ON DISPERSION AND POLARIZATION OF MODES IN A CHIRAL OPTICAL FIBRE

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ABSTRACT

The results of the study of modal properties of an optical fibre with a core of chiral material and nonchiral cladding are presented. The dispersion equation for modes of the chiral optical fibre is obtained and calculated. It is shown that some modes of a chiral optical fibre have cut-off frequencies both in the lower and upper parts of the frequency range. Polarization properties of modes in intervals of frequencies including the points of degeneracy and quasi-degeneracy are studied in details. It is shown, that in the intervals including points of quasi-degeneracy the circular polarization of modes changes: right-handed to left-handed, and vice versa.

INTRODUCTION

In this paper, we presented the results of the study of modal properties of an optical fibre with a core of chiral material and nonchiral cladding. The dispersion equation for modes of the chiral optical fibre is obtained, like for a planar chiral optical waveguide [1,2]. The dispersion characteristics and fields of modes are calculated. It is shown that some modes of a chiral optical fibre have cut-off frequencies both in the lower and upper parts of the frequency range. The approximate scalar analysis of modes of chiral optical fibres with a small difference of the refractive indexes of a core and cladding was presented in the work [3], where circularly polarized (CP) modes for chiral optical fibres were put into operation like linearly polarized (LP) modes for nonchiral optical waveguides [4,5]. Obtained in the present work approximate dispersion equation corresponds to CP-modes and has the similar form as for LP-modes of nonchiral optical fibres. This approximate equation was divided into two independent equations for two classes of modes with the right-handed and left-handed circular polarizations.

The further researches [6,7] showed, that some properties of modes of chiral optical waveguides, namely dispersion and polarization characteristics in the frequency area near to the spectral degeneration points (a cross of the dispersion curves) in scalar approximation analysis are described qualitatively incorrectly. In the locality of these points it is necessary to take into consideration the vector properties of modes and to use a precise dispersion equation for modes of circular chiral dielectric waveguide, that allows to study a fine pattern of dispersion curves in the locality of quasi-degeneracy points. In the present work it is demonstrated, that for one group of circular dielectric waveguide modes these points are degeneracy points and for other group, they are quasi-degeneracy points. The polarization and structure of modes fields are qualitatively transformed in an interval of frequencies including these points.

CIRCULAR CHIRAL DIELECTRIC WAVEGUIDE

The circular chiral dielectric waveguide is schematically shown in Fig. 1, where the chiral core with refractive index n_c, chirality ρ, radius R and nonchiral cladding with refractive index n_o.

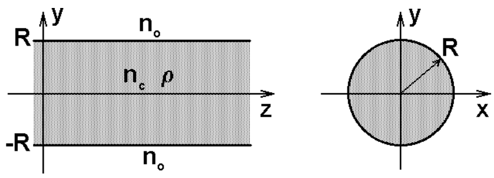

Fig. 1. A chiral optical fibre

For the analysis of modes, we use functions $\vec{F}^{+}$ and $\vec{F}^{-}$ rather than the field functions $\vec{E}$ and $\vec{H}$ that are coupled with the former by the relationships $\vec{F}^{\pm} = \vec{E} \pm i\zeta\vec{H}$ [1-3,6], where $\zeta = (\mu/\varepsilon)^{1/2}$, ε and μ are the permittivity and permeability of the medium. The longitudinal components of functions $F_z^{\pm}$ for waveguide modes are represented as

$F_z^{\pm} = \Psi^{\pm}(r,\varphi)\exp i(\omega t - \gamma z)$, where γ is the propagation constant of the mode. The cross-section components $F_r^{\pm}$ and $F_{\varphi}^{\pm}$ can be expressed of the longitudinal components $F_z^{\pm}$ by the formulas

$$F_r^{\pm} = \frac{1}{k^2 n^2 - \gamma^2}\left(-i\gamma\frac{\partial F_z^{\pm}}{\partial r} \mp kn\frac{\partial F_z^{\pm}}{r\partial\varphi}\right),\; F_{\varphi}^{\pm} = \frac{1}{k^2 n^2 - \gamma^2}\left(\pm kn\frac{\partial F_z^{\pm}}{\partial r} - i\gamma\frac{\partial F_z^{\pm}}{r\partial\varphi}\right). \tag{1}$$

