ELECTRODYNAMIC CRITERION OF SMOOTHNESS AS EXPANDED RAYLEIGH'S CRITERION

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ABSTRACT

On the basis of the theory of a black body at the microwave electromagnetic waves range a criterion of a metallic surface smoothness by asperities as V-figurative and trapezoidal grooves is obtained. The criterion have linked the geometrical sizes of an average profile grooves to wave length and polarization of radiating, electrical and heat-conducting properties of a material. The linearly and chaotically polarized radiating incidence on the aperture of groove is reviewed. Reflective properties comparison of different metals surfaces, that are smooth by Rayleigh criterion and electrodynamic criterion, is produced. The discrepancy of design values of a molybdenum reflectance with experimental data has not exceeded 5 %.

INTRODUCTION

The millimeter-wave parabolic reflectors fabrication technique a grounded on cutting-tool edge working of metals. The working accuracy crude estimate can be received from Rayleigh criterion. In more details a problem on manufacturing tolerance reviewed in [1] with the help of the statistical theory of antennas (STA). However influencing a reflector material and a roughness correlation function on magnitude of tolerance till now was leave out of account. At fabrication of antenna reflectors increased grade of accuracy of reflecting surface carries on to heavy expenses on working. In conditions, when the antenna should mirror the radiation in operating range and disperse in infrared, for example, with the purpose of security of a antenna feed against overheat, the accuracy grade excess becomes invalid.

The real roughness comes nearer to the definite geometrical shape [2]. After surface pretreatment it is possible to approximate an asperity by V-figurative grooves [3]. The surface finishing work results in shearing off of asperities apexes [4] and reshapes trapezoidal corrugations. In this paper we decided a problem about a different polarized radiating incidence on a surface boarded with V-figurative or trapezoidal grooves at the following suppositions. 1. The size of a rough surface is much more than the sizes of one asperity. 2. On an investigated surface we consider one average asperity as groove, unlimited lengthwise, with the flat aperture of constant width. 3. Each facet of the groove mirrors as a flat part, i.e. the diffraction effects are leave out of account. 4. The reflectance of an incident wave on a rough surface sets equal to the same coefficient of one groove.

ELECTRODYNAMIC CRITERION OF SMOOTHNESS FOR E–WAVE INCIDENCE ON V-FIGURATIVE GROOVE

A wave with polarization vector H_y (E-wave) drops from half-space z < 0 on plane *xy* and excites a field inside groove with a V-figurative profile (fig. 1a). It is possible to present the groove sides as an inhomogeneous two-wire line, in which complex voltage U and current I are connected as

$$\text{a) } dU/dz=IZ_1; \qquad \text{b) } dI/dz=UY_1; \tag{1}$$

Linear impedance, conductivity, inductance and capacity in cross-section z are instituted as

$$Z_1=j\omega L_1,\ Y_1=j\omega C_1,\ L_1=L_0D_Z/D,\ C_1=C_0D/D_Z,\ D_Z=D-2z\cdot tg\beta=D(1-z/L)$$

Index "0" is referred to z=0. Let's assume, that losses in a line miss. A wave number and wave impedance are equal to

$$\gamma=\sqrt{Z_1\cdot Y_1}=j\frac{2\pi}{\lambda}, \qquad \rho=\sqrt{Z_1/Y_1}=\rho_0(1-z/L) \tag{2}$$

From (1) and (2) we find

$$\frac{d^2I}{dz^2}+\frac{dI}{dz}\frac{d}{dz}\ln(\rho/\gamma)-I\gamma^2=0, \quad \frac{d^2U}{dz^2}-\frac{dU}{dz}\frac{d\ln(\rho\gamma)}{dz}-U\gamma^2=0 \tag{3}$$

Let's divide a groove on N of elementary bulks by planes parallel *xy*, with spacing interval Δz between them (fig. 1a). Between planes $z=n\Delta z$ and $z=(n+1)\Delta z$ $(n=0, 1, ...N-1)$ we enter local coordinate ζ. Flat walls of the groove are approximated by surfaces with an exponential profile. In everyone elementary bulk the wave impedance is instituted as

$$\rho_n(\zeta)=\rho_n(\zeta=0)\cdot e^{c\zeta}. \tag{4}$$

Owing to a smallness of a spacing Δz inside elementary bulk a longitudinal wave number *c* is considered a constant. Then for V-figurative groove the following equalities are correct

$$\frac{d}{dz}\ln\left(\frac{\rho}{\gamma}\right)=\frac{d\ln(\rho\gamma)}{dz}=\frac{d}{dz}\ln\left(Z_0\left(1-\frac{z}{L}\right)\right)=-\frac{1}{L-z}=c\cdot \tag{5}$$

