NARROW-WAISTED GAUSSIAN BEAM ALGORITHMS AS EFFICIENT SOLVERS FOR IMAGING IN COMPLEX PROPAGATION ENVIRONMENTS *

Vincenzo Galdi (1,2), Haihua Feng (1), Julia Pavlovich (1), David A. Castañon (1), W. Clem Karl (1), and Leopold B. Felsen (1,3)

(1) Dept. of Electrical and Computer Engineering
Boston University, 8 Saint Mary's St., Boston, MA 02215, USA
E-mail: vgaldi@bu.edu, hfeng@bu.edu, oska@bu.edu, dac@bu.edu, wckarl@bu.edu, lfelsen@bu.edu

(2) Waves Group, University of Sannio, Benevento, Italy

(3) Dept. of Aerospace and Mechanical Engineering, Boston University, Boston, MA 02215, USA
Also, University Professor Emeritus, Polytechnic University, Brooklyn, NY 11201, USA

ABSTRACT

In this paper, we explore the use of Gabor-based narrow-waisted quasi-ray Gaussian beam (GB) algorithms as efficient forward solvers in complex inverse scattering scenarios. With particular reference to the problem subsurface sensing of shallowly-buried low-contrast dielectric targets in the presence of moderately rough air-soil interfaces, an *adaptive* strategy is proposed wherein GB algorithms are integrated with standard signal processing and regularization tools. Numerical simulations are presented to assess accuracy, robustness and computational efficiency. The proposed approach has potential applications to antipersonnel land mine remediation.

INTRODUCTION

The problem of determining the properties of objects embedded in complex environments from electromagnetic (EM) scattered field data arises in many important applications, ranging from nondestructive testing to underground imaging. In these EM inverse scattering applications, a (typically discretized) forward scattering model is inverted to estimate the electrical properties of a suitable test domain in order to localize possible anomalies. The inversion procedure usually requires solution of a large number of forward scattering problems, thus making availability of a fast forward solver a key ingredient for overall computational feasibility. In realistic scenarios involving complex propagation environments and/or large computational domains, typical full-wave forward solvers (e.g., moment methods or finite differences) are often not affordable in terms of computing time and resources. Approximate physics-based modelings have been explored in this connection, including Gaussian beam (GB) algorithms which have variously been found to provide a good tradeoff between accuracy and computational burden (see, e.g., [1]). In this paper, we focus on underground imaging problems in the presence of moderately rough interfaces, which arise in many important practical applications, e.g., the clearance of buried unexploded ordinance such as plastic antipersonnel land mines. In this connection, we review and discuss applications of Gabor-based *narrow-waisted* GB algorithms, which were found to be remarkably well-suited for this class of problems [2-4]. In fact, though not suffering from failures near caustics and other ray-field transition regions, these GB algorithms preserve the attractive computational features of standard ray methods in the presence of large computational domains, with minimal memory requirements and computing times, which are typically orders of magnitude below those of conventional full-wave techniques. The availability of such fast forward scattering models enabled us to explore a novel adaptive framework which is described below.

ADAPTIVE APPROACH TO ROUGH SURFACE UNDERGROUND IMAGING

The proposed strategy is illustrated in Fig. 1. A prior (coarse-scale) interface estimation problem is solved, which, by exploiting the GB fast forward models, is posed as a nonlinear optimization problem [5, 6]. This sets the stage for the

* Work supported by ODDR&E under MURI grants ARO DAAG55-97-1-0013 and AFOSR F49620-96-1-0028, and by the Engineering Research Centers Program of the National Science Foundation under award number EEC-9986821. L.B. Felsen also acknowledges partial support from Grant No. 9900448 by the US-Israel Binational Science Foundation, Jerusalem, Israel, and from Polytechnic University, Brooklyn, NY 11201 USA.

actual inverse scattering problem, i.e., the target imaging in the presence of a *known* roughness profile. In particular, we focus on the important and challenging case of shallowly-buried low-contrast mine-like targets. An approximate Born-linearized forward model is utilized for the target scattering, wherein the distortion introduced by the twice-traversed roughness profile is accounted for via the Gabor-based quasi-ray GB fast forward solvers in [2-4]. The resulting reconstructed interface profile is used to correct the raw backscattered field data observed at the receivers so as to compensate for the corresponding clutter; at this stage, statistical models can be invoked to account, in addition, for noise, estimation error and residual unmodeled effects. The forward scattering model is subsequently inverted to retrieve the unknown dielectric permittivity contrast in a suitable test domain. In this connection, use is made of *pixel-based* and *object-based* regularization and reconstruction techniques. Both *frequency-stepped* [7] and *pulsed* [8] excitations have been investigated. The various constituents and representative results are illustrated below.

