FREQUENCY AND TIME DOMAIN PERIODICITY –MATCHED FLOQUET WAVE DIFFRACTION THEORY FOR PLANAR TRUNCATED PHASED ARRAY GREEN'S FUNCTIONS: RECENT DEVELOPMENTS *

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ABSTRACT

This review is concerned with the utility of periodicity-matched Floquet wave (FW) constructs in the exact and high-frequency asymptotic analysis of dyadic Green's functions generated by large truncated linearly phased or sequentially pulsed periodic arrays of dipoles in the frequency or time domains, respectively. These canonical Green's functions form the building blocks for computationally efficient full-wave treatment (via integral-equation-based algorithms) of practical phased array configurations, with phenomenologically incisive interpretations in terms of a periodicity-modulated FW-matched nonuniform and uniform geometrical theory of diffraction.

INTRODUCTION

Large periodic phased array antennas are finding application in a variety of complex propagation environments, e.g., on platforms in the presence of auxiliary structures, as printed elements on or between multilayer substrates, in frequency-selective scenarios, etc. To characterize the behavior of such arrays, the direct brute-force approach of tracking ray-like wave objects from *individual* array elements interactively through the environment becomes computationally intensive and is not matched *a priori* to the constraints imposed by truncated spatial periodicity. These concerns have been addressed through the development of a *collective* approach, whereby the sum of individual element radiations is converted (via the bilaterally truncated Poisson sum formula) into a sum of radiations from equivalent smoothly phased Floquet wave (FW)-modulated field distributions that extend over the truncated array aperture. The global FW parameterization applies to the infinite array geometry, with the effects of truncation taken into account by inclusion of edge and vertex diffractions. The prototype idealized canonical configuration, which can be, and has been, embedded as a kernel within integral equation algorithms for computationally efficient full-wave treatment of realistic finite arrays, is the dyadic Green's function generated by a finite planar array of arbitrarily oriented linearly phased equiamplitude vector dipole currents which, when Poisson-summed, is expressed via truncated FW superposition. Because the FW behavior in the frequency domain (FD) is governed by a dispersion equation that restricts radiation away from the array plane to a finite number of "lower order" propagating "modes" (the modes with large spatial wavenumbers have evanescent decay), the FW parameterization is substantially more efficient than that involving individual elements.

To implement the FW strategy, it has first been necessary to develop a new accurate FW-matched FD diffraction theory for edges, vertices, shadow boundary transitions, etc., that extends previous results of the nonuniform and uniform geometrical theory of diffraction (GTD) for *smooth* perfectly conducting truncated plates to the *truncated dipole lattice* geometry, both in the spatial and wavenumber spectral regimes. This has posed new canonical problems in nonuniform and uniform high frequency (HF) asymptotics, which have been, and are being, dealt with through a progressive escalation in configurational complexity from semi-infinite (single-edge) through sectoral (double edge plus vertex) to polygonal shapes [1]. These free space studies have furnished essential insights into the new phenomenologies of truncated FWs, both in the physical and spectral domains, and for the propagating and evanescent regimes, with special

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attention given to diffraction-induced coupling from evanescent to propagating species. At each phase, the FW-based algorithms have been calibrated for accuracy over specified ranges of problem parameters by comparison with numerical reference solutions obtained through element-by-element summation.
Current investigations are centered on phased planar dipole array configurations on or within infinitely extended multilayer dielectric substrates, starting with semi-infinite planar dipole arrays on a grounded dielectric slab [2].
An entirely new area of investigation has been concerned with the time domain (TD) counterpart of the FD phased array Green's function, i.e., the behavior of FW-parameterized *sequentially short-pulse-excited* dipole array Green's functions in free space. TD-FW dispersion has led to the discovery of novel phenomenologies, which have been extended sequentially from the canonical infinite line dipole array [3] to the semi-infinite line dipole array [4], as well as to the bilaterally infinite planar array [5] and the semi-infinite planar array [6]. The most recent studies [7] deal with TD network representations of the array Green's functions for the geometry in [5].
The basic features and phenomenologies of the FD and TD-FW studies listed above are summarized in the presentation, with conclusions pertaining to future work.

