HYBRID RAY-MODE REPRESENTATIONS REVISITED *

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ABSTRACT

Discretely or continuously transversely stratified physical environments, which support longitudinal waveguiding by reflections at abrupt boundaries and/or by refractions in wave-confining continuous profiles, serve as models for many actual propagation scenarios. In the high frequency (HF) range, progressing (ray-type) and oscillatory (mode-type) field representations have provided alternative wave phenomenologies with complementary local and global (and thereby complementary convergence) properties, respectively. A hybrid ray-mode formalism has been developed which combines these two formulations self-consistently so as to take advantage of the best features of each, as demonstrated by example.

INTRODUCTION

When time-harmonic wave propagation is spatially (transversely) constrained by physical impenetrable boundaries or by “virtual” boundaries (ducts) established through refraction in transversely inhomogeneous media, the resulting source-excited longitudinally guided (ducted) waves have traditionally been described in terms of two alternative phenomenologies: progressing and oscillatory. The progressing formulation views the wavefields as continuous spectra of waves which propagate away from the source to the receiver, through the guiding environment, via multiple transverse reflections and (or) refractions. The oscillatory formulation views the wavefields as discrete (or discrete-continuous) superpositions of (guided mode) eigenspectra which are matched to the entire transversely confining cross section, which are individually independent of source and receiver locations, but whose *amplitudes* of excitation and reception do depend on these locations. At low frequencies (LF), where the transverse dimensions span a few wavelengths, the progressing formulation becomes unwieldy and is not matched to the LF wave physics. Here, the oscillatory formulation prevails because the frequency-dependent eigenmode spectra, while excited in their totality near the source, decay exponentially (are evanescent) longitudinally away from the source when the number of oscillations in their transverse profile exceeds a frequency-dependent cutoff threshold. These higher order modes are filtered out by sufficiently large (in terms of wavelengths) source-receiver separations so that the LF regime is well served by the manageably small number of discrete propagating modes whose eigenspectra are well separated.

On the other hand, at high frequencies (HF) -- the “overmoded” regime where the transverse dimensions span many wavelengths and many modes can propagate -- the oscillatory formulation becomes unwieldy, and is less well-matched to the wave physics for large source-receiver separations since the lower mode eigenspectra there form closely spaced clusters. Now, the progressing wave spectra can be better adapted to the wave physics by approximate tracking of *local* plane waves whose constructive interference maxima coalesce around source-receiver dependent “ray” trajectories which are determined via HF asymptotics. These circumstances have motivated efforts to combine these two *complementary* spectral methodologies, neither of which is convenient for *all* source-receiver locations, in a manner that seeks to exploit the best features of each. The outcome has been a comprehensive rigorously based, selfconsistent *hybrid ray-mode algorithm*, which has clarified the ray-mode interplay through a series of spectral studies and wide-ranging applications to complex waveguiding environments initiated more than two decades ago. These studies are reviewed here because of their potential relevance to HF interaction with complex “new” environments that combine both ray-adaptable and mode-adaptable constituents.

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The formal architecture of the hybrid ray-mode algorithm is based on the truncated Poisson sum formula which converts a finite series of q-indexed elements f_q into a bilaterally infinite series of p-indexed elements F_p plus f-truncation remainders, where F_p is the truncated Fourier transform of the smoothed function f, sampled at (normalized) integer values p. Since the two representations have Fourier-transform-related local-global complementary convergence properties, they can be used to avoid difficulties encountered in one by switching self-consistently to the other. This leads to the basic *ray-mode-equivalence* [1], which allows wave phenomena within a specified spectral interval to be expressed either locally in terms of a specified finite number of multiply reflected ray fields or globally in terms of a self-consistently corresponding finite number of modes, plus remainders that fill possible spectral voids near the truncations.

