THE IONOSPHERIC VERTICAL ELECTRON DENSITY PROFILE RECONSTRUCTION USING SATELLITE SIGNAL DOPPLER SHIFT MEASUREMENTS

O. F. Tyrnov(1), A. G. Tyzhnenko(1)

(1)*Kharkiv V. Karazin National University, Svoboda sq.4, Kharkiv - 61077, Ukraine*
Phone: [38] 0572 471-012, E-mail: Oleg.F.Tyrnov@univer.kharkov.ua

ABSTRACT

A problem of reconstructing a vertical ionospheric electron density profile using measurements of the satellite signal Doppler shift comes to solving an integral equation of the first kind. For this, one uses a new iteration algorithm allowing obtaining a stable in the sense of Hadamard normal solution of the ill-conditioned linear algebraic system, which does not depend on sampling. Accuracy of the solution depends on an irregularity of the experimental data. For used experimental data a relative error does not exceed 4%.

INTRODUCTION

The first serious interest in the problem of reconstructing the electron density, *N(z),* .profile using measurements of the integral effects arising in the ionosphere under radio wave propagation aroused in the early 1970s [1, 2]. In [3–5], there were made attempts to use this possibility for investigating ionospheric parameters. Difficulty in practical use of such an approach is explained by large values of the condition number of the matrix equation (ME), which is to be solved in this case. It is possible to look for a solution of the ME with respect to the parameters of the profile model *N(z)* but a larger interest gives a direct reconstruction of a real profile *N(z)* using experimental data on temporal changes in characteristics of transionospheric propagation of radio waves.

MAIN PRINCIPLES

The problem of finding the ionospheric electron density profile, *N(z),* using the experimental measurements of the satellite signal Doppler shift due to the ionosphere influence comes to solving an integral equation of the 1st kind [6]:

$$\int_{z_0}^{z_c} \frac{(a+z)N(z)dz}{\left[(a+z)^2 - a^2\cos^2\beta\right]^{3/2}} = -\frac{fc\Delta f(\beta)}{2.014\times 10^7 a^2 \beta_t' \sin 2\beta} \qquad (1)$$

where $\Delta f(\beta)$ is the experimental Doppler shift dependence on the satellite elevation angle, β, $a = 6370$ km (Earth radius), f is the signal frequency in MHz, c is the speed of light (km/sec), z_c is the satellite height (1000 km), z_o is the low boundary of the ionosphere. Equation (1) was solved by a collocation method after its reducing to a standard interval on the nonuniform set of points. The integral was discretized by Gauss formula. The ME obtained as a result of discretization is greatly ill-conditioned (the condition number being $\sim 10^{20}$). Moreover, the experimental Doppler shift dependence on the satellite elevation angle is extremely irregular. Under these conditions, the well-known methods for solving ill-conditioned systems are not suitable [7]. Therefore, to solve such systems for a nearly spherically stratified ionosphere, i.e., in the absence of large horizontal gradients of the electron density, it has been worked out a new iteration algorithm [8] for obtaining a normal pseudosolution stable in the sense of Hadamard and independent of sampling. Let us write down a system of equations obtained under discretization of (1) as follows:

$$AX = B. \qquad (2)$$

[1]Acknowledgments. The work is supported by Science and Technology Center in Ukraine Grant 1773 to Kharkiv V. Karazin National University.

Instead of (2), we consider regularized equation

$$\alpha X+AX= B, \tag{3}$$

which minimize the $||AX-B||$ discrepancy at $\alpha \rightarrow 0$. Equation (3) has a unique solution for any $\alpha \neq 0$. If there is an algorithm to solve (3), allowing to find X^* for any small α, and there exists the following limit:

$$X^* = \lim_{\alpha \to 0} X_\alpha^* \tag{4}$$

then this limit solution X^* will be a normal pseudosolution of (2) [9].
As the right-hand side obtained experimentally is extremely irregular, we use the following transformation of integral equation (2), which smooth out the right-hand side:

$$AX = B \Leftrightarrow \gamma X+AX= B + \gamma X, \tag{5}$$

At the same time, γ is chosen so that the matrix

$$L = \gamma E + A \tag{6}$$

would be good-conditioned (here E is the identity matrix), and there would exist an inverse matrix L^{-1}. Multiplying (5) by L^{-1}, we obtain equation

