

RC CHAOTIC OSCILLATORS

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ABSTRACT

Recent investigations show that wireless communication systems are a very promising application area of chaotic dynamics. At the same time synthesis of chaotic oscillators suitable for IC implementation and producing chaotic oscillations in the specified frequency band is a problem. Here the problem of synthesis of RC chaotic oscillator with preassigned spectrum is discussed. Spectral properties of known RC chaotic oscillators are analyzed. Some conjectures regarding the structure of RC chaotic oscillator are formulated.

INTRODUCTION

At present, dynamical chaos attracts attention of researchers all over the world as a new information carrier for wireless communication systems [1, 2]. In particular this interest deals with certain properties of chaotic signals and systems. For example using relatively simple electronic circuits one can obtain wideband chaotic oscillations [3], and a number of different chaotic regimes can be obtained using the same chaotic source.

There is a number of chaotic oscillators models developed by now, which are widely used for different laboratory purposes. At the same time there are restrictions imposed on chaotic oscillators intended for mass production technology. Namely, it is important that chaotic sources produce chaotic signals in the necessary frequency band as in [4] where the regular method for the synthesis of broadband chaotic ring oscillation systems is described.

It is also important that chaotic sources have a prescribed set of technical specifications. In particular, not all chaotic oscillator models can be accomplished in integrated circuits (IC). For example implementation of chaotic oscillator models containing inductor tank resonator is a complex problem. Here we do not take into account a possibility to realize a chaotic oscillator as an "analogue computer" because of its low performance.

Therefore it is important to design RC chaotic oscillators containing only active elements with saturation, capacitors and resistors. However, nowadays there are no regular methods for synthesis of RC chaotic oscillators with preassigned spectral properties. Moreover, data about spectral properties is incomplete even for RC chaotic oscillators, which are well known.

The aim of this report is to analyze of spectral properties of known chaotic oscillators and to discuss synthesis of RC chaotic oscillators with preassigned frequency band.

First, examples of RC chaotic oscillators are described, then some approaches for RC chaotic oscillator design are analyzed, spectral properties of typical RC chaotic oscillators are classified, and finally, possible ways for synthesis of chaotic RC oscillators with preassigned spectral density are proposed.

EXAMPLES OF RC CHAOTIC OSCILLATORS

One of the simplest RC oscillator is Wien-bridge oscillator. Some chaotic Wien-bridge-based oscillators modified to generate chaotic oscillations are described in [5, 6, 8]. For example, bifurcation phenomena in two coupled Wien-bridge oscillators are discussed in [5]. Chua's circuit [7] with tank resonator is represented as an inductorless tank resonator based on Wien-bridge oscillator and capacitor loaded on nonlinear resistor [8] (Fig. 1,c). Developing this approach one may treat the Chua's circuit as a system composed of a linear and a nonlinear parts: a linear oscillator, which can be realized by active RC circuits and a negative nonlinear resistor. This approach lets us consider the tank resonator (linear system) irrelative of its schematic design.

Besides the Wien-bridge oscillator, a phase-shift RC oscillator is widely used in radio engineering. Conditions for chaotic oscillations in phase-shift oscillator are discussed in [9] as a function of parameters of nonlinear element having symmetric transfer characteristic consisting of three linear regions.

The authors of experimental works [5, 10] note that inductorless chaotic oscillators are attractive for IC implementation because realization of an element equivalent to inductance requires a number of transistors. And two nonlinearly coupled inductorless chaotic oscillators based on single transistor phase-shift oscillator are discussed.

Chaotic oscillator suitable for IC implementation can be also realized using hysteretic element [11].

SYNTHESIS OF RC CHAOTIC OSCILLATORS

At present, there are no regular methods for synthesis of chaotic RC oscillators with prescribed set of the following specifications: dimension, Lyapunov exponents and spectrum. Solution of these problems means a different statement of the synthesis problems. For example, the active elements (amplifiers) having saturation type transfer characteristic are preferable due to some technological reasons. So the authors of [12] discuss the problem of chaotic system synthesis on the basis of saturation type integration and summation.

Authors of [13] note that design of electronic circuit relies primarily on linear circuit theory of design, so the design of chaotic oscillators is based on the use of linear elements, such as linear tank resonator and nonlinear elements (transistors, diodes etc.). The authors of [13] illustrate this approach and introduce classes of chaotic RC oscillators representing of oscillator structure in two parts: linear and nonlinear. Oscillation conditions are provided by negative resistors. A schematic diagram of this circuit is depicted in Fig. 1,a,b,c.

The synthesis conception of RC chaotic oscillators with double-scroll-like chaotic attractor was developed in [13].

On the basis of chaotic oscillators discussed in [13] authors generalize the possible structure of the simplest chaotic system of 1.5 degrees of freedom, which can be realized by RC circuits and semiconductors:

$$\ddot{x} + \dot{x} + B\dot{x} + x = 0, \quad (1)$$

where $B = \alpha_1$, if $f(x, \dot{x}) \geq 1$ and $B = \alpha_2$ if $f(x, \dot{x}) < 1$.