EXAMPLE: THREE-DIMENSIONAL GREEN'S FUNCTION FOR A PLANAR RECTANGULAR PHASED ARRAY OF DIPOLES (FREQUENCY DOMAIN) [1]

This example serves to illustrate the phenomenologies and performance of the FW-modulated smoothly phased asymptotic diffraction algorithm for the FD array Green's function (AGF), which replaces the element-by-element sum AGF representation. The canonical problem for the rectangular array geometry is the infinite sector in Fig. 1, which can be used for synthesis of the rectangular configuration by appropriate superposition of sectoral AGFs centered at the remaining three vertexes. With reference to [1] for the analytical details, Table 1 summarizes in periodicity-modulated GTD terminology the final solution for the uniform asymptotic sectoral AGF $g_{00}(\bar{r})$ (see line 1), where the first term on the right-hand side represents summation over the (p,q)-indexed edge-truncated Floquet waves FW_{pq}, with the Heaviside unit functions $U_{pq}^{FW_{1,2}}$ delimiting their domain of existence, and with p and q denoting the FW-matched phasings along the edges z_1 and z_2 in Fig. 1, respectively; the second through fourth terms in line 1 identify truncated-FW-induced edge and vertex diffractions. The remaining lines in Table 1 quantify these diffracted asymptotic field contributions, with inclusion of canonical uniform transition functions that compensate smoothly for the abrupt field discontinuities across the various FW-related shadow boundaries (SB). The corresponding phenomenologies are schematized in Fig. 2. Note that $F(x) \rightarrow 1$ for large x; therefore, "far" from the shadow boundaries, the terms after the first one inside the large parentheses for the (edge 1, 2) and vertex-diffracted contributions in Table 1 can be neglected, thus reducing these contributions to those from the nonuniform FW-modulated GTD.

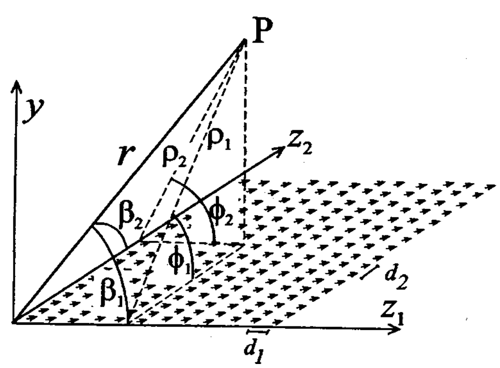

Fig. 1. Geometry and coordinates al array of parallel dipoles oriented along a direction $\hat{u}$. P is the observation point. d_1 and d_2 are the interelement spatial periods along z_1 and z_2, respectively, with linear interelement phase gradients γ_1 and γ_2. A caret $\hat{}$ denotes a vector quantity.

To assess the quality of the asymptotic AGF in Table 1, numerical experiments have been conducted on the square test array in Fig. 3, which is synthesized by superposition of the four relevant sectoral arrays. The problem parameters are listed in the caption of Fig. 3; the array is comprised of 15×15 dipole elements oriented at 45° with respect to the edges, with interelement spacing λ/2 and phasing $\gamma_1 = \gamma_2 = 2/\lambda$, where λ is the free space wavelength. The reference solution is obtained via element-by-element summation over the individual dipole radiators. Excellent agreement is noted between the full asymptotic AGF synthesized from Table 1, and the reference solution. Also shown are results obtained without the vertex contribution in Table 1, thereby highlighting the compensation, due to vertex diffraction, in the field pattern.