These alternative complementary interpretations, which depend quantitatively on the source and receiver locations, are verified by rigorously based saddle point asymptotics that establishes that the ray-field-generated sampled spectra satisfy the modal "transverse resonance" condition. Similarly, an initially specified finite mode bundle, when sampled-Fourier-transformed, yields asymptotically a corresponding bundle of multiply reflected rays, plus truncation remainders. Specific examples are shown of how complex ray field transitions near isolated and accumulated ray caustics can be avoided by filling such spectral intervals with modes, and conversely, how transitions of mode fields into and out of spectral boundaries can be avoided via ray fields. For a review with extensive literature citations, see [2].

In parallel with the FD studies, by Fourier inversion from the FD, the hybrid ray-mode method has been applied in the time domain (TD) [3,4]. More recent investigations have addressed hybrid ray-(leaky mode) parameterization of short-pulse-excited open dielectric waveguides *directly* in the TD [5-7]. A different TD variant of the hybrid ray-mode method, the *hybrid wavefront-resonance* method, was also developed simultaneously with the FD formulation. Here, the hybrid concept is applied to impulse-excited ray fields (wavefronts) and to modal resonances in the *complex frequency domain*, within the framework of SEM (singularity expansion method) [8-10].

SPECTRAL FOUNDATION OF THE HYBRID RAY-MODE ALGORITHM

As noted above, for a hybrid formulation, one seeks to transform a group of elements f_q into a series of elements F_p. through use of the finite Poisson sum formula [2]

$$\sum_{q=q_1}^{q_2} f_q = \frac{1}{2} f_{q1} + \frac{1}{2} f_{q2} + \sum_{p=-\infty}^{\infty} F_p\left(q_1, q_2\right), \tag{1}$$

where $F_p = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\tau)\exp(ip\tau)\,d\tau$, with $f(\tau)$ representing the smooth function generated from the discrete samples f_q, and

$$F_p\left(q_1, q_2\right) = \frac{1}{2\pi}\int_{\tau_1}^{\tau_2} f(\tau)\exp(ip\tau)\,d\tau = F_p + F_p\left(q_1\right) + F_p\left(q_2\right), \tag{2}$$

where $F_p\left(q_{1,2}\right) = \pm\frac{1}{2\pi}\int_{\pm\infty}^{\tau_{1,2}} f(\tau)\exp(ip\tau)\,d\tau$, with $\tau_{1,2} = 2\pi q_{1,2}$ (upper and lower signs go with the first and second subscripts, respectively). When the integrals are evaluated asymptotically by the saddle point method, with τ_p denoting the saddle point, one finds [2]

$$\sum_{q=q_1}^{q_2} f_q \sim \sum_{p=p_1}^{p_2} F_p U\left(\tau_2 - \tau_p\right) U\left(\tau_p - \tau_1\right) + \overline{f}_{q1} + \overline{f}_{q2}, \tag{3}$$

with the truncation remainders given by $\overline{f}_{q2,1} = (1/2)\left(1 \pm \Delta_{q2,1}\right) f_{q2,1}$ and $U(\tau) = 0, \tau < 0,$ but $U(\tau) = 1, \tau > 0.$ Thus, the original group of elements f_q is experessed as a series of its Fourier transformed elements F_p plus elements $\overline{f}_{q2,1}$, which are the collectively weighted last and first elements, respectively, in the original group. These collective remainders account for the spectral voids $\left(\tau_{p1} - \tau_1\right)$ and $\left(\tau_2 - \tau_{p2}\right)$ left unfilled by the F_p elements, where τ_{p2} and τ_{p1} denote the saddle points closest to τ_2 and τ_1, respectively, and contained within the interval $\tau_1 < \tau_p < \tau_2$. When an endpoint $\tau_{1,2}$ lies approximately halfway between the first included $\left(\tau_{p1,2}\right)$ and excluded saddle point, then $\Delta_{q1,2} \approx 0$, and $\overline{f}_{q2,1} \approx (1/2) f_{q2,1}$, i.e., half of the truncating element f_q. However, in the transition region where a saddle point approaches and endpoint, the integral in (2) must be evaluated by uniform asymptotics [2]. When f_q and F_p are chosen as ray fields and modal fields, respectively, the relation in (3) may be regarded as a *ray-mode equivalent*, with the truncation remainder given in terms of *collective rays*. For a schematization, see Figs. 1a and 1b.