According to the fields equation, we derive the equation for $\Psi^{\pm}(r,\varphi)$,

$$\frac{\partial^2\Psi^{\pm}}{\partial r^2} + \frac{\partial\Psi^{\pm}}{r\partial r} + \frac{\partial^2\Psi^{\pm}}{r^2\partial\varphi^2} + \left(k^2 n^2 - \gamma^2\right)\Psi^{\pm} = 0, \tag{2}$$

where $n = n_{\pm} = n_o/(1\pm\delta)$ are for the core, $n = n_o$ is for the cladding, $\delta = kn_c\rho$, $n_c = (\varepsilon\mu)^{1/2}c$, $k = \omega/c$, μ is permeability of free spase, ω is frequency of a field, c is the speed of light in vacuum. It is convenient to determine the solutions of equation (2) in the form,

$$\Psi^{\pm}(r,\varphi) = \begin{cases} a_o^{\pm}K_m(wr/R)\exp(-im\varphi); & r > R \\ a_c^{\pm}J_m(u_{\pm}r/R)\exp(-im\varphi); & r < R \end{cases}. \tag{3}$$

Here, we use the notation $u_{\pm} = \left(k^2 n_{\pm}^2 - \gamma^2\right)^{1/2} R$, $w = \left(\gamma^2 - k^2 n_o^2\right)^{1/2} R$, m= 0,1,2… At the core-cladding boundary the function $\vec{F}^{+}$ and $\vec{F}^{-}$ must satisfy the condinuity conditions

$$F_{\tau o}^{+} + F_{\tau o}^{-} = F_{\tau c}^{+} + F_{\tau c}^{-}, \qquad \left(F_{\tau o}^{+} - F_{\tau o}^{-}\right)\zeta_o^{-1} = \left(F_{\tau c}^{+} - F_{\tau c}^{-}\right)\zeta_c^{-1}, \tag{4}$$

where index $\tau = z,\varphi$ denotes the tangential components of the fields $\vec{E}$ and $\vec{H}$ on the boundary. Substuting functions (3) into boundary conditions (4), we obtain the system of equations for coefficients $a_o^{\pm}$, $a_c^{\pm}$ and propagation constant γ

$$\left(a_o^{+} + a_o^{-}\right)K = a_c^{+}J^{+} + a_c^{-}J^{-},\; n_o\left(a_o^{+} - a_o^{-}\right)K = n_c\left(a_c^{+}J^{+} - a_c^{-}J^{-}\right), \tag{5}$$

$$a_o^{+}A^{+} + a_o^{-}A^{-} = a_c^{+}B^{+} + a_c^{-}B^{-},\; n_o(a_o^{+}A^{+} - a_o^{-}A^{-}) = n_c(a_c^{+}B^{+} - a_c^{-}B^{-}),$$

where $J^{\pm} = J_m(u^{\pm})$, $K = K_m(w)$ are Bessel and modified Bessel functions, $A^{\pm} = \left[m(\gamma - kn_o)K_m(w) \pm kn_o wK_{m\pm1}(w)\right]w^{-2}$, $B^{\pm} = \left[m(kn_{\pm} - \gamma)J_m(u_{\pm}) - kn_{\pm}u_{\pm}J_{m\pm1}(u_{\pm})\right]u_{\pm}^{-2}$.

Eliminating coefficients $a_o^{\pm}$, $a_c^{\pm}$ we obtain the dispersion equation for propagation constant γ on the modes of circular chiral dielectric waveguide

$$d^2\left(A^{+}J^{+} - B^{+}K\right)\left(A^{-}J^{-} - B^{-}K\right) = \Delta^2\left(A^{+}J^{-} - B^{-}K\right)\left(A^{-}J^{+} - B^{+}K\right), \tag{6}$$

where we use the notations $\Delta = (n_c^2 - n_o^2)/2n_c^2$, $d = (n_c + n_o)^2/2n_c^2$ and $0<\Delta<1/2$, $2>d>1/2$.

