DIRECTION OF ARRIVAL ESTIMATION VIA UNITARY TLS -ESPRIT ALGORITHM WITH STRUCTURE WEIGHTING

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ABSTRACT

A new variant of the unitary TLS-ESPRIT Direction -Of-Arrival (DOA) estimation algorithm, which is based on the method of the structure weighting, is presented. Under latter the generalized subarrays overlapping is used, when more than two subarrays are utilized, which incorporate about half as much as sensors of the entire array. The proposed algorithm conserves inessential computational intricacy inherent for the unitary TLS-ESPRIT algorithm, as it is also based on real calculations at all stages. Simulation results verifying the efficiency of the algorithm are presented.

INTRODUCTION

Nowadays the ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) algorithm is one of the widely used algorithms of signals DOA estimation (of their spectrum central frequency). In papers devoted to the algorithm, a main attention is paid to questions of making computation by the algorithm easier and improving its accuracy. One of ways of reducing the ESPRIT algorithm complexity is the use of the unitary transformation method [1]. In this case a decrease of the computational load is achieved due to the eigendecomposition of a real (not complex) covariance matrix of the received by the antenna array signals mixture [2,3]. In the paper, a new variant of the unitary TLS-ESPRIT algorithm is proposed, which is based on the structure weighting method [4]. The proposed variant makes possible to increase an accuracy of DOA estimation of radiation sources in the domain of mean and great values of SNR (being of practical interest, for example, when solving a problem of the noise jammer finding). The essence of the algorithm is considered by the example of a uniform linear array (ULA), that is of an antenna array with symmetrical location of identical in pairs elements.

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Let ULA be composed of M identical elements, which receive the impinging from directions $\theta_1,\cdots,\theta_V$ narrow-band signals emitted by V far-field sources. From the measured output of array the objective is to estimate the sources DOAs.

The $M\times 1$ array output vector can be expressed as

$$\mathbf{x}(t)=\mathbf{A}(\theta)\mathbf{s}(t)+\mathbf{n}(t), \tag{1}$$

where $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1),...,\mathbf{a}(\theta_V)]$ is the $M\times V$ matrix of the source steering vectors, $\mathbf{a}(\theta)$ is the $M\times 1$ steering vector, $\mathbf{s}(t)$ is the $V\times 1$ vector of source complex envelopes, $\mathbf{n}(t)$ is the $M\times 1$ vector of sensor noise. The number of sources V is assumed to be known. Also is assumed, that source signals are zero-mean, complex Gaussian, temporally white processes with the covariance matrix $\mathbf{S}=E[\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)]$, where $E[\cdot]$ and $(\cdot)^H$ stand for expectation operator and hermitian transpose, respectively. The sensor noise $\mathbf{n}(t)$ is also the zero-mean complex Gaussian process and is assumed to be both temporally and spatially white with the variance σ^2.

Let $\mathbf{R}$ denote the covariance matrix of $\mathbf{x}(t)$. In accordance with the made assumptions, $\mathbf{R}$ is given by

$$\mathbf{R}=E[\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)]=\mathbf{A}(\theta)\mathbf{S}\mathbf{A}^{\mathrm{H}}(\theta)+\sigma^2\mathbf{I}, \tag{2}$$

where $\mathbf{I}$ is the $M\times M$ identity matrix. The sample covariance matrix obtained from N snapshots is determined as

$$\hat{\mathbf{R}}=\frac{1}{N}\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t). \tag{3}$$

The eigendecomposition (ED) of $\hat{\mathbf{R}}$ has the following form

$$\hat{\mathbf{R}}=\hat{\mathbf{E}}_s\hat{\mathbf{\Lambda}}_s\hat{\mathbf{E}}_s^{\mathrm{H}}+\hat{\mathbf{E}}_n\hat{\mathbf{\Lambda}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}, \tag{4}$$

where $\hat{V}\times\hat{V}$ and $(M-\hat{V})\times(M-\hat{V})$ diagonal matrices $\hat{\mathbf{\Lambda}}_s$ and $\hat{\mathbf{\Lambda}}_n$ contain $\hat{V}$ and $M-\hat{V}$ signal and noise subspace eigenvalues, whereas $M\times\hat{V}$ matrix $\hat{\mathbf{E}}_s=[\hat{\mathbf{e}}_1,\cdots,\hat{\mathbf{e}}_{\hat{V}}]$ and $M\times(M-\hat{V})$ matrix $\hat{\mathbf{E}}_n=[\hat{\mathbf{e}}_{\hat{V}+1},\cdots,\hat{\mathbf{e}}_M]$ contain the corresponding eigenvectors, i.e. signal - and noise - subspace eigenvectors.