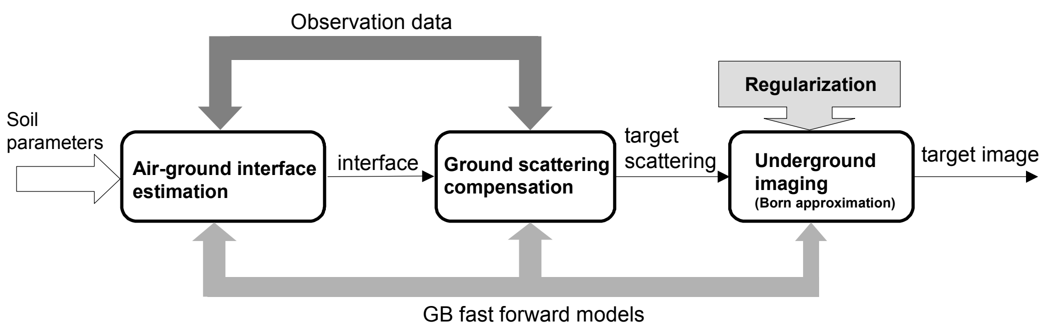

Fig. 1. Schematic flow-chart of the proposed adaptive strategy.

Fast Forward Scattering Models

The approximate Gabor-based quasi-ray GB fast forward solvers constitute an essential ingredient of the proposed adaptive framework. Starting from the original applications to transmission of aperture-excited time-harmonic wavefields through curved dielectric layers [9, 10], the GB solvers have been extended to deal with reflection from, and transmission through, moderately rough slightly lossy dielectric interfaces for both time-harmonic [2] and pulsed [3] excitations. These forward solvers have been validated and calibrated over relevant ranges of parameters against rigorous full-wave numerical reference solutions, and have been shown to furnish robust and reliable predictions with computing time and resource requirements considerably cheaper than those of typical full-wave solvers. The above results have so far been restricted to 2-D configurations; generalization to 3-D (vector) configurations is in progress [11].

Interface Profile Reconstruction

The reconstruction of the (coarse scale) rough interface profile from reflected field observations is addressed in [5, 6] for both frequency-stepped and pulsed excitations. In this connection, a compact low-dimensional spline parameterization of the roughness profile is utilized in conjunction with the GB forward scattering model in [2-4] to pose the interface reconstruction problem as a nonlinear optimization problem (see [5, 6] for implementation and computational issues). The above approach was tested in realistic applications involving sparsely-sampled noisy observations and imperfect knowledge of soil parameters, yielding accurate and robust reconstructions with reasonable computing times (1 min. on a 700 MHz PC) and resources.

Target Imaging

As shown in Fig. 1, the coarse-scale interface profile reconstruction is used to generate predictions of the ground scattering, which is a major corruptor of the observed signal. This formalizes strong suppression of the rough-ground-related clutter, leaving the observed data primarily representative of the underground target scattering contribution. For the low-contrast targets of interest here, the Born approximation is invoked to linearize the forward scattering model. Further rough-interface-induced distortion in the observed signal is accounted for via the use of an approximate GB-parameterized half-space Green's function [7, 8]. Various *pixel-based* and *object-based* regularization techniques have been investigated in order to achieve reliable inversion of the forward scattering model and to cope with its inherent *ill-posedness*. Pixel-based approaches made use of L_p ($0 < p \leq 1$) norm regularizations [6]; object-based approaches made use of curve evolution techniques [7, 8], wherein one homes in on *direct* estimation of robust target features (permittivity contrast and boundary). The reader is referred to [7, 8] for theoretical and implementation details.

