ANALYTICAL INVESTIGATIONS OF DISPERSION AND REFLECTION IN TWO-DIMENSIONAL ELECTROMAGNETIC CRYSTALS FORMED BY THIN INFINITE LOADED WIRES

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ABSTRACT

2D electromagnetic crystals formed by rectangular lattices of thin ideally conducting cylinders loaded by bulk reactive impedances are considered. An analytical-numerical theory of dispersion and reflection from this medium is presented. The consideration is based on the local field approach. The transcendental dispersion equation is obtained in closed form and solved numerically. Different types of the loads like inductive, capacitive, serial and parallel LC circuits are considered. Typical dispersion curves and reflection coefficients are calculated, plotted and analyzed.

INTRODUCTION

In recent years, many new exciting applications have been suggested for periodical structures. In the microwave regime, stop band structures can be used, for example, as elements of microwave filters and antenna reflectors. One of the most attractive features of these artificial materials is a possibility to design materials with desired properties, which can be, in some instances, electrically controlled. Recently, a new structure for the use in antennas has been proposed in [1]. Here, the material is formed as a two-dimensional lattice of thin conducting wires which are loaded by bulk capacitances. Tuning capacitive loads (together with a proper choice of the wire radius and array period), desired electromagnetic properties can be obtained. Other applications of periodical arrays of conducting wires are in the design of artificial dielectrics with the relative permittivity smaller than unity and novel metamaterials with negative permittivity and permeability values.

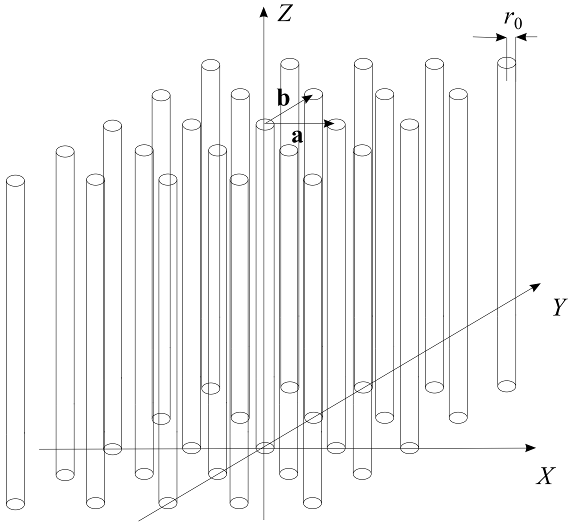

Fig. 1. Geometry of the loaded wire medum. All the wires are loaded by distributed reactive impedances.

Usually, periodical structures with stop bands are usually analyzed only numerically, and few analytical studies are available. This concerns also two-dimensional arrays of cylinders. Even for the simple case of thin ideally conducting wires, no simple analytical model is available, except for the low-frequency range. The need of understanding electromagnetic properties of novel composites for new applications motivates to develop analytical models. In this presentation we briefly overview our analytical studies of dispersion in media formed by lattices of ideally conducting loaded cylinders (see Fig. 1) using the local field approach. Also, reflection from an interface between such lattices and free space is studied. Our technique is similar to the theory [2] describing dispersion and reflection properties of three-dimensional lattice of point scatterers. At the first stage we derive a transcendental dispersion equation in the impedance form like in [2,3,4]. We solve this dispersion equation in order to obtain dispersion curves for such a structure. The behavior of dispersion curves dramatically depends on the value of the loads.

Having completed the analysis of dispersion characteristics of the medium it becomes possible to study the reflection properties of a half space filled by an electromagnetic crystal of our interest. In order to solve the reflection problem we solve the dispersion equation numerically by determining the normal components of the propagation vector in terms of the incident wave vector and find all modes that can exist in the structure including evanescent ones. Following [2], we can then determine the reflection coefficient from a half space. It is very important to take into account evanescent modes. These modes do not change the absolute value of the reflection coefficient but they influence its phase. Interesting effects of changing of the reflection coefficient within the band gap between –1 and +1 are revealed. The introduced theory gives an exact solution of the reflection problem in a simple and physically clear form. In the presentation, typical dispersion curves, graphs for decaying modes and for the reflection coefficient are given and discussed.