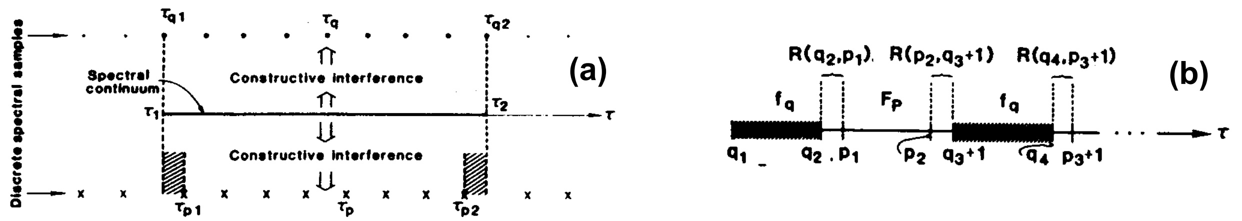

Fig. 1. (a): Ray-mode equivalence by alternative constructive spectral interference. Finite spectral wavenumber interval $\tau_1 \leq \tau \leq \tau_2$ is filled completely with rays $\tau_{q1} \leq \tau_q \leq \tau_{q2}$ (dark dots), with $\tau_{q1,2} = \tau_{1,2}$. The equivalent mode group $\tau_{p1} \leq \tau_p \leq \tau_{p2}$ (dark crosses), with $\tau_{p1,2} \neq \tau_{1,2}$, does not fill the interval completely. The spectral voids (shaded) are filled with spectral remainders. Light dots and crosses identify ray fields and mode fields outside the interval. (b): Partitioning of spectral range for hybrid formulation. q_i and p_i denote spectral values delimiting intervals occupied by f_q and F_p elements, respectively. $R(q,p)$ and $R(p,q)$ are remainders accounting for spectral voids.

EXAMPLE: SURFACE-GUIDED RAY AND MODE FIELDS IN AN INHOMOGENEOUS MEDIUM WITH EXPONENTIAL REFRACTIVE INDEX PROFILE: $n(x) = \exp(-x/d)$ [11]

The physical configuration of the refractive index profile along the normalized height coordinate (x/d) is shown in Fig. 2a, with the *j*-indexed ray species for each *q*-indexed ray traveling from *S* to *P* along the normalized duct coordinate z/d shown in Fig. 2b, and the *p*-indexed ducted mode profile shown in Fig. 2c. Figure 3a, for the problem parameters specified in the figure caption, shows the caustics for the *j*-indexed species corresponding to various *q*-indexed rays, whereas Fig. 3b shows the signal strength as a function of height at a fixed range z/d, computed via a rigorous mode sum reference solution and via various asymptotic hybrid ray-mode combinations depicted in the inset; very good agreement is observed. For example, at $x/d = 0.05$, the hybrid representation involves two rays of species j=1 plus one mode, one ray of species 2 plus eight modes, three rays of species 3 plus one mode and two rays of species 4 plus on mode. A ray representation alone (with caustic correction) is utilized beyond $x/d = 0.115$; it is found to coincide completely with the reference solution. In this example, only ray fields with one or two reflections may be identified; this corresponds to $(x/d) \geq 0.1$. The ray method is inapplicable for rays with more than two reflections because of the pile-up of higher order *q* caustics along the source height near the boundary. The locations of the caustics and cusps for the once and twice reflected rays, as inferred from Fig. 4a with $z/d = 0.192$ and variable x/d, are indicated below the abscissa. Note that the caustic corrections required in the ray formalism are avoided by the properly selected hybrid mix.

CONCLUSIONS

The hybrid ray-mode formulation of high frequency wave propagation in longitudinally ducting guiding environments has been reviewed, and illustrated with an example. Although developed more than two decades ago, this physically appealing self-consistent "optimal" complementary partitioning of the wave spectral domain so as to fill poorly convergent ray intervals with modes, and vice versa, has not yet been widely utilized. It is hoped that this presentation may generate interest for possible application to multimode propagation channels of current concern (in the HF time-harmonic or the short-pulse transient regimes).