SPECTRAL PROPERTIES OF CHAOTIC RC OSCILLATORS

Let us consider the spectral properties of some chaotic RC oscillators mentioned above.

In Fig. 1,a,b,c the spectrums of systems described in [13] are presented. Although systems mentioned in [13] produce broadband oscillations, their spectrums are not smooth and the main power is concentrated near resonance frequencies of oscillators. In Fig. 1,c the spectrum of a system realized as Chua's RC circuit is depicted [8, 13]. The resonance frequency exceeds other spectrum by ~20 dB. In this case the power spectrum is controlled by means of tuning resonance frequency of the tank oscillator.

System with 1.5 degrees of freedom with a piece-wise characteristic of nonlinear element can be described as follows:

$$\ddot{x} + A\dot{x} + Bx = G(x) \quad (2)$$

In particular, if $A=1$ and $G=-1$ system (2) coincides with system (1). As is shown in [13], a chaotic system with double-scroll-like attractor can be rewritten in this way and adequate schematic implementations using active RC circuits can be found. That is why it is interesting to evaluate the spectral characteristics of signals generated by (2) for different A , B and $G(x)$. The family of chaotic systems (2) for a number of piece-wise functions $G(x)$ and parameters A and B is described in [14].

The waveform and power spectrum of a chaotic signal produced by (2) with $A=0.44$, $B=2$ and $G(x)=x^2-1$ is depicted in Fig. 2,a. In Fig. 2,b the waveform and spectrum of signal correspond to $A=0.6$, $B=2$ and $G(x) = |x| - 1$. The waveform and power spectrum in Fig. 2,c correspond to $A=0.3$, $B=0.3$ and $G = -(|x| + x)/2 + 1$, and in Fig. 2,d to $A=0.5$, $B=1$ and $G = -(x - \text{sign}(x))$.

In Fig. 1,d waveform and spectrum correspond system [9]. As is shown in [9], though oscillations are formally chaotic their spectrum is similar to spectrum of quasiperiodic oscillations. Besides, system [9] has a narrow window of chaos in parameter space.

SYNTHESIS OF RC CHAOTIC OSCILLATORS WITH THE PREASSIGNED SPECTRUM

In this paper, as in other, oscillators with 1.5 degrees of freedom are discussed. As is shown in [4], chaotic oscillations with preassigned spectrum cannot be produced by lowdimensional systems. This is also confirmed by direct evaluation of the spectrum discussed in previous section. So to syntheses chaotic oscillators with preassigned spectrum, it is necessary to consider oscillators of higher dimensions as in [3, 4].

A chaotic source produced of oscillations in ~0.5...3 Mhz band is described in [3]. Chaotic oscillator and frequency-selective network forming the oscillation spectrum make this chaotic source. So, to generate wideband chaotic oscillations, it is necessary to have high dimensional frequency-selective network (4.5–6.5 degrees of freedom).

As is shown in [4] wideband chaotic oscillations can be also obtained using nonlinear element, whose transfer characteristic has both positive and negative slopes as in ring oscillation systems. Therefore, both systems with complex nonlinear element or with high-dimensional frequency-selective network can probably produce chaotic oscillations in specified frequency band.

So it can be concluded that there is a gap between spectral properties of above RC chaotic oscillators and the necessary ones. There are models of RC chaotic oscillators RC that can be implemented in IC technology but they cannot form oscillation spectrum in specified frequency band. On the other hand there are known chaotic oscillators that produce oscillations with prescribed characteristics, but is not obvious how they can be implemented in IC technology. It should be noted that the synthesis of chaotic source with prescribed set of specifications is a hard problem irrelative of technology of chaotic source implementation.

A possible approach to synthesis of RC chaotic oscillators having preassigned spectrum and implemented in integrated circuit is based on using of elements, which are typical for IC technology. First-order RC circuit (delay elements), tank resonators, RC periodical oscillators and amplifiers with saturation can be relatively easy realized in IC. All of these elements are basic blocks for synthesis of chaotic RC oscillators. Operation of such oscillators is identical to interaction between physical devices having probably different mathematical description so that such interaction produces a chaotic motion. An essential feature of this approach is the description of chaotic oscillator, which is irrelative of certain schematic design.

CONCLUSIONS

On the basis of known results regarding the synthesis of chaotic RC oscillators gaps in the synthesis of chaotic oscillators with controlled spectrum are found. Spectral properties of known RC chaotic oscillators are studied.

Synthesis of RC chaotic oscillator model with preassigned spectrum is shown to be an actual problem, since the chaotic oscillators do not have features, necessary for chaotic communication systems. Possible structures of chaotic RC oscillator models with controlled spectrum are proposed.

