TRANSIENT ANALYSIS OF A SINGLE WIRE TRANSMISSION LINE ABOVE A DIELECTRIC HALF-SPACE

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ABSTRACT

Abstract: A time domain analysis of a single wire transmission line located above a dielectric half-space and illuminated by the electromagnetic pulse (EMP) is presented in the paper. The analysis is based on the solution of the corresponding space-time Hallen integral equation and it is carried out directly in the time domain. The influence of a two media configuration is taken into account via the space time reflection coefficient arising from the modified image theory. The corresponding integral equation is handled via the time domain Galerkin-Bubnov variant of the boundary element method (GB-BEM).A root-mean-square *(rms*) energy measure for the transient response of thin wires is also calculated. Some illustrative numerical results are presented in the paper.

INTRODUCTION

Time domain (TD) analysis of electromagnetic coupling to overhead wires can be performed by using the transmission line or the wire scatterer model. The transmission line model is sufficient approximation for the longer lines but fails to predict resonances and the end effects when finite length wires are considered. A time domain wire scatterer model of the transmission line above a perfectly conducting (PEC) ground based on the Pocklington integral equation has been proposed in [1] and [2]. However, numerical treatment of TD Pocklington integral equation via moment method/marching–on-in-time algorithm suffers from serious drawback regarding the stability of numerical results [3]. As it was shown in [4]-[6] the problem can be sufficiently overcome using the Hallen integral equation approach, hence a time domain wire antenna model of a straight areal line of finite length located horizontally above a perfect ground based on the Hallen integral equation has been reported in [4].
A time domain simulation of the overhead wire above dielectric half-space exposed to the transient excitation based on the scattering theory is presented in this work. The problem is formulated via Hallen integral equation for half-space problems and the influence of a lower medium is taken into account via space-time dependent reflection coefficient [4]-[8]. A root-mean-square (*rms*) value as an energy measuer of the transient response of a thin wire is used in this work. As the *rms* value is an energy measure of the transient current flowing along the wire this parameter can be important in various EMC applications.

TIME DOMAIN FORMULATION

The time domain study of the electromagnetic pulse (EMP) coupling to the an overhead wire is based on a thin wire antenna model. The transient response of the horizontal wire in the air above a dielectric medium is governed by the corresponding half-space integral equation which can be derived as an extension of the free space equation. For simplicity, firstly the free space Hallen equation is derived.

It is well-known that the tangential electric field components at the wire surface satisfy the equation:

$$E_x^{inc} + E_x^{sct} = R_L i \qquad (1)$$

where R_L denotes a resistive loading per unit of wire length, while i is the transient current along the wire.

Starting from Maxwell's equations, enforcing the condition (1) and taking into account the thin wire approximation [8] one obtains the space-time Pocklington integro-differential equation:

$$-\varepsilon \frac{\partial E_x^{inc}}{\partial t} = \left[\frac{\partial^2}{\partial x^2} - \frac{1}{c} \frac{\partial^2}{\partial t^2} \right] \int_0^L \frac{i(x^{'}, t - R/c)}{4\pi R} dx^{'} - R_L \frac{\partial i(x,t)}{\partial t} \tag{2}$$

The Hallen integral equation counterpart is derived by integrating the equation (2), in a quite similar manner as was done in [4]:

$$\begin{aligned} \int_0^L \frac{i(x^{'}, t - R/c)}{4\pi R} dx^{'} &= F_0(t - \frac{x}{c}) + F_L(t - \frac{L - x}{c}) + \\ &+ \frac{1}{2Z_0} \int_0^L E_x^{inc}(x^{'}, t - \frac{|x - x^{'}|}{c}) d x^{'} - \frac{1}{2Z_0} \int_0^L R_L(x^{'}) i(x^{'}, t - \frac{|x - x^{'}|}{c}) dx^{'} \end{aligned} \tag{3}$$

where I(x', t-R/c) is the unknown space-time dependent current to be determined, c is the velocity of light, and Z_0 is the wave impedance of a free space. The unknown functions $F_0(t)$ and $F_L(t)$ are related with the multiple reflections of the current wave from the wire ends. From the modified image theory [ ] it follows:

