DESCRIPTION OF GAUSSIAN BEAMS PROPAGATION ON THE BASIS OF COMPLEX GEOMETRICAL OPTICS

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ABSTRACT

We present efficient numerical algorithms for calculation of Gaussian beam wave fields in inhomogeneous media. The proposed algorithms are based on complex geometrical optics (CGO). The use of the perturbation theory for complex rays significantly shortens the calculations in comparison with the complete numerical algorithm of CGO, to put aside the parabolic equation method. Examples are presented to illustrate the efficiency of the proposed method.

1. INTRODUCTION

The capability of complex geometrical optics (CGO) to describe the diffraction of Gaussian beams has been known for as long as 30 years [1–2]. However, the numerical algorithms describing such diffraction were not developed until recently [3, 4]. These cover a wider scope of problems: besides the calculation of Gaussian beam fields, the developed algorithms can solve almost all the CGO problems, as applied to Gaussian beams there are surplus. In this work, the numerical algorithm of wave field calculation by the CGO method, applied to Gaussian beams only, is much simplified on the account of the perturbation theory.
The simplified calculation of wave fields of a point source according to the perturbation theory is outlined in Sec. 2. Section 3 is dedicated to the perturbation theory for Gaussian beams. In Sec. 4, the proposed procedure is applied to the case of Gaussian beam propagation in a homogeneous medium. The domains of applicability of the proposed method are discussed in Sec. 5. Section 6 finally presents our examples of numerical calculation of wave fields for various cases of Gaussian beam propagation in inhomogeneous media.
In contrast to the general scheme which requires sighting of complex rays to each observational point, the simplified scheme carries out sighting only to one or two points of the beam cross section in two- or three-dimensional problems, respectively.

2. PERTURBATION THEORY FOR A POINT SOURCE

A general numerical scheme of the CGO presupposes determination of all the complex rays coming to a given real observational point [3, 4]. In the case of Gaussian beams, the general scheme of search for complex rays becomes simpler in two respects. First, due to weak divergence of Gaussian beams, a set of complex solutions can be restricted to only complex rays $\mathbf{r}_1(\tau)$ close to the central real ray $\mathbf{r}_{\rm ref} = \mathbf{r}_{\rm ref}(\tau)$ of the beam, where τ is the parameter along the beam. The second opportunity to diminish calculations in the case of Gaussian beams is related to the perturbation theory for the rays leaving the point source. To explain the calculation idea, we consider the reference ray $\mathbf{r}_{\rm ref}(\tau)$ and a ray $\mathbf{r}_\beta(\tau)$ close to it, leaving source O at a small angle β to the reference ray (see Fig. 1). We select a certain point M_0 and consider point M closest to it in the perturbed ray $\mathbf{r}_\beta(\tau)$. Let the distance between M and M_0 be δ. Within the linear perturbation theory, the value δ is proportional to β, while the difference of eikonals at points M and M_0 is proportional to δ^2,

$$\psi(M) - \psi(M_0) \approx \frac{1}{2} K \delta^2, \tag{1}$$

where K is the local curvature of the phase front at point M_0.

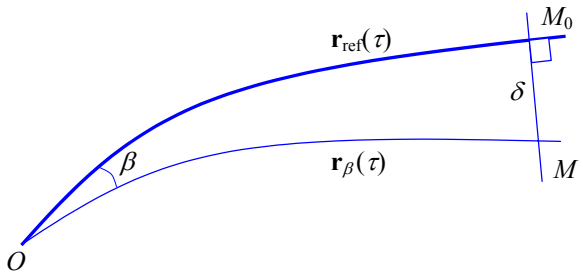

Fig. 1. To the calculation of the difference of eikonals from two close rays leaving a point source. The central ray is shown by the bold curve, the perturbed ray exits at the small angle β to the central ray. The eikonal at point M in the perturbed ray differs from that at the closest point M_0 in the central ray by the value proportional to the squared distance δ between these points.

