}{r}\frac{\partial}{\partial r}\left(r|\alpha_y||E_x|^2\right)\mathbf{i}_\phi \simeq \frac{\epsilon_0\kappa|\alpha_y|}{kn_e}\frac{|E_x|^2}{\Delta L}\mathbf{i}_\phi; \quad \alpha_y = -i\frac{E_y}{E_x}, \tag{7}$$

and directed in the plane perpendicular to the ambient magnetic field along the azimuthal unit vector $\mathbf{i}_\phi$. Such a force exists in the region where κ is not equal to zero, presumably near the reflection level, where abnormal absorption is observed. Thus, we see that in an anisotropic medium, such a plasma in a magnetic field, several forces exist that are greater than usually used for estimates the simple expression $f \simeq \nabla\mathcal{E}/n_e \simeq \mathcal{E}/n_e L$, which is valid only for a isotropic, uniform and nonabsorbing medium.

PLASMA MOTION

The force acting on the electrons in the plasma causes them to move. The plasma polarizes, an electric field $-\nabla\varphi$ appears, and the plasma ions also begin to move. The neutral atmosphere can also be involved in the motion. But the neutral gas viscosity η is high, $\eta/L^2 \gg \nu_{in}$ (ν_{in} is the frequency of collisions of ions with atoms), that we can consider the gas velocity to be zero, $\mathbf{v}_n = 0$. The electrons and ions collide with each other and also with atoms. Under the stationary conditions we can find the ponderomotive action of the electromagnetic wave in a magnetized plasma resulting in it's polarization and appearance of a electric field

$$e\varphi = -\frac{\gamma_i T_i}{\gamma_e T_e + \gamma_i T_i}\int^z f_{e\parallel}dz' + w\int^z\int\left(\nabla\times\mathbf{f}_{e\perp}\right)_z dz'dz''. \tag{8}$$

The plasma motion is

$$\mathbf{v}_{i\perp} = -w\frac{1}{eB^2}\int^z\int \nabla_\perp\left(\nabla\times\mathbf{f}_{e\perp}\right)_z dz'dz''\times\mathbf{B}. \tag{9}$$

The value of w is the ratio of Hall electron conductivity to the longitudinal conductivity, $w = \sigma_\wedge/\sigma_\|$. The electric current perpendicular to the ambient magnetic field also appears

$$\mathbf{j}_\perp = -\frac{n_e}{B^2}\left(\int^z \nabla_\perp f_{e\|}dz' - \mathbf{f}_{e\perp}\right)\times\mathbf{B}. \tag{10}$$

This excites the additional magnetic field as longitudinal, as azimuthal.

DISCUSSION

First one can emphasize that the plasma motion across the magnetic field is not equal to the electric drift as is seen from the expressions (8,9). This is true only when the ponderomotive force has no component along the magnetic field, $f_{e\|} < wf_{e\perp}$. The longitudinal component of the ponderomotive force $f_{e\|}$ generates the electric potential $e\varphi \simeq -f_{e\|}L_\|$ but does not induce plasma motion. The transverse component of the ponderomotive force induces an electric potential and a plasma motion, but they are small by the factor $w(L_\|/L_\perp)$. This explains the observations of the electric field made by the rocket over the Arecibo heating facility [3], and the plasma drift motion measured by Bernhard *et al.* [4]. If we attribute the observed electric field to the plasma crossed drift it turns out to be as high as $100m/sec$. The observed motion is at least ten times less. From that we can conclude that the parameter $w(L_\|/L_\perp) \leq 0.1$. We will see below the ratio $L_\|/L_\perp \simeq 50$, and we can estimate the value of w, $w \leq 2\cdot 10^{-3}$. In a magnetized turbulent plasma the effective frequency of scattering of electrons by waves can be estimated as $\nu_{eff} \simeq \omega_c(\mathcal{E}_T/n_eT_e)$, where $\mathcal{E}_T$ is the energy density of turbulent electromagnetic waves. Considering $n_eT_e \simeq 3\cdot 10^{-9}Jm^{-3}$ we obtain $\mathcal{E}_T \leq 6\cdot 10^{-12}Jm^{-3}$. This gives an estimate for the turbulent electric field $E_T = (2\epsilon_0^{-1}\mathcal{E}_T)^{1/2} \leq 1V/m$. Thus, from our estimate it follows that the turbulent field is of the order of the field of the pumping electromagnetic wave $E_0 \simeq 1V/m$. That is believed to be very natural.