$$\int_0^L \frac{I(x', t - R/c)}{4\pi R} dx^{'} - \int_0^L r_{TM}(\theta) \frac{I(x', t - R'/c)}{4\pi R'} dx^{'} = F_0(t - \frac{x}{c}) + F_L(t - \frac{L - x}{c}) + \frac{1}{2Z_0} \int_0^L E_x^{exc}(x', t - \frac{|x - x'|}{c}) dz' \tag{4}$$

where r_{TM} is the reflection coefficient given by:

$$r_{TM} = \frac{\varepsilon_r \cos\theta - \sqrt{\varepsilon_r - \sin^2\theta}}{\varepsilon_r \cos\theta + \sqrt{\varepsilon_r - \sin^2\theta}} \tag{5}$$

and E^{exc} is the total field tangential to the wire surface given by:

$$E_x^{exc} = E_x^{inc}(t) - r_{TM}(\theta) E_x^{inc}(t - \frac{2h}{c}) \tag{6}$$

Transient current induced along the wire due to the EMP excitation can be obtained as a solution of the time domain Hallen integral equation (4) by the time domain Galerkin-Bubnov scheme of the boundary element method [4]-[8].

Root-Mean-Square Value of the Wire Current

It is well-known from the basic theory of electric circuits that transient current delivers an average power to resistive load. The amount of delivered power strongly depends on the particular waveform. A measure of comparing the power delivered by different waveforms is root-mean-square (*rms*) value of a transient current. The *rms* value of a time-varying current is a constant that is equal to the direct current value that would deliver the same average power to a given resistence R_L.

Instantaneous power delivered to a resistence R_L by a transient current $i(t)$ is:

$$p(t) = R_L i^2(t) \tag{7}$$

while the corresponding average power P_{av} is given by:

$$P_{av} = \frac{1}{T_0} \int_0^{T_0} p(t) dt = \frac{1}{T_0} \int_0^{T_0} R_L \, i^2(t) dt = R_L \, I_{rms}^2 \tag{8}$$

from which the *rms* current is then:

$$I_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} i^2(t)dt} \tag{9}$$

and the *rms* value of the thin wire space-time current distribution can be obtained by relation:

$$I_{rms}(x) = \sqrt{\frac{1}{T_0}\int_0^{T_0} i^2(x,t)dt} \tag{10}$$

where T_0 is the time interval of interest.

NUMERICAL RESULTS

Figure 1 shows the transient current induced at the center of the horizontal line of length *L*=10m, and radius *a*=6.74cm above dielectric half-space (εr=10) at the height h=5m illuminated by a tangential electric field in the form of the standard double-exponential electromagnetic pulse (EMP):

$$E_x^{inc}(t) = E_0(e^{-at} - e^{-bt}) \tag{11}$$

where E_0=1kVm^{-1}, a=4*10^6, b=4.76*10^8.

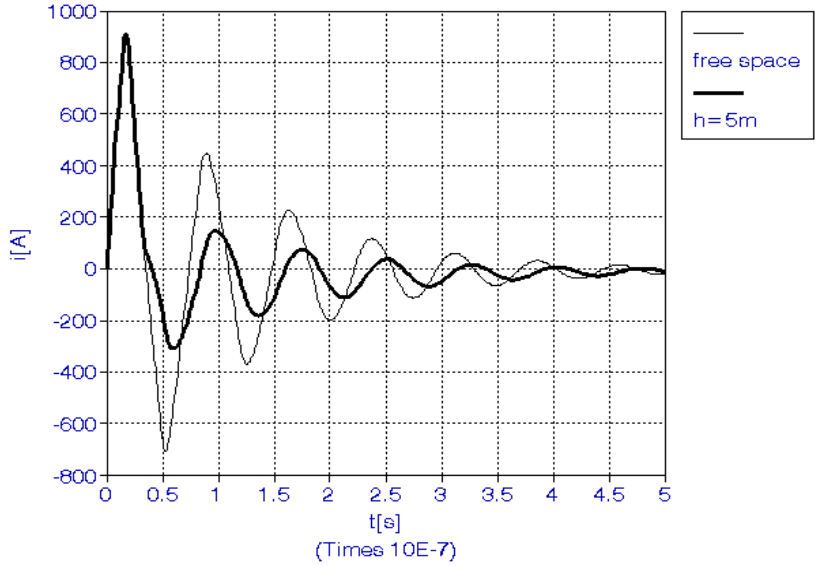

Fig. 1. Transient current induced at the center of the horizontal wire above dielectric half-space (L=1m, a=6.74cm, h=5m, ε_r=10