3. PERTURBATION THEORY FOR GAUSSIAN BEAMS

As is known, the Gaussian beam field can be considered as a field of a point source located in an expanded complex space [2]. Relation (1) retains its validity in the case of Gaussian beam if eikonals $\psi(M)$ and $\psi(M_0)$, as well as curvature K, are considered as complex values. In this case, the real $\mathrm{Re}\,K$ and imaginary $\mathrm{Im}\,K$ parts of the complex curvature are related, respectively, to the curvature radius R of the phase front and the Gaussian beam width a, $K = 1/R + i/ka^2$, where k is the wavenumber. Thereafter, we distinguish these two values, interpreting the curvature K as its real part only.

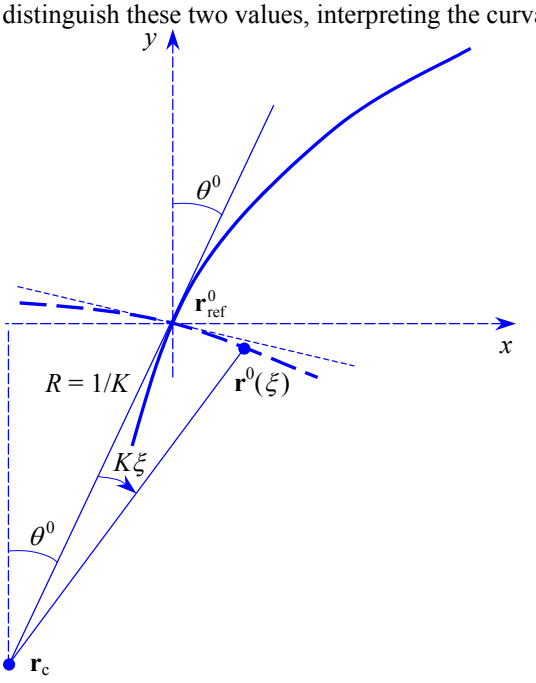

Fig. 2. Approximation of the field phase front of the paraxial Gaussian beam by a parabola. The bold solid and bold dashed curves respectively indicate the central ray and the phase front left by complex rays; $\mathbf{r}_{\mathrm{c}}$ is the beam curvature center (our considered point), where the parameters of the ray $\mathbf{r}^0_{\mathrm{ref}}$ are specified; $\mathbf{r}^0(\xi)$ is any phase front point characterized by the parameter ξ. The central ray exits from the phase front at the point $\mathbf{r}^0(0) = \mathbf{r}^0_{\mathrm{ref}}$ at the angle θ^0.

We describe the calculation algorithm for the Gaussian beam field in the vicinity of the central ray. Let the exit point $\mathbf{r}^0_{\mathrm{ref}}$ of the central ray, the initial beam propagation direction characterized by the angle θ^0 with the vertical axis, the phase front curvature K^0, and the initial beam width a^0 be given (see Fig. 2), as well as the medium permittivity as the

function $\varepsilon(\mathbf{r})$. The values $K(\tau)$ and $a(\tau)$ are to be determined along the trajectory of the central ray $\mathbf{r}(\tau)$ of the Gaussian beam. Within the perturbation theory, we suppose the beam shape to remain Gaussian along the whole central ray.
To numerically solve the ray equations, we find an equation of the initial exit line of the rays, coinciding with the phase front parametrized by the variable ξ. Let $\mathbf{r}_c$ be the radius vector of the beam curvature center and $R = 1/K$ be the curvature radius. Then the point with the parameter ξ at the phase front has the radius vector

$$\mathbf{r}^0(\xi) = \mathbf{r}_c + R\begin{bmatrix}\cos(\theta^0 + K\xi)\\ \sin(\theta^0 + K\xi)\end{bmatrix}. \tag{2}$$

Supposing the beam to be paraxial, we expand (2) in a power series of small parameter $K\xi << \theta^0$, neglecting all the terms above the second order,

$$\mathbf{r}^0(\xi) = \mathbf{r}^0_{\text{ref}} + \begin{pmatrix}\cos\theta^0\\ -\sin\theta^0\end{pmatrix}\xi + \frac{K}{2}\begin{pmatrix}-\sin\theta^0\\ -\cos\theta^0\end{pmatrix}\xi^2. \tag{3}$$