STUDY OF DISPERSION AND REFLECTION FOR DIFFERENT LOADS

At first we consider arrays of wires loaded by capacitances. This means that the wire is periodically interrupted (period is much smaller than the wavelength) and lumped capacitances are inserted in every gap. If the load capacitance tends to infinity (which corresponds to unloaded cylinders) we obtain the classical dispersion curves [3,5] with complete coincidence. Also we observe a classical wide band gap [6] at low frequencies.

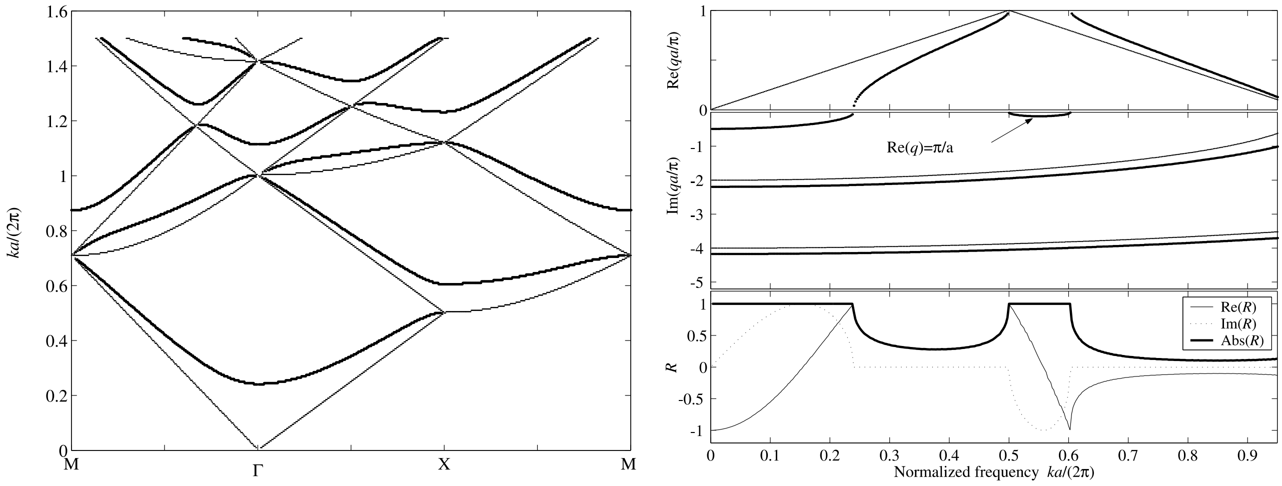

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Fig. 2. Dispersion plot (a), and dependencies of the reflection coefficient and propagation factor on the normalized frequency (b) for unloaded wire media with 0.001 filling ratio. Dispersion curves for the media are presented as thick lines. Thin lines represent dispersion curves for free space.

In the case when the capacitance is very small (which corresponds to interrupted cylinders with wide gaps), a loaded cylinder can be viewed as a line of dipoles, and the medium behaves as a three-dimensional lattice of dipole scatterers. In this case our general results reduce to that of [6] (see Fig. 3a). Thus, we have verified our theory in two limiting cases. If we have some large but finite capacitive loads, a new small pass band at very low frequencies is formed, but the dispersion properties at high frequencies are practically not influenced by the wire loads (see Fig. 4). Making load capacitance smaller, we reduce the band gap at low frequencies. If the self-resonance frequency of the loaded cylinder is high enough, this band gap completely disappears and the first branch of the dispersion curves takes the same form as for a three-dimensional lattice of point scatterers.