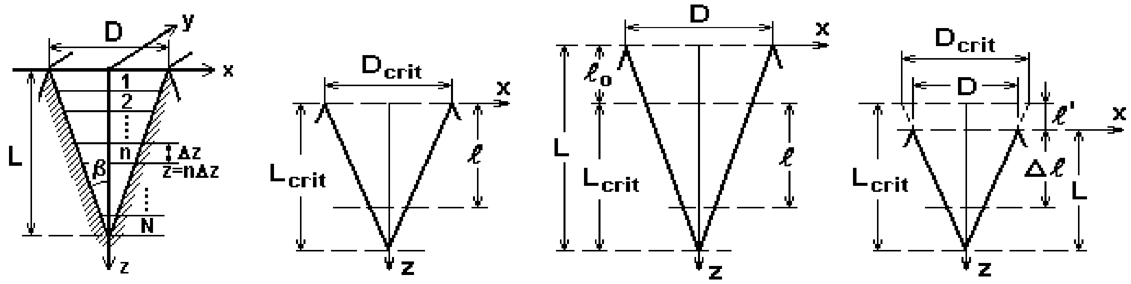

Fig. 1. V-figurative groove.

With (4), (5) solutions of a system (3) we find as the sum of voltage and current waves streaming inside Δz in directions $+z$, $-z$

$$U = U'\exp\Gamma'\zeta + U''\exp\Gamma''\zeta \ , \ I = I'\exp G'\zeta - I''\exp G''\zeta \tag{6}$$

where U', U'', I', I'' are amplitudes voltage and current waves streaming inside Δz; Γ', Γ'', G', G'' are propagation constants, those are roots of characteristic equations.

$$\Gamma' = c/2 + jp\,,\ \ \Gamma'' = c/2 - jp\,,\ \ G' = -c/2 + jp\,,\ \ G'' = -c/2 - jp\,, \text{ where } p = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \frac{1}{4(L-n\Delta z)^2}}\,. \tag{7}$$

From a condition p=0 we find $z=n\Delta z=L-\lambda/4\pi$. In groove with "«extreme" depth L=$\lambda/4\pi$ a propagated wave is absent.
In telegraph equation (1) we substitute (6), (7) and differentiate it on ζ considering z by a constant.

$$\left[-U'\left(\frac{c}{2}+jp\right) - I'\gamma\rho_0\left(1-\frac{z}{L}\right)\right]\exp\left[\left(\frac{c}{2}+jp\right)\zeta\right] = \left[U''\left(\frac{c}{2}-jp\right) + I''\gamma\rho_0\left(1-\frac{z}{L}\right)\right]\exp\left[\left(\frac{c}{2}-jp\right)\zeta\right]. \tag{8}$$

Equation (8) is identically contented at anyone ζ, therefore both factors at the exponential member equal to zero. Two equations for amplitude of currents I', I'' in n-th bulk result. The functions U', U'' are instituted from boundary conditions: if the number of elementary bulks on length L is equal to N, in plane $z=L$ equation $U'_N + U''_N = 0$ results; from here a voltage reflection factor is receivable.

Let's enter losses at the expense of final conductivity of groove walls. Let's consider n-th elementary bulk of groove as a cut of the flat waveguide with spacing interval D_Z between planes and with wave TEM (E00). In the flat lossy waveguide the variation of the mean for period energy stream along axis z is characterized as

$$P = P_0 e^{-2\chi_E z}\,, \text{ where } \chi_E = \frac{\sqrt{\pi f \mu \sigma^{-1}}}{D_z W_0}\,, \tag{9}$$

where f *is* a frequency, *Hz*; μ is the absolute magnetic permeability, *H/m*; σ is a metal conductivity, *1/(Ohm·m)*; W_0 is the free space resistance, *Ohm*.

For taking into account of a wave attenuation in a material of groove walls instead of (7) is received

$$\Gamma' \to \Gamma' - \chi_E\,,\ \ \Gamma'' \to \Gamma'' + \chi_E\,,\ \ G' \to G' - \chi_E\,,\ \ G'' \to G'' + \chi_E\,. \tag{10}$$

Let's write an energy stream variation inside groove to direction +z from cross-section $z=m\Delta z$ up to cross-section $z=n\Delta z$ as

$$\frac{P_n}{U'_m I'_m} = \exp\left(2\Delta z \sum_{k=m}^{n-1}\left(jp_k - \chi_{Ek}\right)\right). \tag{11}$$

For a quantitative assessment of a depth, in which walls of groove effectively reflect, coefficient of a smoothness is entered

$$A = s/S, \tag{12}$$

where s is a groove aperture area; S is an effective reflective internal surface area of the groove.

Let's investigate of different depth grooves.