This relation approximates the phase front by a parabola near the reference ray shown in Fig. 3 by bold dashed line. Let the eikonal at the phase front be $\psi^0(\xi) = i\xi^2/2k(a^0)^2$ [2]. Then the initial wave field is given by $u^0 = \exp(ik\psi^0) = \exp[-\xi^2/2(a^0)^2]$, where a^0 is the beam width at point $\mathbf{r}^0_{\text{ref}}$, and k is the wavenumber. We find two components of the ray momentum $\mathbf{p}^0(\xi)$ at point $\mathbf{r}^0(\xi)$ at the phase front from the equation

$$\frac{\partial\psi^0}{\partial\xi} = \mathbf{p}^0(\xi)\frac{\partial\mathbf{r}^0}{\partial\xi} \tag{4}$$

and the eikonal equation

$$\left(\mathbf{p}^0\right)^2 = \varepsilon\left[\mathbf{r}^0(\xi)\right]. \tag{5}$$

Thus, from $\mathbf{r}^0_{\text{ref}}$, θ^0, K^0, and a^0, we find the functions $\mathbf{r}_0(\xi)$ and $\mathbf{p}^0(\xi)$, which are in this case the initial conditions for the ray equations

$$\frac{d\mathbf{r}}{d\tau} = \mathbf{p}, \qquad \frac{d\mathbf{p}}{d\tau} = \frac{\nabla\varepsilon(\mathbf{r})}{2}. \tag{6}$$

The latter is to numerically solved together with the differential equation for the eikonal

$$\frac{d\psi}{d\tau} = \varepsilon(\mathbf{r}). \tag{7}$$

Let $\mathbf{F}(\xi, \tau) \equiv (\mathbf{r}, \mathbf{p}, \psi)$ be the five-dimensional complex vector with components F_j. The first four components are the coordinates $(\mathbf{r}, \mathbf{p}) \equiv (x, y, p_x, p_y)$ of the complex ray in the phase space $\{\mathbf{r}, \mathbf{p}\}$, and the fifth component is the eikonal ψ of the corresponding complex ray. Then ray equations (6) in combination with (7) take on the form convenient for numerical integration,

$$\frac{d\mathbf{F}}{d\tau} = \mathbf{R}(\mathbf{F}), \tag{8}$$

where $\mathbf{R}(\mathbf{F})$ is the complex vector function of the right-hand sides of the set of ordinary differential equations (6) and (7),

$$\mathbf{R}(\mathbf{F}) \equiv \left[F_3, F_4, \frac{\partial\varepsilon}{\partial x}(F_1, F_2), \frac{\partial\varepsilon}{\partial y}(F_1, F_2), \varepsilon(F_1, F_2)\right]. \tag{9}$$

A solution to these equations has to meet the initial conditions written as the unified vector relation

$$\mathbf{F}^0(\xi) \equiv \mathbf{F}(\xi, \tau = 0) = \left[\mathbf{r}^0(\xi), \mathbf{p}^0(\xi), \psi^0(\xi)\right]. \tag{10}$$

We suppose that substituting $\xi = 0$ into initial conditions (10), we have numerically integrated set (8) for the central ray up to any real parameter τ_{ref} to find the values $\mathbf{F}^1(\tau_{\text{ref}}) = (\mathbf{r}^1_{\text{ref}}, \mathbf{p}^1_{\text{ref}}, \psi^1_{\text{ref}})$. Then the corresponding curvature $K^1(\tau_{\text{ref}})$ and beam width $a^1(\tau_{\text{ref}})$ are to be calculated.