When realizing the ESPRIT algorithm the selection matrices $\mathbf{J}_1=[\mathbf{I}_{m\times m}\ \mathbf{0}_{m\times 1}]$ and $\mathbf{J}_2=[\mathbf{0}_{m\times 1}\ \mathbf{I}_{m\times m}]$ are formed, where $\mathbf{0}_{m\times 1}$ is the zero matrix of size $m\times 1$, m is the number of elements at each subarray and in the maximum overlapping case $m=M-1$. In [4] more fully exploits the ULA structure by reason of using more than one subarray pairs (but subarray is of smaller size) is proposed. This is accomplished by means of corresponding selection matrices $\mathbf{J}_{1s}=[\overline{\mathbf{J}}_{q\times m}\ \mathbf{0}_{q\times 1}]$ and $\mathbf{J}_{2s}=[\mathbf{0}_{q\times 1}\ \overline{\mathbf{J}}_{q\times m}]$, where $q=m_s(M-m_s)$ is the total number of elements in each subarray for this case, $\overline{\mathbf{J}}=[\overline{\mathbf{J}}_1^T,\overline{\mathbf{J}}_2^T,\ldots,\overline{\mathbf{J}}_{M-m_s}^T]$, $\overline{\mathbf{J}}_i=[\mathbf{0}_{m_s\times(i-1)}\ \mathbf{I}_{m_s\times m_s}\ \mathbf{0}_{m_s\times(M-i-m_s)}]$, and $(\cdot)^{\mathrm{T}}$ denote transpose. Here m_s is the number of elements in a subarray of smaller size ($m_s<m$). In this case, $m_s\times m$ matrix $\overline{\mathbf{J}}_i$ picks m_s contiguous rows of the matrix $\mathbf{E}_s$: $i, i+1,\ldots,i+m_s-1$. This overlapping is named in [4] as generalized and obviously that a choice of $m_s=1$ leads back to the initial overlapping structure.

In the TLS-ESPRIT algorithm with generalized overlapping under estimation of DOAs the eigendecomposition of the following matrix multiplication is calculated

$$[\mathbf{J}_{1s}\mathbf{E}_s\,\mathbf{J}_{2s}\mathbf{E}_s]^H[\mathbf{J}_{1s}\mathbf{E}_s\,\mathbf{J}_{2s}\mathbf{E}_s]=[\mathbf{J}_1\mathbf{E}_s\,\mathbf{J}_2\mathbf{E}_s]^H\mathbf{\Sigma}[\mathbf{J}_1\mathbf{E}_s\,\mathbf{J}_2\mathbf{E}_s], \tag{5}$$

where $\mathbf{\Sigma}=\overline{\mathbf{J}}^T\overline{\mathbf{J}}$ is the $(M-1)\times(M-1)$ weighting matrix. At this point instead of both immediate using of matrices $\mathbf{J}_{1s},\mathbf{J}_{2s}$ and work with the left side of (5) it is appropriate to use the right side of (5). The matrix $\mathbf{\Sigma}$ is the diagonal matrix, $\mathbf{\Sigma}=\mathrm{diag}[1\ 2\ldots \mathrm{w}\ldots \mathrm{w}\ldots 2\ 1]$, $\mathrm{w}=\min(m_s, M-m_s)$. As for parameter m_s, it under maximum overlapping of subarrays in [4] is recommended $m_s\approx(M/2)$. The elements of the matrix $\mathbf{\Sigma}$ indicate how many times each row of matrix $\mathbf{A}$ is used (in other words, how many times the every element of first subarray of size $M-1$ in a subarray pairs is used). This approach is named in [5] as structure weighting (in [4] the approach is named as row weighting).