The groove depth $L=L_{crit}$ (depth, which with taking into account of attenuation is more $\lambda/4\pi$) (fig. 1b). Inside groove there is a propagating wave. With growth of depth z the field power drops pursuant to (11). Having equated the left-hand part (11) to magnitude e^{-2} is receivable depth $z=\ell$, on which the current density drops by e. By analogy with the theory of a skin–effect we suppose, that all electromagnetic energy fallen outside in groove, effectively operates in bulk between planes $z=0$ $(m=0)$ and $z=\ell$. From (12) smoothness coefficient of groove with depth $L=L_{crit}$ is receivable

$$A_{critE} = \frac{L\sin(\beta)}{L\sin(\beta) + \ell(1-\sin(\beta))}. \tag{13}$$

The groove depth $L=\ell_0+L_{crit}$ (fig. 1c). The critical cross-section is on depth ℓ_0. Having substituted in (11) $L=L-\ell_0$, we calculate depth $z=\ell$, on which a current amplitude flowing to direction $+z$, drops by e in comparison with an amplitude of the same current in the critical cross-section ($m=m_{crit}$). The effective groove depth is equal to $\ell_0+\ell$. A smoothness coefficient is receivable as

$$A_{\uparrow E} = \frac{L\sin(\beta)}{L\sin(\beta) + (\ell_0+\ell)(1-\sin(\beta))}. \tag{14}$$

The groove depth $L<L_{crit}$ (рис. 1d). We calculate the dimension of groove aperture outline D_{crit}. On coefficient $k=D/D_{crit}$ is instituted how many time an energy level which drops on the groove aperture is diminishes with decreasing of depth from L_{crit} to L. We discover effective depth $\Delta\ell$ from (11), having multiplied a right part on k $(m=0)$. If $k\leq e^{-2}$, the surface is considered smooth. A smoothness coefficient is receivable as

$$A_{\downarrow E} = \frac{L\sin(\beta)}{L+\Delta\ell(1-\sin(\beta))}. \tag{15}$$

Let's formulate *the criterion of a smoothness*: for E-wave the surface with V-figurative grooves will be smooth if

$$L \leq L_{\text{crit}}-\ell. \tag{16}$$

ELECTRODYNAMIC CRITERION OF SMOOTHNESS FOR H–WAVE INCIDENCE ON V-FIGURATIVE GROOVE

The wave with polarization vector E_y (H-wave) drops from half-space z<0 on plane xy and excites a field inside groove with a V-figurative profile (fig. 1a). Each elementary bulk, for example, with number n is introduced as a cut of the rectangular waveguide of length Δz and width D_z. The incident H-wave excites in cuts, since maiden, a wave such as H_{01}. In case $D=m\lambda$, $m>1/2$ in direction $+z$ wave H_{01} is propagated with constant of propagation

$$\gamma_n = j\sqrt{\left(\frac{2\pi}{\lambda}\right)^2-[\frac{\pi L}{D_0[L-(n-1)\Delta z]}]^2-\chi_H}, \quad \text{where} \quad \chi_H=\frac{\sqrt{\pi\cdot f\cdot\mu\cdot\sigma^{-1}}}{W_0}\cdot\frac{\lambda^2}{D_z^{\,2}\sqrt{(2D_n/\lambda)^2-1}} \tag{17}$$

Equation (17) institute an wave H_{01} attenuation in cuts of the flat waveguide with spacing interval D_z between plates. With growing depth z the groove width and parameter γ are moderated, at $\gamma=0$ (in "critical" cross-section) the wave ceases to be propagated. From equation g=0 "critical" cross-section depth $z=\ell_0=L-\lambda/(4tg\beta)$ and groove width in the "critical" cross-section $D_{crit}=\lambda/2$ is receivable. Equations (13)–(16) are similar to expressions for the E–wave case.

ELECTRODYNAMIC CRITERION OF SMOOTHNESS FOR LINEARLY AND CHAOTIC POLARIZED WAVES INCIDENCE ON TRAPEZOIDAL GROOVE

Trapezoidal profile (fig. 2) may be receivable, If between the apertures of adjacent V-figurative grooves to enter flat parts of a surface with $s_1=tD$, where t is a coefficient of proportionality. Smoothness coefficients of a surface for H (E) wave is instituted under formulas

$$A_{t\uparrow H(E)} = \frac{(1+t)L\sin\beta}{L(1+t\cdot\sin(\beta))-\ell(1-\sin\beta)}, \quad A_{t\downarrow H(E)} = \frac{(1+t)L\sin\beta}{L\cdot t\cdot\sin(\beta)+\ell(1+\sin\beta)}. \tag{18}$$

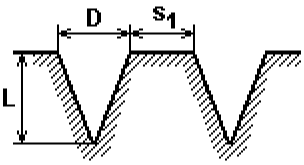

Fig. 2. Trapezoidal grooves.