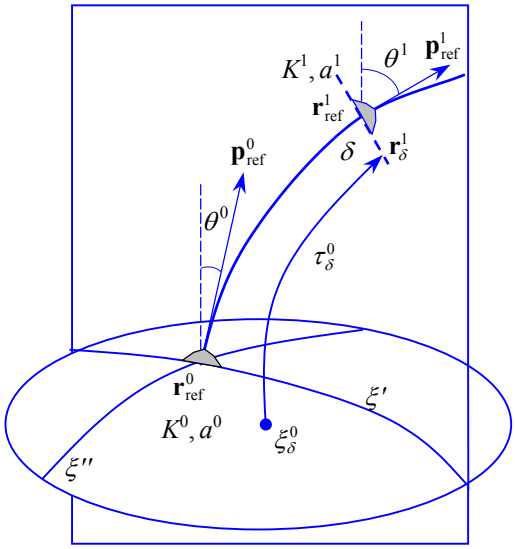

Fig. 3. To the explanation of the algorithm of search for the parameters a^1 and K^1 from the initial characteristics a^0 and K^0 of the Gaussian beam at the given "distance" τ_{ref} along the central ray. One complex ray that left the point ξ^0_δ of the phase fronts hits the point of our concern, spaced by the distance δ from the central ray (bold curve). The phase front is analytically continued to the complex region, where $\xi' = \operatorname{Re}\xi$ and $\xi'' = \operatorname{Im}\xi$. Within the linear perturbation theory, this complex ray is found by two iterations by the Newton method.

First, we draw a normal to the ray at point $\mathbf{r}^1_{\text{ref}}$ (Fig. 3). It is defined by the unit vector

$$\mathbf{n}^1 = \frac{(p^1_y, p^1_x)}{|\mathbf{p}^1|} \tag{11}$$

In normal $\mathbf{n}^1$, we select point $\mathbf{r}^1_\delta$ spaced from the reference ray by the short distance δ,

$$\mathbf{r}^1_\delta = \mathbf{r}^1_{\text{ref}} + \delta\mathbf{n}^1. \tag{12}$$

Second, we find the complex ray coming to a real point, characterized by the complex exit point ξ^0_δ and "length" τ^0_δ by sighting the complex rays by the Newton method (see [3, 4]). For initial approximation, we select the reference ray, for which $\xi_0 = 0$ and $\tau_0 = \tau_{\text{ref}}$.

Due to smallness of δ, we employ the Jacobi matrix $\hat{D}$ in the Newton iterative process

$$\begin{pmatrix} \xi_{\alpha+1} \\ \tau_{\alpha+1} \end{pmatrix} = \begin{pmatrix} \xi_\alpha \\ \tau_\alpha \end{pmatrix} - \hat{D}^{-1}(\xi_\alpha, \tau_\alpha)\left[\mathbf{r}(\xi_\alpha, \tau_\alpha) - \mathbf{r}^1_\delta\right], \tag{13}$$

calculated not at the next αth iteration ξ_α, τ_α, as it is accepted in sighting of complex beams [3, 4], but at the reference ray $\xi_0 = 0$, $\tau_0 = \tau_{\text{ref}}$,

$$\hat{D}(0, \tau) = \begin{pmatrix} \left.\dfrac{\partial x^1}{\partial \xi}\right|_{\text{ref}} & \left(\mathbf{p}^1_{\text{ref}}\right)_x \\ \left.\dfrac{\partial y^1}{\partial \xi}\right|_{\text{ref}} & \left(\mathbf{p}^1_{\text{ref}}\right)_y \end{pmatrix}, \tag{14}$$

where $(x^1(\xi), y^1(\xi)) = \mathbf{r}^1$. Then the sought-for values ξ_δ^0 and τ_δ^0 are calculated in two iterations,

$$\begin{pmatrix} \xi_1 \\ \tau_1 \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_{\text{ref}} \end{pmatrix} - \hat{D}^{-1}\left(\mathbf{r}_{\text{ref}}^1 - \mathbf{r}_\delta^1\right),$$
$$\begin{pmatrix} \xi_2 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} \xi_1 \\ \tau_1 \end{pmatrix} - \hat{D}^{-1}\left[\mathbf{r}\left(\xi_1, \tau_1\right) - \mathbf{r}_\delta^1\right] = \begin{pmatrix} \xi_\delta^0 \\ \tau_\delta^0 \end{pmatrix}. \tag{15}$$