The useful property of array with symmetrical arrangement of identical in pairs sensors consists in that if to take the array center as phase reference, then the source steering vector will of the conjugate-centrosymmetric form [1]:

$$\overline{\mathbf{a}}(\omega)=[\exp(-j(\frac{M-1}{2}\omega)),\exp(-j(\frac{M-3}{2}\omega)),\ldots,\exp(j(\frac{M-3}{2}\omega)),\exp(j(\frac{M-1}{2}\omega))]^T, \tag{6}$$

where $\omega=2\pi d\sin\theta/\lambda$ is a so called spatial frequency, d is the interelement spacing, λ is the wave length. Under using of the unitary transformation method the complex-valued vector $\overline{\mathbf{a}}(\omega)$ is transformed into real-valued vector $\mathbf{d}(\omega)=\mathbf{U}^H\overline{\mathbf{a}}(\omega)$ of the same size by the matrix

$$\mathbf{U}_{2K+1}=(1/\sqrt{2})\begin{bmatrix}\mathbf{I}_K & \mathbf{0} & j\mathbf{I}_K\\ \mathbf{0}^T & \sqrt{2} & \mathbf{0}^T\\ \tilde{\mathbf{I}}_K & \mathbf{0} & -j\tilde{\mathbf{I}}_K\end{bmatrix} \tag{7}$$

if $M=2K+1$ (i.e. the number of sensors is odd), where matrix $\tilde{\mathbf{I}}_K$ is an $(M-1)/2\times(M-1)/2$ exchange matrix (with ones on its antidiagonal and zeros elsewhere) and by matrix $\mathbf{U}_{2K}$, which is obtained from (7) by dropping its center row and center column, for the case of $M=2K$.

The source steering vector satisfies the shift invariance property [3], that for the generalized overlapping is expressed as

$$\exp(j\omega)\mathbf{J}_{1s}\overline{\mathbf{a}}(\omega)=\mathbf{J}_{2s}\overline{\mathbf{a}}(\omega) \tag{8}$$

Since $M\times M$ matrix $\mathbf{U}$ is unitary, i.e. $\mathbf{U}\mathbf{U}^H=\mathbf{U}^H\mathbf{U}=\mathbf{I}$, then (8) may be presented as

$$\exp(j\omega)\mathbf{J}_{1s}\mathbf{U}\mathbf{d}(\omega)=\mathbf{J}_{2s}\mathbf{U}\mathbf{d}(\omega). \tag{9}$$

Premultiplying both sides of the above mentioned expression by $q\times q$ matrix $\mathbf{U}_q^H$, we obtain the following formula

$$\exp(j\omega)\mathbf{U}_q^H\mathbf{J}_{1s}\mathbf{U}\mathbf{d}(\omega)=\mathbf{U}_q^H\mathbf{J}_{2s}\mathbf{U}\mathbf{d}(\omega). \tag{10}$$

Note, that matrix $\mathbf{U}_q$ is formed analogously to the matrix $\mathbf{U}$ by (7), and matrices $\mathbf{J}_{1s}$ and $\mathbf{J}_{2s}$ satisfy the equality $\tilde{\mathbf{I}}_q \mathbf{J}_{2s} \tilde{\mathbf{I}} = \mathbf{J}_{1s}$, where $\tilde{\mathbf{I}}_q$ and $\tilde{\mathbf{I}}$ are exchange matrices of sizes $q \times q$ and $M \times M$, respectively. As a result, we have

$$\mathbf{U}_q^H \mathbf{J}_{2s} \mathbf{U} = \mathbf{U}_q^H \tilde{\mathbf{I}}_q \tilde{\mathbf{I}}_q \mathbf{J}_{2s} \tilde{\mathbf{I}} \tilde{\mathbf{I}} \mathbf{U} = \mathbf{U}_q^T \mathbf{J}_{1s} \mathbf{U}^* = (\mathbf{U}_q^H \mathbf{J}_{1s} \mathbf{U})^* . \quad (11)$$

Mark the real and imaginary parts of matrix multiplication $\mathbf{U}_q^H \mathbf{J}_{2s} \mathbf{U}$ as $\mathbf{K}_{1s}$ and $\mathbf{K}_{2s}$ (they are $q \times M$ real-valued matrices):

$$\mathbf{K}_{1s} = \mathrm{Re}(\mathbf{U}_q^H \mathbf{J}_{2s} \mathbf{U}),\ \mathbf{K}_{2s} = \mathrm{Im}(\mathbf{U}_q^H \mathbf{J}_{2s} \mathbf{U}) . \quad (12)$$

According to this definition, (10) may be presented as

$$\exp(j\omega/2)(\mathbf{K}_{1s} - j\mathbf{K}_{2s})\mathbf{d}(\omega) = \exp(-j\omega/2)(\mathbf{K}_{1s} + j\mathbf{K}_{2s})\mathbf{d}(\omega) , \quad (13)$$