CHIRAL OPTICAL FIBRE

The dispersion equation for modes of a chiral optical fibre with a small difference between the refractive indexes of a core and cladding, i.e., $\Delta \approx (n_c - n_o)/n_c << 1$ and $\delta << 1$, it is possible to simplify. Using standard notation applied for optical waveguides, we introduce the reduced propagation constant and frequency and chirality: $b = w/V$, $V = kRn_c(2\Delta)^{1/2}$, $\kappa = \rho/\left(R\Delta\sqrt{2\Delta}\right)$, where b is coupled with γ by the relationship $\gamma \approx kn_o(1 + \Delta b^2)$. Using the reduced parameters, we have $w = bV$, $u_{\pm} = c_{\pm}V$, $c_{\pm} = \left(1 \mp \kappa V - b^2\right)^{1/2}$. Approximately dispersion equation (6) at $\Delta << 1$, is reduced to two independent equations

$$\left(A^{+}J^{+} - B^{+}K\right) = 0,\; \left(A^{-}J^{-} - B^{-}K\right) = 0. \tag{7}$$

Under conditions $\gamma/kn_o - 1$, $kn_{\pm}/\gamma - 1$, $n_{\pm}/n_o - 1 \le \Delta << 1$ for $\delta << 1$, after some transformations, we obtain

$$bK_{l\mp1}(bV)/K_l(bV) \pm c_{\pm}J_{l\mp1}(c_{\pm}V)/J_l(c_{\pm}V) = 0, \tag{8}$$

where $l=m\pm1$. Discarding quantities Δ and δ that are small as compared with unity, and retaining their ratio $\kappa V=\delta/\Delta$ into $c_{\pm}$, allows to obtain a form (8) of the dispersion equation. Obtained in the present work approximate dispersion equation (8) corresponds to CP-modes [3] and has the similar form as for LP-modes of nonchiral optical fibres [4,5]. This approximate equation was divided into two independent equations (7) for two classes of modes with the right-handed and left-handed circular polarizations. The dispersion curves introduced on Fig.2 have identification $CP^{\pm}_{lp}$, i.e., circularly polarized modes with indexes l and p, meaning the azimuthal and radial orders of a variation of their cross fields.

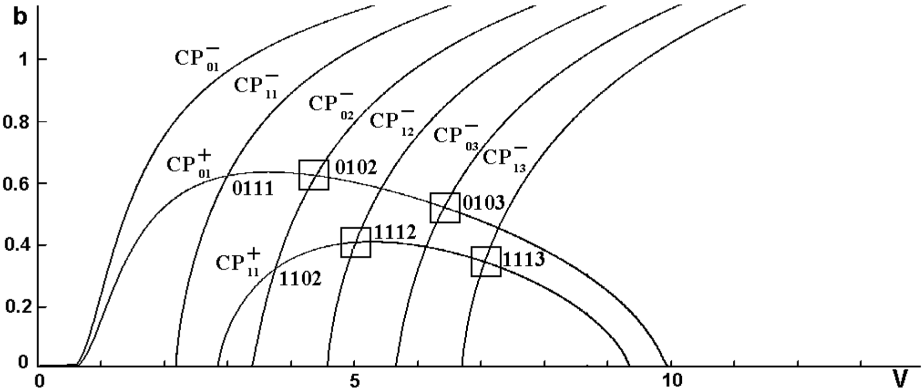

Fig. 2. The dispersion characteristics of modes of a chiral optical fibre

In the present work in details it is demonstrated, that one group of circular dielectric waveguide modes have degeneracy points (a cross of the dispersion curves) and other group have quasi-degeneracy points [6,7].
The dispersion equation was solved by the way, described in the work [7], where the method of analysis of distinctive spectral features is given by the solution of a dispersion equation for modes of chiral dielectric waveguide in local area near points of degeneracy and quasi-degeneracy. In Fig. 3 the dispersion characteristics *b(V)* accordingly near to points (0102), (0103) are show for κ=0,1 and Δ=0,04.

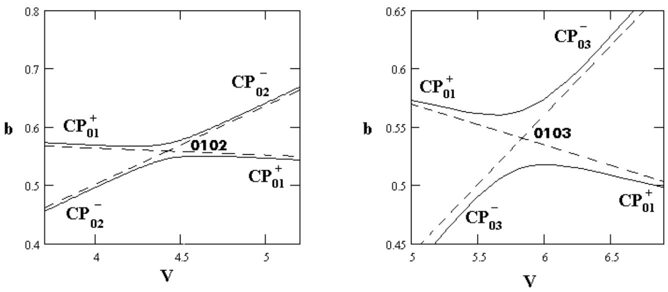

Fig. 3. The dispersion curves near the quasi-degeneracy points (0102), (0103)

The features of dispersion curves are visible near to the of suspected spectral degeneration points. For dispersion curves with identical indexes *l*, they are points quasi-degeneration (Fig.3), i.e. curves approach, but are not intercepted. The curves with miscellaneous indexes *l* are intercepted (Fig.2) and are points of spectral degeneration. The curves with identical indexes *l* are evaluated by the solution of one equation (6). The curves with different indexes *l* are evaluated by the solution of two different independent equations (6).