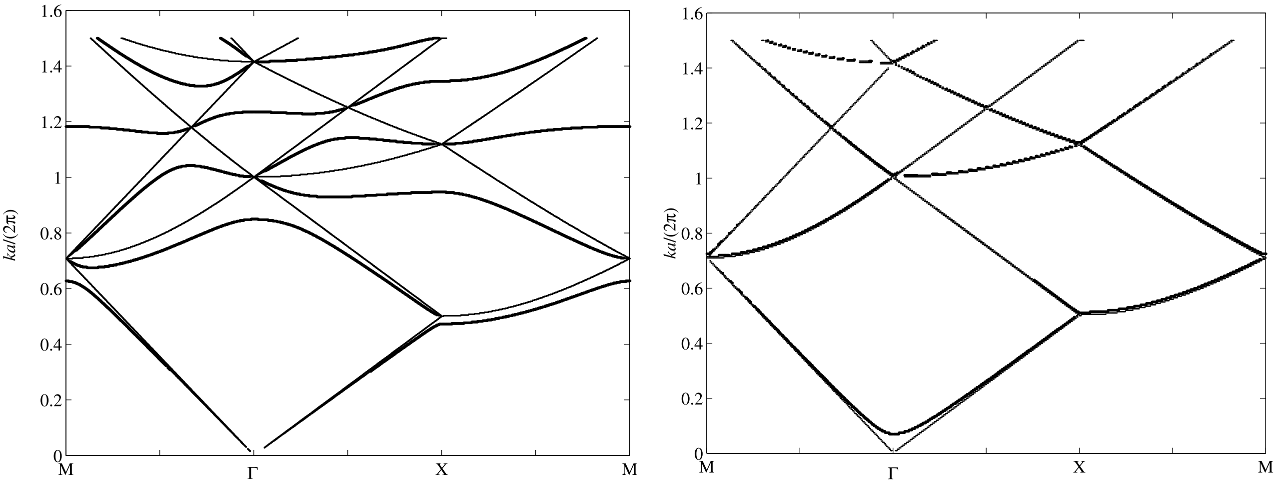

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Fig. 3. Dispersion plots for wire media loaded by capacitive impedance per unit length with $C=0.05\pi\varepsilon_0$ (a) and by inductive impedance per unit length with $L=20\pi\mu_0$ (b). Filling ratio is 0.001. Dispersion curves for the media are presented as thick lines. Thin lines represent dispersion curves for free space.

Inductive loads connected in series with wire sections allow us to make the low frequency band gap narrower (see Fig. 3b). This can be interpreted as an effectively decreasing of the diameter of wire. This way we can make the low frequency stop band very thin. At the upper edge of this stop band the interface of the wire lattice behaves as a magnetic wall. So, we can create an artificial magnetic wall with such a lattice at very low (and tunable) frequencies.