Far from the reflecting region, below the wave amplitude maximum on the distance of several d, the largest ponderomotive force is f_e^3 (4) because it is inversely proportional to the wave length, but not to the beam's scale. For $n_e = 2\cdot 10^{11}m^{-3}$, $\omega_c/\omega \simeq 0.2$, $E_x \simeq E_0 \simeq 1V/m$, $\lambda \simeq 60m$ the ponderomotive force is of $5\sin\theta 10^{-25}N$. This is not small - for an electron it gives a large acceleration $a \simeq 5\sin\theta 10^5 m/sec^2$. However, the generated transverse electric field is small $E_\perp \simeq 3w\sin^{-1}\theta 10^{-6}V/m$ because it is purely transverse to the magnetic force.

Another transverse ponderomotive force f_e^a (7) is the force owing to absorption of wave's angular momentum. But it is smaller than usual striction force f_e^1 (2) by a factor $\kappa/k < 1$. It is possible to enhance the wave's angular momentum by generating the beam with a spiral wave front [5], not flat as usually used. For that the fields in the beam have to have the dependence on the azimuthal angle ϕ - $\exp\{il\phi\}$. Then the total angular momentum for a circular polarized wave becomes proportional to $1 + l$.

Finally, the highest ponderomotive force is that which arises at the reflection of the wave when it transfers it's momentum to the plasma. It is larger than (10) by at least a factor of 2 (when the reflection coefficient R=0). It is proportional to the total energy density $\mathcal{E}$ but not the part of it. The forces (5) and (4) are a similar type taking into account the beam turning near the reflection level, $\mathbf{f}_e^r \simeq 2\mathbf{h}\mathcal{E}/n_ed$. This force has a longitudinal, as well as perpendicular component. For Arecibo they are of the same order ($180^\circ - \theta = 42^\circ$), but for Tromsø the longitudinal one is much greater ($180^\circ - \theta = 13^\circ$). The value of $\mathcal{E}$ in the reflection region grows significantly due to the slowing of the wave. The longitudinal electric field increases in several times. According to the calculations of the wave propagation for Arecibo and Tromsø made by Thidé and Lundborg [6, 2], $\mathcal{E} \simeq 20\mathcal{E}_0$. $\mathcal{E}_0$ is the wave's density energy of the electromagnetic wave of $1V/m$ amplitude, $\mathcal{E}_0 \simeq 5\cdot 10^{-12}Jm^{-3}$. The ponderomotive force is of $f_e^r \simeq 5\cdot 10^{-24}N$. The longitudinal component of this force produces according to formula (8) the transverse electric field $E_\perp \simeq f_e^rL/ed \simeq 1.5\cdot 10^{-3}V/m$. This is directed to the south. The east-west field is $d/L \simeq 50^{-1}$ times less. Just about the such electric field ($4mV/m$) observed in *in situ* measurements in Arecibo [3].

References

[1] L.D. Landau, E.M. Lifshitz, L.P. Pitaevskii, *Electrodynamics of continuous media*, 2nd ed., Pergamon Press, p. 59, 1984.

[2] B. Lundborg, B. Thidé, "Standing wave pattern of HF radio waves in the ionospheric reflection region 2. Applications," *Radio Science*, vol. 21, p. 486-500, 1986.

[3] W.J. Peria, M.C. Kelley,”Convection electric field observations near the Arecibo HF heater beam,” *JGR*, vol. 106, pp. 18,517-524, 2001.

[4] P.A. Bernhard, L.M. Duncan, C.A. Tepley, ”Artificial airglow excited by hight-power radio waves,” *Science*, vol. 242, pp. 1022-1027, 1988.

[5] M. Padgett, L. Allen, ”Light with a twist in its tail,” *Contemporary Physics*, vol. 41, pp. 275-285, 2000.

[6] B. Thidé, B. Lundborg, ”Structure of HF pump in ionospheric modification experiments. Linear treatment,” *Phys. Scripta*, vol. 33, pp. 475-479, 1986.