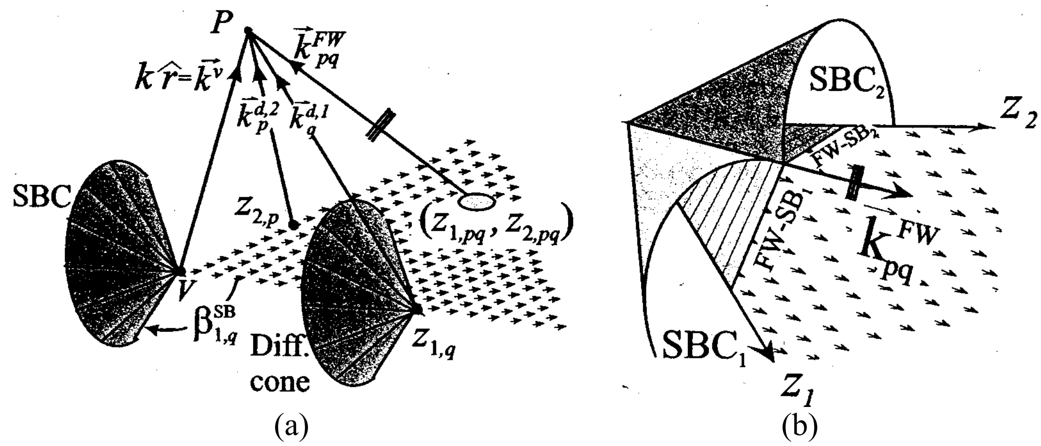

Fig. 2. FW-matched ray description of the field radiated to *P* by the sectoral array of dipoles. (a): The ray with wavevector $\vec{k}_{pq}$ from the bulk of the array identifies the FW_{pq} contribution. The *q*-indexed (edge 1)-diffracted ray to *P* originates at point $z_{1,q}$ and lies on a diffraction cone with aperture angle $\beta_{1,q}$. The shadow boundary cone (SBC), which truncates the domain of existence of this ray species, has the same aperture angle $\beta_{1,q}$ and is centered at the vertex. Analogous considerations apply to the *p*-indexed (edge 2)-diffracted rays, with (1,*q*) replaced by (2,*p*). (b): Schematic of the shadow boundaries (SBs). The FW-SB planes truncate the domain of existence of a FW_{pq}, which occupies the region under the "roof" formed by the intersection of the two FW-SBs. The domains of existence of the diffracted rays from edges 1 and 2 are truncated at their SBCs, which intersect along the intersection line of the two FW-SBs; this intersection line coincides with the propagation vector $\vec{k}_{pq}$ of the *pq*-FW, with $z_{12}-$ plane components $k_{z_1,q} = \gamma_1 + 2\pi q/d_1$, $k_{z_2,p} = \gamma_2 + 2\pi p/d_2$, and *y*-component $k_{ypq} = \left(k^2 - k^2_{z_1,q} - k^2_{z_2,p}\right)$.

Table 1. High-frequency uniform representation of the AGF (for notation, see caption of Fig. 2).

Total AGF	$g_{0,0}(\vec{r}) \sim \sum_{p,q} U^{FW,1}_{pq} U^{FW,2}_{pq} g^{FW}_{pq}(\vec{r}) + \sum_q U^{d,1}_q g^{d,1}_q(\vec{r}) + \sum_p U^{d,2}_p g^{d,2}_p(\vec{r}) + g^v(\vec{r})$
Edge-1 diffraction	$g^{d,1}_q(\vec{r}) \sim \frac{e^{-j\vec{k}^{d,1}_q \cdot \vec{r}}}{2d_1\sqrt{2\pi j \rho_1 k_{\rho 1,q}}} \left(B_2(k_{s2,q}) + \sum_p \frac{(F(\delta^2_{1,pq}) - 1)}{jd_2(k_{z2,p} - k_{s2,q})} \right), B_2(k_{s2,q}) = [1 - e^{jd_2(k_{s2,q} - \gamma_2)}]^{-1}$
Edge-2 diffraction	$g^{d,2}_p(\vec{r}) \sim \frac{e^{-j\vec{k}^{d,2}_p \cdot \vec{r}}}{2d_2\sqrt{2\pi j \rho_2 k_{\rho 2,p}}} \left(B_1(k_{s1,p}) + \sum_q \frac{(F(\delta^2_{2,pq}) - 1)}{jd_1(k_{z1,q} - k_{s1,p})} \right), B_1(k_{s1,p}) = [1 - e^{jd_1(k_{s1,p} - \gamma_1)}]^{-1}$
Canonical transition function	$F(x) = 2j\sqrt{x}\, e^{jx} \int_{\sqrt{x}}^{\infty} e^{-jt^2}\, dt; \begin{pmatrix} \delta_{1,pq} = \sqrt{2k_{\rho 1,q}\rho_1} \sin(\frac{1}{2}(\phi_{1,pq} - \phi_1)) \\ \delta_{2,pq} = \sqrt{2k_{\rho 2,p}\rho_2} \sin(\frac{1}{2}(\phi_{2,pq} - \phi_2)) \end{pmatrix}$
Vertex diffraction	$g^v(\vec{r}) \sim \frac{e^{-jkr}}{4\pi r} \left(B_1(\bar{k}_{z1s})B_2(\bar{k}_{z2s}) + \frac{B_2(\bar{k}_{z2s})(F(a^2_q) - 1)}{jd_1(k_{z1,q} - \bar{k}_{z1s})} + \frac{B_1(\bar{k}_{z1s})(F(b^2_p) - 1)]}{jd_2(k_{z2,p} - \bar{k}_{z2s})} + \frac{(T(a_q, b_p, w) - F(a^2_q) - F(b^2_p) + 1)}{-d_1 d_2(k_{z1,q} - \bar{k}_{z1s})(k_{z2,p} - \bar{k}_{z2s})} \right)$
Canonical transition function	$T(a_q, b_p, W) = \frac{a_q b_p}{j\pi\sqrt{1 - W^2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{j(\xi^2 + 2\omega\xi\eta + \eta^2)}}{(\xi - (a_q/\sqrt{1 - W^2}))(\eta - (b_p/\sqrt{1 - W^2}))}\, d\xi\, d\eta\, a_q = \sqrt{2kr} \sin(\frac{1}{2}(\beta_{1,q} - \beta_1)), b_p = \sqrt{2kr} \sin(\frac{1}{2}(\beta_{2,p} - \beta_2)); \omega = \cot\beta_1 \cot\beta_2 = \cos\phi_1 \cos\phi_2$