The nonpolarized wave can be decomposed on a pair of uncorrelated waves with orthogonal polarizations and identical average amplitudes. The total power of the nonpolarized wave equal to the sum of powers of the polarized waves. Therefore a smoothness coefficient for the nonpolarized wave on known ones of two orthogonally polarized waves is equal to

$$A=(A_H+A_E)/2. \tag{19}$$

SIMULATION RESULTS

Let's express smoothness coefficient A through an integral hemispherical reflectance R_S

$$R_S = \int_0^\infty R_\lambda U_\lambda d\lambda \cdot \left(\int_0^\infty U_\lambda d\lambda\right)^{-1}, \tag{20}$$

where U_λ is Plank's equation;

$$R_\lambda = rA/[1-r(1-A)] \tag{21}$$

R_λ is spectral hemispherical reflectance [2], r is reflectance of a smooth metallic surface.

At $\lambda > 1$ μm for heat-resistant metals [5]

$$r = 1-5.8\cdot 10^{-5}\sqrt{\frac{T}{\lambda\cdot\delta}}, \tag{22}$$

where T is a temperature, K; δ is heat conductivity factor, W/m·K.

In the table the results of calculations of magnitude R_S (20) and experimental data [6] are shown.

Table

Grade of roughness	R_S									
	T=1200 K		T=1400 K		T=1600 K		T=1800 K		T=2000 K	
	Trial	Count	Trial	Count	Trial	Count	Trial	Count	Trial	Count
10 b	0,885	0,885	0,86	0,862	0,835	0,84	0,81	0,818	0,786	0,798
9 a	0,882	0,882	0,858	0,862	0,833	0,844	0,808	0,827	0,784	0,811
6 a	0,874	0,874	0,85	0,855	0,824	0,836	0,799	0,819	0,775	0,803

The calculations are made for the case of E-wave incidence on a surface with V-figurative grooves, class of roughness was 6a and 9a, β = 45.5°; and on a surface with trapezoidal grooves, class of roughness was 10b, t = 0.25, β = 26°; λ =1–13 microns. The measurements of the molybdenum integral hemispherical radiant emittance were made by a calorimetric method with electronic heating of a sample at temperature range 1200-2000 K. Surfaces of two samples were treated by a sandpaper, the check sample was polished [6].

The minor discrepancy of calculated and measured results at temperature increasing is conditioned by the shear of Planck's curve maximum towards more short wavelengths, inaccuracy of (21) on the boundary of adaptability and errors of the calorimetric measurement method. The ratio of measured integral hemispherical radiant emittance to counted one differs less, than on 5 % from a ratio of full integral surface density of a black-body radiation to integral surface density of radiating at the waveband 1-13 microns.

In fig. 3 dependences of a discrepancy r-R_S of smooth and rough surface reflectance from wavelength are shown at different parameters of roughness.

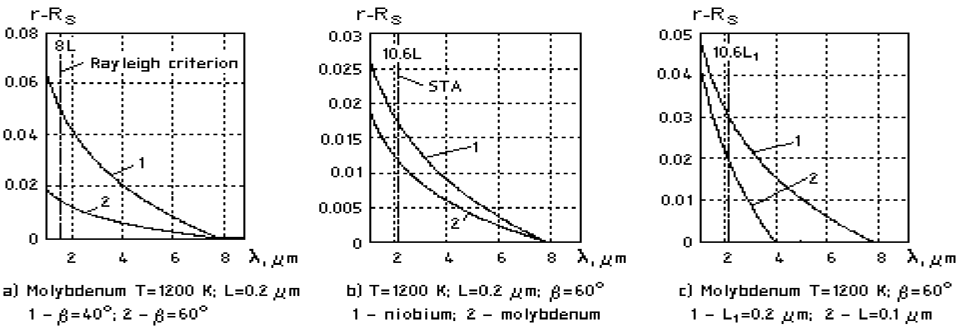

Fig. 3. Discrepancies of smooth and rough surface reflectance from wavelength at different parameters of roughness.

CONCLUSION

Reflectivity R_{sm} of smooth surface for waves $\lambda \geq \lambda_{sm}$ is instituted by (21) with A=1. For $\lambda < \lambda_{sm}$ smoothness coefficient A < 1 and the value of reflectivity of a rough surface is less than R_{sm}. So, for a smooth molybdenum surface by Rayleigh criterion ($L \leq \lambda/16$) – grade of roughness is 9a, T=1200 K – the reflectivity is less by 17 % than counted one according to (21). For $L \leq \lambda/20$, $L \leq \lambda/32$, $L \leq \lambda/39$ and $L \leq \lambda/80$ the difference is equal to 10; 2; 1 and 0,05 % correspondingly, for $L \leq \lambda/125$ the difference is less than 0,01 %. It is practically established, that the increase of the requirements to a working accuracy of a pillbox antenna reflector from $\lambda/20$ up to $\lambda/80$ carries on to decrease a level side lobes by 5 dB, the further enhancement of working accuracy does not give noticeable results [7].

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