Then, $\mathbf{F}_\delta^1 \equiv \mathbf{F}(\xi_\delta^0, \tau_\delta^0) = (\mathbf{r}_\delta^1, \mathbf{p}_\delta^1, \psi_\delta^1)$ calculating via numerical integration (8), we find the beam parameters at trajectory point τ_{ref} we were interested in: the angle θ^1 of the central beam tilt to the vertical axis, the phase front curvature K^1, and the width a^1,

$$\theta^1 = \arctan\left[\frac{\left(\mathbf{p}_\delta^1\right)_x}{\left(\mathbf{p}_\delta^1\right)_y}\right],$$
$$K^1 = \frac{2\,\text{Re}\left(\psi_\delta^1 - \psi_{\text{ref}}^1\right)}{\delta^2}, \quad a^1 = \frac{\delta}{\sqrt{k\,\text{Im}\,\psi_\delta^1}}. \tag{16}$$

If the beam shape remains Gaussian, these values are independent of the parameter δ. Besides, we have all the values necessary to calculate the beam amplitude,

$$A^1 = \sqrt{\frac{\det \hat{D}\left(\tau = 0\right)}{\det \hat{D}\left(\tau = \tau_{\text{ref}}\right)}}, \tag{17}$$

so that the Gaussian beam field in the vicinity of point $\mathbf{r}_{\text{ref}}^1$ is given by

$$u^1\left(\tau_{\text{ref}}, \delta\right) = A^1 \exp\left\{ik\left[\psi'\left(\tau_{\text{ref}}\right) + i\psi''\left(\delta\right)\right]\right\}$$
$$= A^1 \exp\left[-\frac{\delta^2}{2\left(a^1\right)^2}\right] \exp\left(ik\frac{\delta^2}{2R^1}\right), \tag{18}$$

where $\psi' = \text{Re}\,\psi_\delta^1$, $\psi'' = \text{Im}\,\psi_\delta^1$, and $R^1 = 1/K^1$ is the curvature radius of the beam phase front at the considered trajectory point.

Then the iterative process is repeated. Having the new beam parameters θ^1, K^1, and a^1, we carry out sighting to the new point $\mathbf{r}_\delta^2$ and find the new exit point ξ_δ^1 of the complex beam at the phase front $\mathbf{r}^1(\xi)$ and its new "length" τ_δ^1. Similarly we find θ^2, K^2, a^2, and A^2, moving via the reference ray to the new real value τ_{ref}.

An important advantage of this method for calculating the beam parameters along its trajectory is an opportunity to check applicability of the employed procedure. If the found complex ray has not hit the point $\mathbf{r}^1$ in two iterations (15), three and more iterations are necessary. Under these conditions, the assumption on the square-law dependence of the complex phase ψ_δ^1 on the shift δ fails to put the complete CGO numerical algorithm [3, 4] into use.

4. PERTURBATION THEORY FOR COMPLEX RAYS IN A HOMOGENEOUS MEDIUM

Let us illustrate the run of the described procedure for a homogeneous two-dimensional space, $\varepsilon = 1$. We set the initial beam coming out of the coordinate origin along the axis y, supposing the initial front to be plane. Then $\theta^0 = 0$, $K^0 = 0$, and $a^0 = a$. The phase front left by the rays represents line $x^0 = \xi$, on which the initial field is specified as $u^0 = \exp(ik\psi^0)$, $\psi^0 = i\xi^2/2a^2$. An analytical solution to ray equations (8) represents complex trajectories of the set of rays defined by the complex exit point ξ and parametrized by the complex length τ (see [2]),

$$x(\xi,\tau)=\left(1+\frac{i\tau}{b}\right)\xi,$$
$$y(\xi,\tau)=\left(1+\frac{\xi^2}{2b^2}\right)\tau, \tag{19}$$

where $b = ka^2$. The Jacobi matrix of the Cartesian-to-ray coordinate transformation is given by

$$\hat{D}(\xi,\tau)=\begin{pmatrix}1+i\tau/b & i\xi/b\\ \xi\tau/b^2 & 1+\xi^2/2b^2\end{pmatrix}. \tag{20}$$