After simple manipulations the following expression can be obtained

$$\mathrm{tg}(\omega/2)\mathbf{K}_{1s}\mathbf{d}(\omega) = \mathbf{K}_{2s}\mathbf{d}(\omega) . \quad (14)$$

Let us define the transformed steering matrix as $\mathbf{D} = \mathbf{U}^H \overline{\mathbf{A}}$. Then for all $\hat{V}$ sources the real-valued relation (14) may be expressed as

$$\mathbf{K}_{1s}\mathbf{D}\mathbf{\Omega}_\omega = \mathbf{K}_{2s}\mathbf{D} , \quad (15)$$

where $\mathbf{\Omega}_\omega = \mathrm{diag}[\tan(\omega_i/2)]_{i=1}^{V}$ contains the desired information about DOAs, $\mathbf{D} = [\mathbf{d}(\omega_1), \ldots, \mathbf{d}(\omega_{\hat{V}})]$.

It should be noted, that analogously to TLS-ESPRIT with structure weighting in this case is necessary to calculate eigendecomposition of matrix multiplication

$$[\mathbf{K}_{1s}\mathbf{E}_{su}\ \mathbf{K}_{2s}\mathbf{E}_{su}]^H [\mathbf{K}_{1s}\mathbf{E}_{su}\ \mathbf{K}_{2s}\mathbf{E}_{su}] = [\mathbf{K}_1\mathbf{E}_{su}\ \mathbf{K}_2\mathbf{E}_{su}]^H \mathbf{U}_{M-1}^H \mathbf{\Sigma} \mathbf{U}_{M-1}\ [\mathbf{K}_1\mathbf{E}_{su}\ \mathbf{K}_2\mathbf{E}_{su}] , \quad (16)$$

where matrix $\hat{\mathbf{E}}_{su}$ consists of signal-subspace eigenvectors of matrix $\mathrm{Re}(\mathbf{U}^H \hat{\mathbf{R}} \mathbf{U})$, $\mathbf{K}_1 = \mathrm{Re}(\mathbf{U}_{M-1}^H \mathbf{J}_2 \mathbf{U})$, $\mathbf{K}_2 = \mathrm{Im}(\mathbf{U}_{M-1}^H \mathbf{J}_2 \mathbf{U})$, and obtained weighting matrix $\mathbf{U}_{M-1}^H \mathbf{\Sigma} \mathbf{U}_{M-1} = \mathbf{\Sigma}_{un}$ is also diagonal, such that $\mathbf{\Sigma}_{un} = \mathrm{diag}[1\,2 \ldots \mathrm{w}\,\mathrm{w}\,1\,2 \ldots \mathrm{w}]$.

Thus, a sequence of steps for realization of the unitary TLS-ESPRIT algorithm with structure weighting and maximum overlapping is as follows:

1) computation of the ED of matrix $\mathrm{Re}(\mathbf{U}^H \hat{\mathbf{R}} \mathbf{U})$ and obtaining of the $M \times \hat{V}$ matrix $\hat{\mathbf{E}}_{su}$, whose columns are eigenvectors of $\mathrm{Re}(\mathbf{U}^H \hat{\mathbf{R}} \mathbf{U})$, that correspond to the $\hat{V}$ greatest eigenvalues of $\mathrm{Re}(\mathbf{U}^H \hat{\mathbf{R}} \mathbf{U})$;

2) definition of $(M-1) \times M$ matrices $\mathbf{K}_1$, $\mathbf{K}_2$ and calculation of the ED of matrix multiplication

$$[\mathbf{K}_1\mathbf{E}_{su}\ \mathbf{K}_2\mathbf{E}_{su}]^H \mathbf{U}_{M-1}^H \mathbf{\Sigma} \mathbf{U}_{M-1}\ [\mathbf{K}_1\mathbf{E}_{su}\ \mathbf{K}_2\mathbf{E}_{su}] = \mathbf{E}_0 \mathbf{\Lambda} \mathbf{E}_0^H ,$$

where $\mathbf{\Lambda}$ is the eigenvalue matrix of this multiplication, and $\mathbf{E}_0$ is the eigenvector matrix ;

3) partition of the matrix $\mathbf{E}_0$ into submatrices of size $\hat{V} \times \hat{V}$

$$\mathbf{E}_0 = \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix}.$$

4) calculation of the eigenvalues $\lambda_\mathrm{v}, \mathrm{v} = 1, \ldots, \hat{V}$ of matrix $\mathbf{\Psi}_{TLS}$, $\mathbf{\Psi}_{TLS} = (-\mathbf{E}_{12}\mathbf{E}_{22}^{-1})$;

5) determination of the spatial frequency sources as $\omega_i = 2\,\mathrm{atan}(\lambda_i), i = 1, \ldots, \hat{V}$.