$$X - \gamma L^{-1}X = L^{-1}B. \tag{7}$$

The condition number of the matrix E-γ L^{-1} is the same as the initial one, but the right-hand side is multiplied by the matrix L^{-1} which, for some γ values, smoothes out the right-hand side and stabilizes the solving. We designate as follows:

$$A_1= E - \gamma L^{-1},\ B_1= L^{-1}B\ . \tag{8}$$

In order to solve regularized (7),

$$\alpha X + A_1X = B_1, \tag{9}$$

we use the following factorization [8]:

$$k= \alpha E + A_1 = L_1 + U_1, \tag{10}$$

where we have L_1, and U_1,

$$L_1 = \begin{pmatrix} \alpha_1 & 0 & 0 & \dots \\ k_{21} & \alpha_1 & 0 & \dots \\ k_{31} & k_{21} & \alpha_1 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}, \quad U_1 = \begin{pmatrix} k_{11}-\alpha_1 & k_{12} & k_{13} & \dots \\ 0 & k_{22}-\alpha_1 & k_{23} & \dots \\ 0 & 0 & k_{33}-\alpha_1 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

and α_1 is the parameter of the factorization allowing to put down (9) in iteration form

$$X= L_1^{-1} B_1 - L_1^{-1}U_1X. \tag{11}$$

A norm of the iterating operator $||L^{-1}U_1||$ is not smaller than unity but is very close to it. Therefore, the iteration process

$$X^{(n)}= L_1^{-1} B_1 - L_1^{-1} U_1X^{(n-1)} \tag{12}$$

does not always converge. Its convergence depends on a type of the right-hand side and the factorization parameter (α_1). If the iteration process (12) converges, then its subsequent solutions $X^{(n)}$ form an increasing limited subsequence, then they constitute a compact set in a Banach space. Corresponding discrepancies form a subsequence decreasing to zero. In this case, as was shown in [10], $X^{(n)}$ solutions tend to a normal pseudosolution of (9).Iteration algorithm (12) used in our paper allows to obtain a solution for any small value of regularization parameter α, including $\alpha = 0$. At the same time, there is a limit (4) allowing to obtain a normal pseudosolution of (2).

NUMERICAL SOLUTIONS

Testing of the proposed algorithm was carried out using both the ionosphere models and experimental data on temporal changes in the ionospheric component of the Doppler frequency shift, Δf, of navigation satellite signals, which were obtained within satellite elevation angles of 8–62^{o} (ascending branch), Fig. 1.

In order to obtain a numerical solution, the integral equation (1) was reduced to a standard integration interval. Under discretization, there was used a quadrature Gauss formula together with roots of Legendre polynomial as nodes. The number of nodes varied from 32 to 100. As a result, the solution does not practically change, its accuracy increasing slightly with the number of nodes.

A solution error of the ill-conditioned ME depends on an irregularity degree of the experimental data. For very irregular experimental data of the Doppler satellite signal shift (Fig. 1), used in calculations, a relative error is not more than 4%. A convergence rate of the iteration process (12) used for solving the ill-conditioned ME is extremely high. The time of solving the system of the 35th order with the ME condition number of ~10^{20} is 3 sec when using PC Pentium Pro 200 MHz (the program being written in Mathlab). Fig. 2 shows results of reconstructing the $N(z)$ profile, conforming to the ionosphere models. Curve 1 is the solution of algorithm integral equation (1) — $N(z)$ profile reconstructed using initial experimental temporal changes of Δf (dots). Curve 2 presents the $N(z)$ profile reconstructed by means of smoothing out the Δf dependence. When comparing these profiles, it is seen that the equation system solution is stable.