REPRESENTATIVE RESULTS

The above strategy has been validated and calibrated via an extensive series of numerical simulations, for both frequency-stepped and pulsed excitations. Here, we present typical results for the case of pulsed excitation, with reference to the simulation geometry and parameters in Fig. 2(a), where a plastic mine-like 10cm × 6cm elliptic target with relative permittivity $\varepsilon_{r2} = 3.5$ is buried in a homogeneous dielectric half-space with constitutive parameters chosen so as to simulate a class of realistic clay-loam soils ($\varepsilon_{r1} = 4, \sigma_1 = 0.01\,\mathrm{S/m}$) with a randomly-generated moderate roughness realization (~3-4 cm peak-to-peak, maximum slope ~30°). A cosine-tapered quasi-plane-wave pulsed excitation with 80cm aperture width and a fourth-order Rayleigh wide-band time profile (2.45 GHz center frequency, 1.4 GHz bandwidth) is assumed, and the backscattered field is observed at 11 receivers located 30cm above nominal ground. A full-wave simulation algorithm [7] is utilized to generate synthetic observation data, which are subsequently corrupted by 10% additive uniform noise. For this configuration, Fig. 2(b) shows the comparison between the total field e (soil+target) and the background field e^b (soil only) waveforms at a fixed receiver location, computed via the full-wave algorithm. Their difference (the target contribution e^s), which is unappreciable on the scale of Fig. 2(b), is displayed in Fig. 2(c) and is shown to be in good agreement with our Born-GB synthesis. Interface profile reconstruction results via the algorithm in [6] are shown in Fig. 2(d); except for the poorly illuminated side regions, good accuracy is observed. Adaptive ground clutter compensation is performed as in [7]. A representative example of object-based (curve evolution) target reconstruction is shown in Fig. 2(e). Rather accurate target boundary and dielectric contrast estimations are observed. Similar results were obtained for the frequency-stepped configuration [8]. Computational features and intrinsic algorithmic limitations are discussed in [7, 8]. Here we only point out that overall computing times are on the order of a few minutes, thus leaving room for optimism that extensions to more realistic 3-D configurations (currently under investigation) will remain computationally feasible.

CONCLUSIONS

The use of Gabor-based narrow-waisted Gaussian beam algorithms as fast forward scattering models in complex inverse scattering scenarios has been discussed here in connection with the problem of subsurface sensing in the presence of moderately rough air-soil interfaces. Preliminary 2-D results, restricted to slightly lossy soils and low-contrast targets, show that quite accurate estimations of the coarse scale roughness profile can be obtained from reflected field sparse data, and can be fruitfully exploited to enhance underground imaging, with reasonable computing time and resources.

REFERENCES

[1] B. Rao and L. Carin, “Beam-tracing-based inverse scattering for general aperture antennas,” *J. Opt. Soc. Am. A*, vol. 16, No. 9, pp. 2219-2231, Sept. 1999

[2] V. Galdi, L.B. Felsen, and D.A. Castañon, “Quasi-ray Gaussian beam algorithm for time-harmonic two-dimensional scattering by moderately rough interfaces,” *IEEE Trans. Antennas and Propagat.*, vol. 49, No. 9, pp. 1305-1314, Sept. 2001.

[3] V. Galdi, L.B. Felsen, and D.A. Castañon, “Quasi-ray Gaussian beam algorithm for short-pulse two-dimensional scattering by moderately rough dielectric interfaces,” *IEEE Trans. Antennas and Propagat.*, vol. 50, No. 12, Dec. 2002, in press.

[4] V. Galdi, L.B. Felsen, and D.A. Castañon, “Narrow-waisted Gaussian beam algorithms as efficient forward solvers in complex propagation and scattering scenarios: Refinement and further calibrations of previously obtained preliminary algorithms,” this Conference.

[5] V. Galdi, D.A. Castañon, and L.B. Felsen, “Multifrequency reconstruction of moderately rough interfaces via quasi-ray Gaussian beams,” *IEEE Trans. Geosci. and Remote Sensing*, vol. 40, No. 2, pp. 453-460, Feb. 2002.

[6] V. Galdi, J. Pavlovich, D.A. Castañon, W.C. Karl, and L.B. Felsen, “Moderately rough dielectric interface reconstruction via short-pulse quasi-ray Gaussian beams,” *IEEE Trans. Antennas and Propagat*, vol. 51, No. 3, Mar. 2003, in press.

[7] V. Galdi, H. Feng, D.A. Castañon, W.C. Karl, and L.B. Felsen, "Multifrequency subsurface sensing in the presence of a moderately rough air-soil interface via quasi-ray Gaussian beams," *Radio Science*, Special Issue on URSI 2001 EMT Symp., 2002, in press.

[8] V. Galdi, H. Feng, D.A. Castañon, W.C. Karl, and L.B. Felsen, "Moderately rough surface underground imaging via short-pulse quasi-ray Gaussian beams," submitted to *IEEE Trans. Antennas and Propagat.*, Oct. 2001.

[9] J.J. Maciel and L.B. Felsen, Gaussian beam analysis of propagation from an extended aperture distribution through dielectric layers, Part II - circular cylindrical layer *IEEE Trans. Antennas Propagat.*, vol. 38, No. 10, pp. 1618-1624, Oct. 1990.

[10] J.J. Maciel and L.B. Felsen, "Gabor-based narrow-waisted Gaussian beam algorithm for transmission of aperture-excited 3D vector fields through arbitrarily shaped 3D dielectric layers," submitted to *Radio Science*, Oct. 2001.