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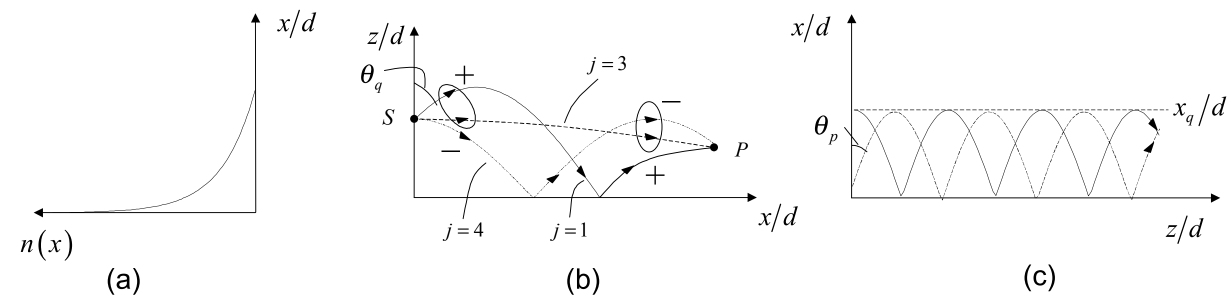

Fig. 2. (a): Refractive index profile. (b): Ducted multiply reflected ray species from source (*S*) to observer (*P*). +: upgoing; -: downgoing. $j=1: S^+, P^+$; $j=2: S^-, P^+$; $j=3: S^+, P^-$; $j=4: S^-, P^-$. Ray index q is determined by the ray departure angle θ_q. (c): Modal rays for the *p*-th mode with modal duct width x_p/d, and eigenangle θ_p.

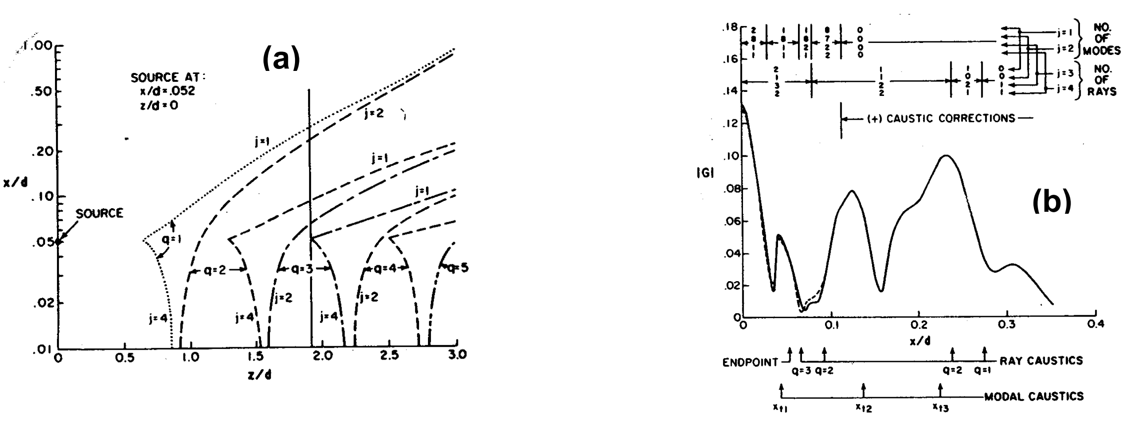

Figure 3. Surface duct with exponential refractive index $n(x)=\exp(-x/d)$. A time-harmonic line source with normalized frequency $kd=100$ is located at the normalized coordinates $x/d=0.052, z/d=0$. (a): Caustics associated with rays of species $j=1,...,4$, which have undergone q reflections on the boundary $x=0$. (b): Variation of signal strength $|G|$ for observation points located at a range $z/d=1.92$, with variable depth x/d. Solid curve —— exact solution (rigorous guided mode sum); dashed curve - - - hybrid ray-mode solution in (3); the number of rays and modes for each ray species is indicated in the inset.