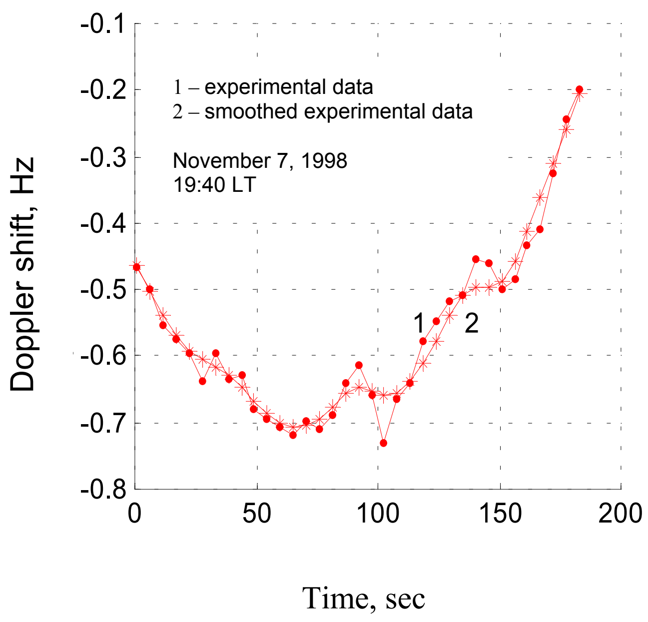

Fig. 1 Experimental temporal dependence of satellite signal Doppler shift on 150 MHz.

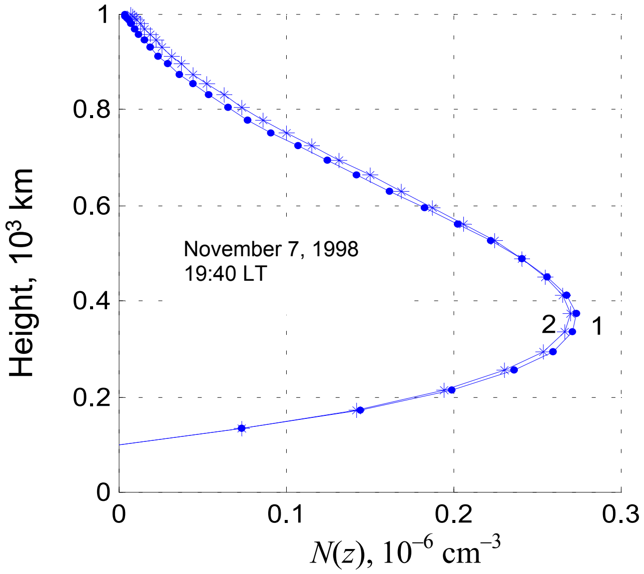

Fig. 2 Reconstructed electron density profiles.

CONCLUSION

A new iteration algorithm of reconstructing the $N(z)$ profile using records of the temporal changes in characteristics of transionospheric radio wave propagation is suggested. The reconstruction is made by solving the ill-conditioned linear algebraic system. The solution of this system is stable in the sense of Hadamard and immune to the sampling. Such an algorithm may be used for investigating dynamics of the ionospheric F2 region

REFERENCES

[1] F. Bertin and J. Papet-Lepine, "Determination du profil vertical de l'ionosphere a partir de la measure du contenu total d'electrons," *Comptes rendus Acad. Sc. Paris, Serie B*, vol. 269, no. 3, pp. 149–152 (1969).
[2] F. Bertin and J. Papet-Lepine, "Resolution des equations integrales reliant une serie de mesures obliques a une repartition verticale: appllication a la geophysic externe," *Annales de Geophysique*, vol. 26, no. 4, pp. 863–879 (1970).
[3] V. Gonzales, "A method for the determination of ionospheric electron density profiles from Faraday rotation of satellite borne radio signals," Electrical Engineering Research Laboratory/Univ/Illinois, NASA Grant No. Ns 624–54, (Sept. 1972).
[4] P. F. Checcacci and A. M. Scheggi, "Use of transfer function in ionospheric investigation". – In: Proc Symp. on the Future Application of Satellite Beacon Experiments, D-3411, Lindau/Harz, W. Germany. (Nov. 1970).
[5] P. F. Checcacci and A. M. Scheggi, "Transfer function of some ionospheric models," *IEEE Trans. Antennas Propagat*. (Commen.), No. AP-21, pp. 400–401 (1973).
[6] N. A. Armand, V. A. Andrianov, and V. M. Smirnov, *Radio Engineering and Electronics*, vol. 32, 4, p. 673 (1987) (in Russian).
[7] D. J. Donohue, H.-C. Ku, and D. R. Thompson, *IEEE Trans. Antennas Propagat*. AP-46, No. 1, p. 121 (1998).
[8] A. G. Tyzhnenko, Proc. Int. Conf. Mathematical Methods in EM Theory (MMET 2000), Kharkov, p. 417 (2000)
[9] A. F. Verlan and V. S. Sizikov, *Integral Equations*, Kyiv, 1986 (in Russian).
[10] A. N. Tikhonov and V. Ya. Arsenin, *Techniques for solving ill-conditioned problems*, Moscow, 1986 (in Russian).