Our concern is the complex ray hitting the real point $(x,y) = (\delta,y)$. We choose initial sighting values in the straight reference ray coinciding with axis y: $\xi_0 = 0$, $\tau = y$. The first iteration (15) yields

$$\begin{pmatrix}\xi_1\\ \tau_1\end{pmatrix}=\begin{pmatrix}0\\ y\end{pmatrix}-\begin{pmatrix}1 & 0\\ 0 & 1/g\end{pmatrix}\begin{pmatrix}-x\\ 0\end{pmatrix}=\begin{pmatrix}x/g\\ y\end{pmatrix}, \tag{21}$$

where $g = (1 + iy)/b$. Comparing the found ξ and τ to the corresponding values from review [2], we conclude that even the first iteration yields the true exit point ξ, while the second iteration yields the ray "length" τ,

$$\begin{pmatrix}\xi_2\\ \tau_2\end{pmatrix}=\begin{pmatrix}x/g\\ y\end{pmatrix}-\begin{pmatrix}1 & 0\\ 0 & 1/g\end{pmatrix}\begin{pmatrix}0\\ \dfrac{yx^2}{2b^2g^2}\end{pmatrix}=\begin{pmatrix}x/g\\ y\left[1-\dfrac{(x/g)^2}{2b^2}\right]\end{pmatrix}. \tag{22}$$

Thus, even two sighting iterations by the Newton method result in exactly the complex ray hitting the specified real point (x,y). Repeating the further calculations of review [2], we arrive at the exact expression for the wave field of the Gaussian beam,

$$u=\frac{1}{g}\exp\left(iky-\frac{x^2}{2ga^2}\right). \tag{23}$$

which can be found from the parabolic equation.

5. LIMITS OF APPLICABILITY OF THE PROPOSED PROCEDURE. FEATURES OF THE BEAM BEHAVIOR NEAR CAUSTICS

The assumption on the linear growth of the shift δ with the angular perturbation β, as well as a square-law growth of the eikonal as receding from the reference ray is valid only in the vicinity of the reference ray, whose width can be estimated as μL, where μ is the perturbation and L is the characteristic scale of the medium property variations.

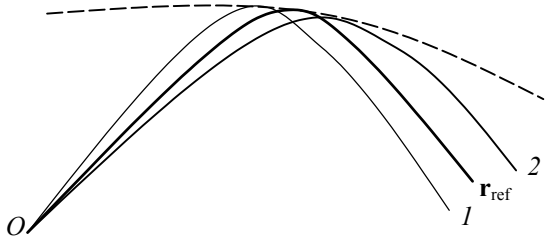

Fig. 4. Central ray $\mathbf{r}_{\text{ref}}$ shown by bold curve and two marginal rays *1* and *2* left a point source. Both marginal rays *1* and *2* appear lower than the central ray near the ray–caustic tangency point (dashed curve).

The applicability conditions of the linear perturbation theory are also violated near the caustics. To qualitatively describe the Gaussian beam passage near the caustic, we consider a reference ray $\mathbf{r}_{\text{ref}}$ and two marginal rays *1* and *2* coming out of a point source (Fig. 4). Both marginal rays appear lower than the reference ray near the reference ray–caustic tangency point. One can see in Fig. 4 that two following effects can be observed in the vicinity of the caustic. First, the beam narrows about twice near the caustic. Second, the beam becomes asymmetric, it appears entirely on one side of the reference ray.

Both effects considered in the third example of Sec. 6 are described by the complete (not simplified) CGO algorithm.

6. EXAMPLES

In this Section, we present examples illustrating the proposed algorithm efficiency.
In the first example, we consider a Gaussian beam passing along the axis y in a waveguide with a quadratic profile of the permittivity,

$$e = e_0\left(1 - x^2/L^2\right), \tag{24}$$

where ε_0 is the permittivity of waveguide axis and L is the effective thickness of the waveguide. In the numerical calculations, we put $\varepsilon_0 = 1.1$ and $L = 200\sqrt{2}\lambda$, where λ is the wavelength. The beam width corresponding to the fundamental mode is $a_c = kLe_0^{-1/2}$.Therefore, such a Gaussian beam is to propagate along the waveguide with no change.