The results obtained for the given half-space problem are accompanied with the results obtained for the same wire insulated in free space and illuminated by the same excitation. Fig. 2 shows the spatial distributions of the associated *rms* values.

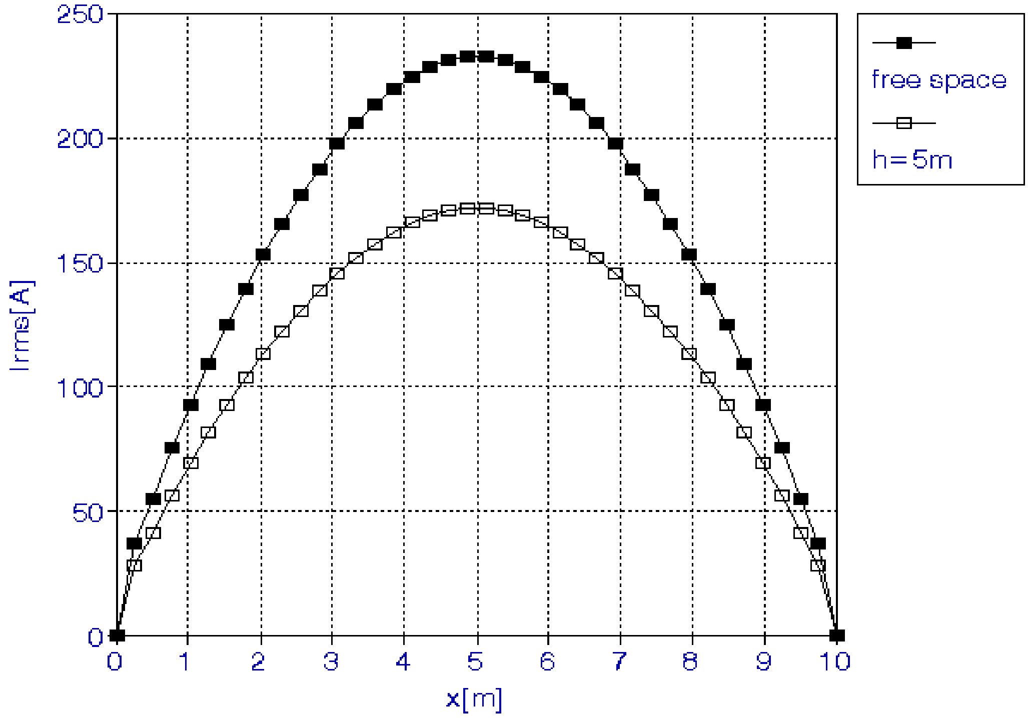

Fig.2. Spatial distribution of the *rms* value of the transient current along the horizontal wire (L=1m, a=6.74cm, h=5m, ε_r=10) excited by the standards EMP waveform

FINAL REMARKS

A time domain antenna model of a single wire transmission line exposed to the electromagnetic pulse is presented in the paper. The problem is formulated by the space-time Hallen integral equation. The influence of the lower reflecting medium is taken into account via the simplified reflection coefficient arising from the modified image theory. This is the extension of the previous work dealing with the transmission lines in the presence of the perfect ground. The transient current induced on the overhead wire is obtained by solving the Hallen integral equation via the time domain variant of the Galerkin Bubnov boundary element method (GB-BEM). Transient responses for double-exponential pulse are presented.
A root-mean-square (*rms*) measure of non-linear effects to the transient response of thin wire structures is presented. Since the rms value of the time varying current is a measure of the average power delivered to the resistive load it could be very useful parameter in design of lightning protection devices.

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