SIMULATION RESULTS

The experimental investigations of the obtained DOA estimation algorithm were conducted by computer simulation. The simulation result is presented in Figs.1,2. In both cases two uncorrelated equally powered sources with angular coordinates $\theta_1 = 10^\circ$, $\theta_2 = 14^\circ$ are assumed. In the first case the number of ULA sensors is equal to $M = 12$, in the second case $M = 20$. The number of data samples was taken $N = 100$, and $L = 1000$ independent simulation runs were performed to obtain each simulation point. Curves of root-mean square errors for two sources do not differ significantly and on figures plotted for source with DOA $\theta_1 = 10^\circ$. Figs.1,2 display DOA RMSE (root mean square error) of the unitary TLS-ESPRIT algorithm (solid line), the modified unitary TLS-ESPRIT algorithm [6] with subarray shift equal to three sensors (dashed lines) and unitary TLS-ESPRIT algorithm with structure weighting ($m_s = 5$ in Fig.1 and $m_s = 9$ in Fig.2) versus SNR. In this figures the stochastic Cramer-Rao bound (CRB) [7] is also shown.

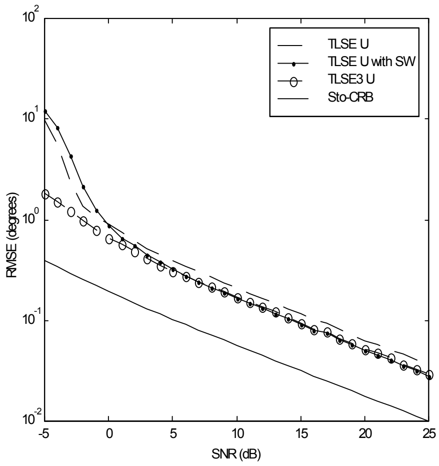

Fig.1 RMSE of $\theta_1 = 10^\circ$ versus SNR ($M = 12$, $N = 100$, $L = 1000$).

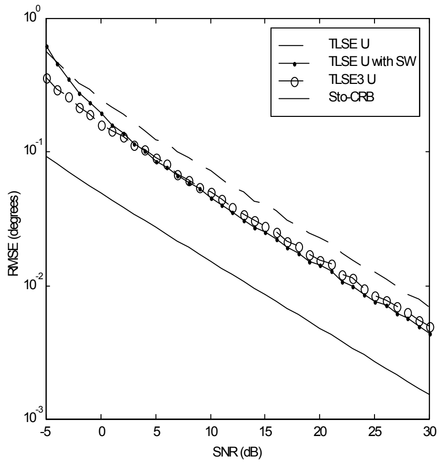

Fig.2 RMSE of $\theta_1 = 10^\circ$ versus SNR ($M = 20$, $N = 100$, $L = 1000$).

In the domain of mean and great values of SNR being of practical interest (for example, a problem of the noise sources finding), the modified unitary TLS-ESPRIT algorithm with subarrays shifted by 3 elements and the proposed algorithm has higher accuracy characteristics as compared with the unitary TLS-ESPRIT algorithm. However, in the modified unitary TLS-ESPRIT algorithm this is achieved by decrease a sector of the unambiguous DOA estimation of radiation sources (sector is approximately $(-20^\circ, 20^\circ)$). There is no such a disadvantage in the proposed algorithm. When applying the method of the unitary transformation, the efficiency of the structure weighting increases as a number of sensors increases. This is stipulated by the possibility of using a great number of subarrays into which antenna is being divided (by convention) when realizing the structure weighting method. At small SNR the obtained algorithm does not yield a gain. It is explained by the inadequacy (when realizing it) of the used subarray aperture for the high accuracy DOA estimation with a given distance between sources.

CONCLUSION

A new unitary formulation of the popular TLS-ESPRIT algorithm, in which the structure weighting method is used, has been considered. This algorithm has a reduced computational complexity, because it is based on the real-valued computations at all stages. The obtained algorithm can be easily extended to the 2D case through the result of [3]. The results can be used in the aim of obtaining a variant of the unitary TLS-ESPRIT algorithm with structure weighting in beamspace for 1D and 2D cases, and for using the DFT matrix as the beamforming one.

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