POLARIZATION OF MODES FIELDS

The polarization properties of modes have a peculiarity near the quasi-degeneracy points. The polarization and structure of modes fields are qualitatively transformed in an interval of frequencies including these points. The polarization of modes in cross section of a chiral waveguide is not linear (it is circular or elliptical) and inhomogeneous. For its analysis, in an interval of frequencies including quasi-degeneracy points, there was a necessity of the use of integral factors of polarization on a wavefront. In [8] factors of polarization were put into operation: Φ is the factor of polarization orientation and S is the factor of ellipticity,

$$\Phi = \arctan\left(\frac{|B|^2 - 1 + |B^2 + 1|}{2B'} \right), \ S = \frac{|B|^2 + 1 - |B^2 + 1|}{2B''}, \tag{9}$$

where $B = B' + iB'' = E_\varphi / E_r = (F_\varphi^+ + F_\varphi^-)/(F_r^+ + F_r^-)$. The geometrical meaning of a factor Φ (9) is equal to angle of a slope of a large axis of an ellipse of polarization to a radial coordinate direction, and modulus $|S| = b/a$, where b and a are small and large axes of the polarization ellipse. The sign S means the direction of the components of vector of a field $\vec{E}$ rotations: right (right ellipse) or left (left ellipse) cross-section to of the optical waveguide. For the description of polarization transformation of modes at frequency change near the quasi-degeneracy points the factors $\overline{\Phi}$, $\overline{S}$ are utilised as average on cross-sectional area s of a fiber core [8]

$$\overline{\Phi} = \frac{1}{s}\int_s \Phi(r,\varphi) ds = \frac{2}{R^2}\int_0^R \Phi(r) r dr, \ \overline{S} = \frac{1}{s}\int_s S(r,\varphi) ds = \frac{2}{R^2}\int_0^R S(r) r dr. \tag{10}$$

The dependence of the factor $\overline{S}$ from reduced frequency of a field V near to quasi-degeneracy points (0102), (0103) are presented in Fig.4.

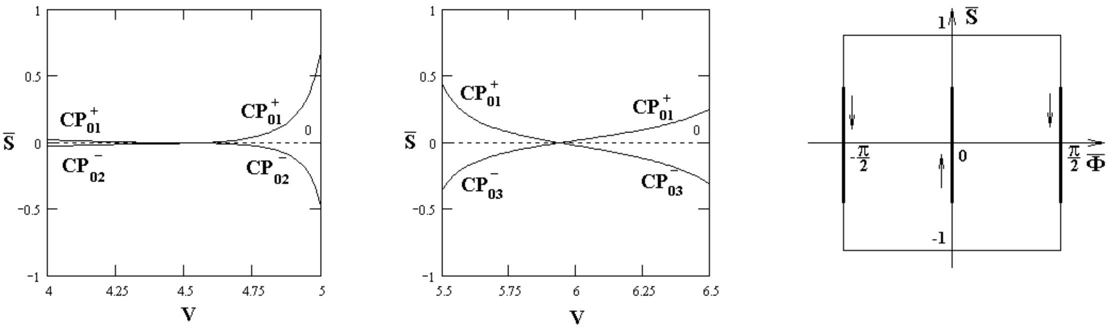

Fig. 4. The dispersion characteristics of an average factor $\overline{S}$ near a quasi-degeneracy points (0102), (0103) and the transformation of polarizations of modes in the $\overline{\Phi}\overline{S}$ - plane

The rotation of their polarization changes on opposite: right-handed to left-handed, and vice versa, at the passing of dispersion curves near a point of quasi-degeneracy (the sign for a factor $\overline{S}$ changes). The calculation of values $\overline{\Phi}$ for $B' = 0$ has a feature. These values are calculated according to expression $\overline{\Phi}(B' = 0) = \overline{\Phi}_0$, where $\overline{\Phi}_0 = 0$ for $|\overline{\Phi}| < \pi/4$ and $\overline{\Phi}_0 = \pm\pi/2$ for $|\overline{\Phi}| > \pi/4$. The $\overline{\Phi}\overline{S}$ - plane near the points of quasi-degeneracy are presented in Fig.4.

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