CONCLUSIONS

The evolution and current status of the frequency or time domain Floquet-wave (FW)-based analytic treatment of radiation from linearly phased or sequentially pulsed large planar truncated periodic arrays of dipoles has been reviewed here. The utility of these algorithms for integral equation full-wave treatment of operational practical arrays has been emphasized, as have the cogent physical insights which are provided by their high-frequency asymptotic reduction, and interpretation in terms of a periodicity-matched FW-modulated geometrical theory of diffraction (GTD). FD results have been presented for the canonical sectoral array, and the performance of the GTD-based asymptotics has been illustrated on a numerical example. Future efforts will be directed toward extensions of these techniques to emerging applications of phased arrays in complex platform or other environments.

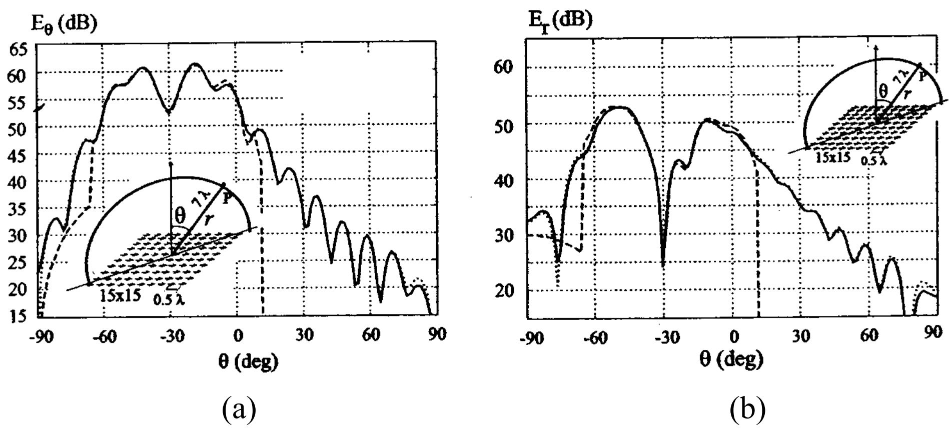

Fig. 3. Electric field for a rectangular array ($\gamma_1 = \gamma_2 = 2/\lambda$). The solution is based on Table 1. Scan close to a vertex in a plane normal to the array at 45° between the edges, containing the array normal. Asymptotic solution (continuous curve), reference solution (dotted curve), asymptotic solution without vertex contribution (dashed curve). (a) angular and (b) radial field component.

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