[11] V. Galdi, L.B. Felsen, and D.A. Castañon, "3-D short pulse scattering by moderately rough dielectric interfaces via quasi-ray Gaussian beams," to be presented at 2002 IEEE Antennas and Propagat. Int. Symposium, San Antonio, TX, USA, June 16-21, 2002.

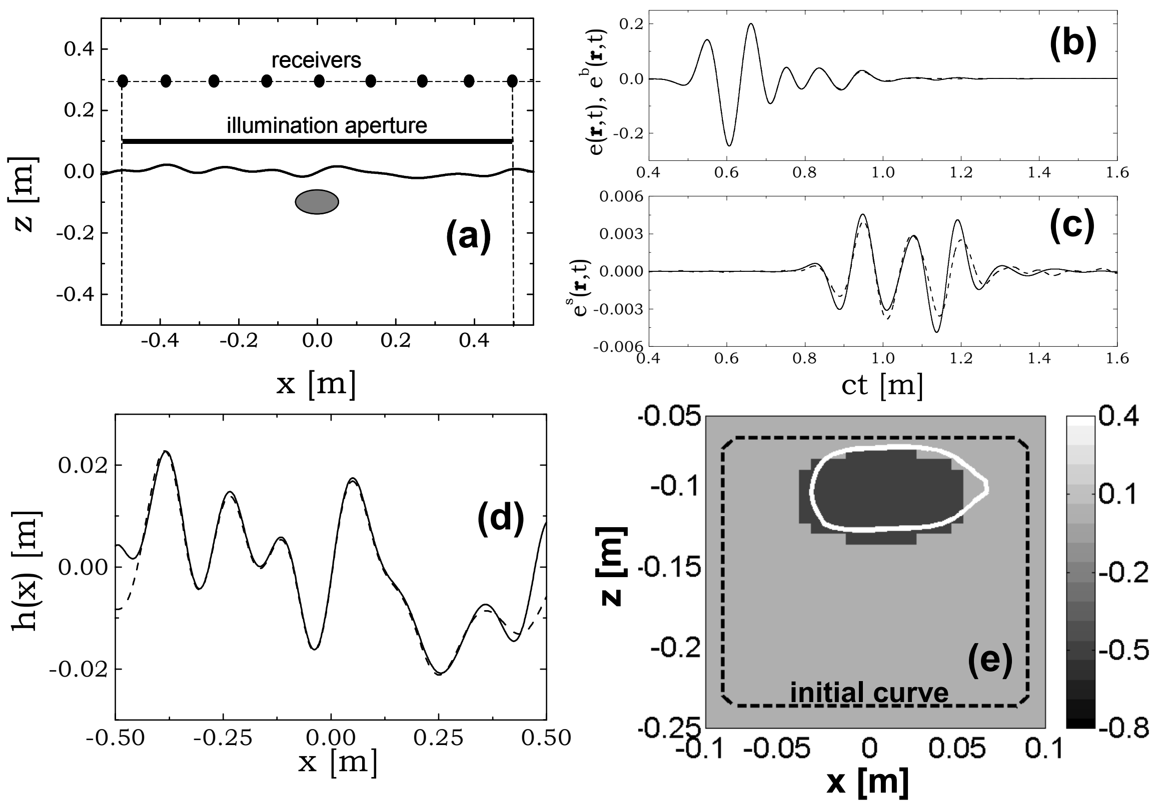

Fig. 2. Numerical results. (a): Simulation geometry and parameters. Soil: $\varepsilon_{r1} = 4, \sigma_1 = 0.01\,\mathrm{S/m}$. Target: 10cm × 6cm ellipse with center at 10cm below nominal ground, $\varepsilon_{r2} = 3.5, \sigma_2 = 0$. Excitation: cosine-tapered quasi-plane-wave with 80cm aperture width, and fourth-order Rayleigh wide-band excitation (2.45 GHz center frequency, 1.4 GHz bandwidth). The backscattered field is observed at 11 equispaced receivers located 30cm above nominal ground and corrupted with 10% additive uniform noise. (b) Full-wave predictions for fields observed at (x=z=0.3m) (c denotes the free-space wavespeed): —— Total field e; ----- Background field e^b. Both are coincident on this drawing. (c) Target-scattered field $e^s = e - e^b$ at (x=z=0.3m): —— Full-wave prediction; ---- Born-GB approximation. (d): Example of reconstruction of the rough interface profile $z = h(x)$: —— Actual profile; ----- Reconstruction. (e): Curve evolution target reconstruction (white curve) is overlayed on ground truth (relative permittivity contrast gray-scale plot). The estimated target relative permittivity is $\varepsilon_{r2} = 3.56$ (1.7% error).