PLASMA DIELECTRIC FOR A DUSTY PLASMA WITH MASS DISTRIBUTION AND CHARGE FLUCTUATIONS

Michael A Raadu[1]

[1]*Royal Institute of Technology, Alfvén Laboratory, Division of Plasma Physics, SE-100 44 Stockholm, Sweden, E-mail: raadu@plasma.kth.se*

ABSTRACT

The dynamical charging of grains in a dusty plasma leads to wave damping. A distribution of grain sizes is generally expected in actual dusty plasmas, and the contribution of thermalised dust to the plasma dielectric may in special cases be equivalent to that for a kappa (generalised Lorentzian) distribution of mono-sized particles. The plasma electrostatic dispersion relation combining these effects is derived. An enhanced shielding is found.

CHARGING DYNAMICS WITH SIZE DISTRIBUTION

Plasma dust grains are assumed to have a thermal velocity distribution with temperature T, and size distribution $h(a)$ which has a power law for small radii, a, and for large masses, $m(a)$, decreases with an exponential factor [2]:

$$dn_d = f(a,v)\,da\,dv = h(a) f_T(v)\,da\,dv = Ca^{\beta} e^{-\alpha^3 a^3} \exp\left[-m(a)v^2/2kT\right] da\,dv \tag{1}$$

The distribution $h(a)$ has a maximum for a grain radius $a_m=(\beta/3)^{1/3}a^{-1}$ and tends to a delta-function at a_m (mono-sized distribution) if α and β go to infinity. A pertubation analysis for electrostatic modes using the linearised Vlasov and Poisson equations gives a dispersion relation in the form:

$$k^2 + \lambda_{De}^{-2} + \lambda_{Di}^{-2} + X_d(\omega/k) + Y_d(\omega) = 0 \tag{2}$$

where for low frequencies the ions and electrons give Debye shielding. The dust response term X_d for fixed charges is equivalent to that of an additional massive ion component with a Lorentzian distribution [2] with kappa = $(2\beta+5)/6$, and with a plasma frequency given by a weighted mean over grain sizes. Using this equivalence, known results [3] give an analytical expression for X_d. The dust charging dynamics gives an additional term Y_d which is a weighted mean over the corresponding terms for mono-sized dust [1]. Using the notation of [1]:

$$Y_d = \int_0^{\infty} 4\pi a \Omega_{V_0}(a) / \left[\Omega_{U_0}(a) - i\omega\right] h(a)\,da \tag{3}$$

where the charging frequencies may be assumed to be linearly proportional to the grain size a [1]. With the $h(a)$ used in [2] integration of (3) gives an analytic (lengthy) expression for Y_d. In general this involves incomplete gamma functions, and for integer β an infinite series including digamma functions. At zero frequency this represents an extra shielding term:

$$Y_d(0) = 4\pi n_d \alpha^{-1} \Gamma\left[(\beta+2)/3\right] / \Gamma\left[(\beta+1)/3\right] \Omega_{V_0}/\Omega_{U_0} \tag{4}$$

which combined with the Debye shielding terms in (2) gives an effective shielding distance:

$$\lambda_{Deff} = (\lambda_{De}^{-2} + \lambda_{Di}^{-2} + Y_d(0))^{-1/2} \tag{5}$$

For low frequencies Y_d may be expressed as a power series, for example to first order:

$$Y_d(\omega) \approx Y_d(0) + 4\pi n_d \Omega_{V_0} / (\Omega_{U_0} \Omega'_{U_0}) i\omega + O(\omega^2) \tag{6}$$

For high frequencies the asymptotic form may be use:

$$Y_d(\omega) \approx 4\pi n_d \alpha^{-2} \Gamma\left[\beta/3 + 1\right] / \Gamma\left[(\beta+1)/3\right] \Omega'_{V_0} / (-i\omega) \tag{7}$$

These results may now be used and extended to treat problems of electrostatic waves in dusty plasmas including both the effect of charging dynamics and a distribution of grain size.

REFERENCES

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