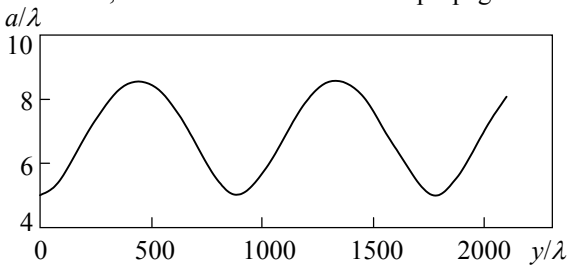

Fig. 5. Dependence of the Gaussian beam width a/λ on the longitudinal coordinate y/λ in the waveguide with a parabolic profile of the permittivity; λ is the wavelength.

Figure 5 shows the width $a(y)$ variation for a beam with an initial width narrower than a_c. Not only the beam width $a(y)$ varies along axis y, but also its other parameters: the phase front curvatures $K(y)$ and amplitude $A(y)$ [5]. Such variations are caused by interference between the two first waveguide eigenmodes. The solution found by the CGO method well conforms to the exact solution to the Helmholtz equation for this case.
The second example illustrates oblique incidence of the Gaussian beam onto a linear plasma layer,

$$e(x, y) = 1 - Cx - Dy, \tag{25}$$

with the parameters $D = 1/1000\lambda$ and $C = -D/2$. The initial beam width is 5λ. The permittivity lines are shown in Fig. 6*a* (dotted line). The beam initially expands along the central ray shown in Fig. 6*a* by bold curve, but becomes narrow again near the caustic. The described perturbation theory is inapplicable for complex rays; therefore, we calculated the beam field by the complete numerical CGO algorithm. In the caustic region, the beam center is shifted with respect to the central ray, and the beam loses its Gaussian shape. Figure 6*b* shows the profile (absolute value) of the beam wave field in the cross section of the central ray. In this case, the shift d of the wave field maximum is about 20 % of the beam width. We also note that the beam tracing method [5, 6] is inapplicable in the caustic region.

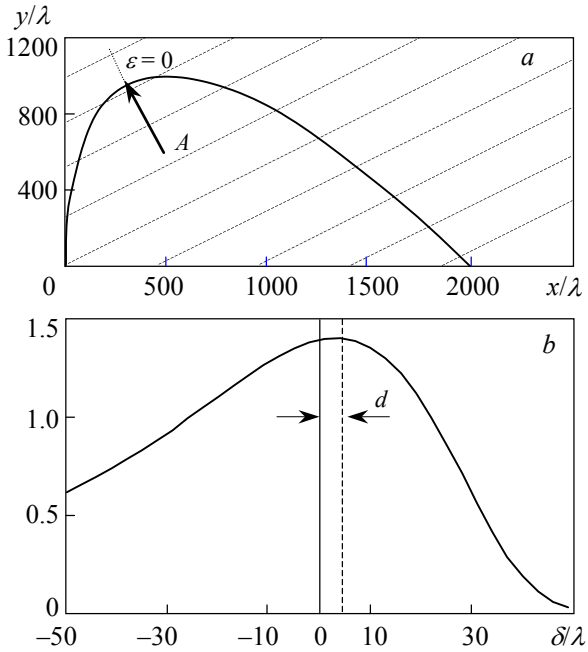

Fig. 6. (*a*) Central ray of the Gaussian beam propagating in a linear plasma layer (bold curve). The permittivity levels are shown by dotted lines. (*b*) The asymmetric profile of the beam wave field modulus in cross section *A* showed by bold arrow. The dashed and solid lines show, respectively, the maximum of the wave field magnitude $A(y)$ and the central ray position. The shift is characterized by the spacing *d* between the lines

Other examples illustrating the efficiency of the CGO numerical algorithms for the calculation of wave fields of Gaussian beams in inhomogeneous media are given in [3, 4].

7. CONCLUSION

The proposed method efficiently describes the propagation of Gaussian beams in inhomogeneous media, as was shown in the above examples. An important advantage of the method is the simplicity of its numerical version and an appreciably smaller calculations not only in comparison with the complete wave problem, but also with the complete numerical CGO algorithm. Another useful feature of the method is an opportunity to check its applicability. The ideas of the method can make a basis of numerical algorithms to be derived to solve more complicated problems, such as propagation of Gaussian beams in inhomogeneous anisotropic plasma, taking into account polarization effects.
Undeniably, development of numerical algorithms makes CGO an efficient method of the wave theory.

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