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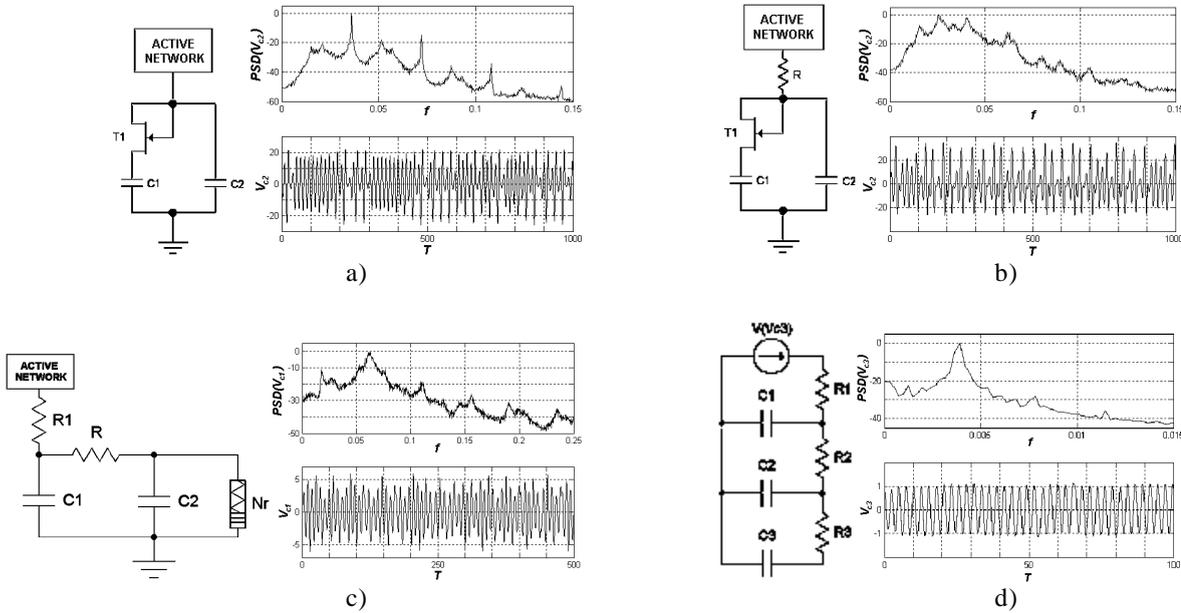


Fig. 1. Schematic circuits, waveforms and spectrums of RC chaotic oscillators: a) chaotic oscillator using FET-C composite with a parallel RC network [13]; b) chaotic oscillator using FET-C composite with a series RC network [13]; c) generic RC implementation of Chua's circuit based on the sinusoidal oscillator [13]; d) third order RC-ladder phase-shift oscillator [9].

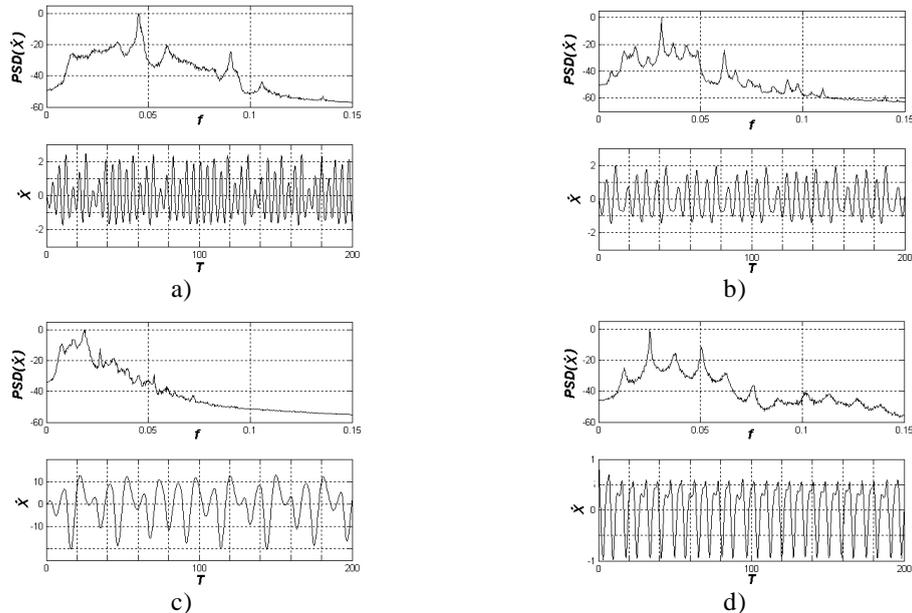


Fig. 2. Waveforms and spectrums of some simple third order (2) chaotic systems [14]: a) $A=0.44, B=2, G=x^2-1$, initial conditions are $(0,0,0)$; b) $A=0.6, B=2$ and $G=|x|-1$, initial conditions are $(0,0,0)$; c) $A=0.3, B=0.3$ and $G=-(|x|+x)/2+1$, initial conditions are $(0,0,0)$; d) $A=0.5, B=1$ and $G=-(x-sign(x))$, initial conditions are $(0,0,0)$.