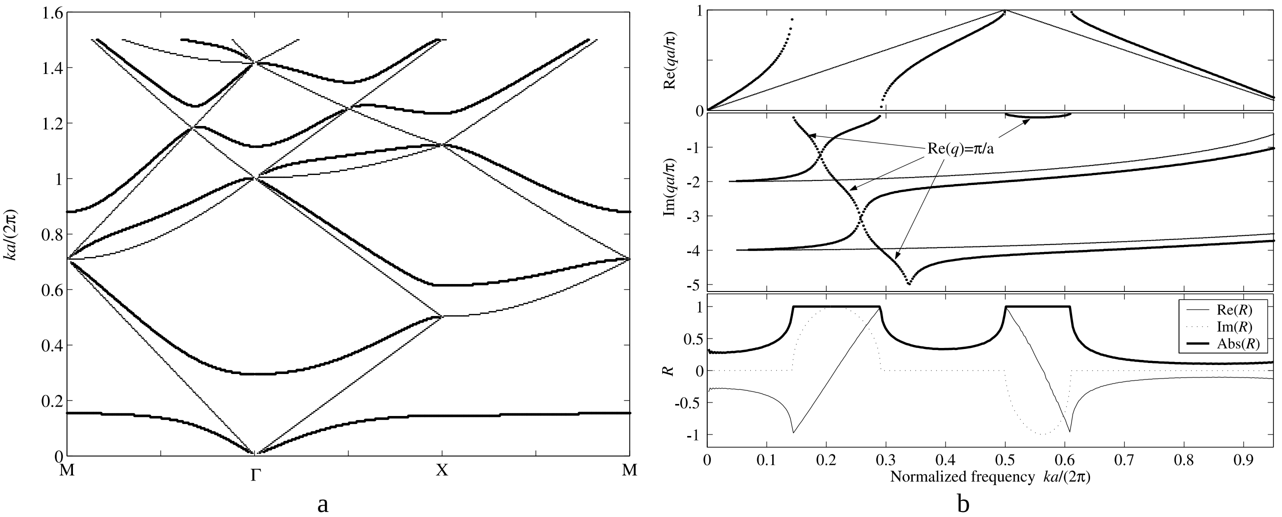

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Fig. 4. Dispersion plot (a) and dependencies of the reflection coefficient and propagation factor on the normalized frequency (b) for wire media loaded by capacitive impedance per unit length with $C=2\pi\varepsilon_0$. Filling ratio is 0.001. Dispersion curves for the media are presented as thick lines. Thin lines represents dispersion curves for free space.

Combinations of inductive and capacitive loads give us additional opportunities of tuning dispersion characteristics. If the bulk loads contain a parallel connection of an inductance and a capacitance, the main effect is in the appearance of a new pass band near the resonance frequency of the load circuit (see Fig. 5). One can tune dispersion characteristics by positioning the self-resonance pass band at any required frequency. Furthermore, the band gap structure in this case is modified only very near to the load resonance. Far from that frequency the waves are naturally not affected.

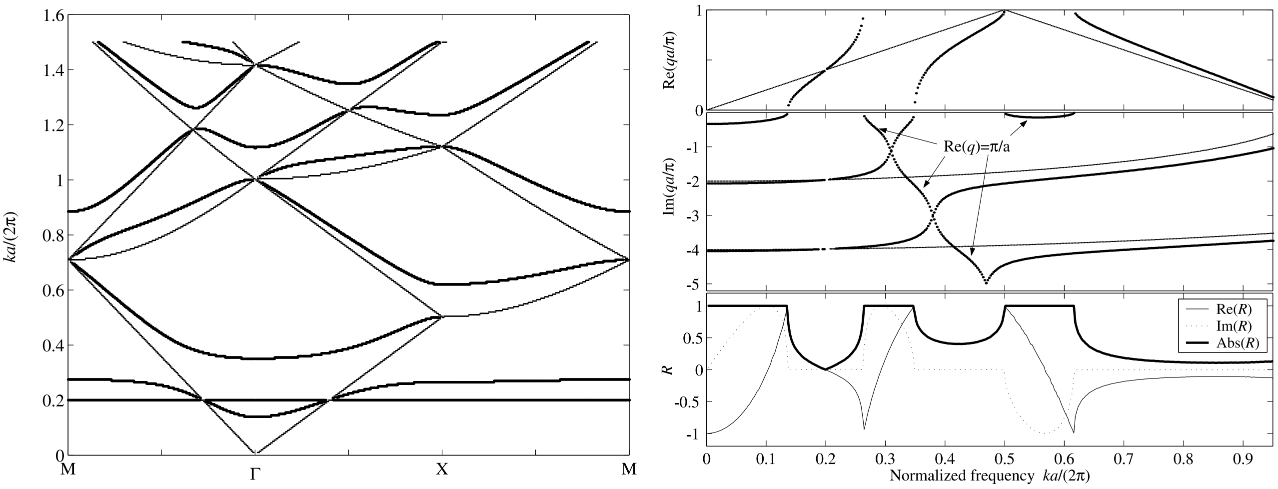

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Fig. 5. Dispersion plot (a) and the dependencies of the reflection coefficient and propagation factor on the normalized frequency (b) for wire media loaded by parallel resonant circuits with $L=2\pi\mu_0$ per unit length and tuned to the resonant frequency with $k_0=0.4\pi/a$. Filling ratio is 0.001. Dispersion curves for the media are presented as thick lines. Thin lines represents dispersion curves for free space.

For the low-frequency regime it is possible to introduce effective parameters (frequency dependent permittivity in this case) of the material. In the classical case of ideally conducting and not loaded wires [6] we have a negative permittivity. In the case of capacitive and parallel circuit loads the permittivity is resonant.

CONCLUSION

We have developed an analytical dispersion theory together with a theory of reflection for the analysis of electromagnetic crystals formed by rectangular lattices of infinite ideally conducting loaded wires. Calculated dispersion curves and reflection coefficients for some typical cases have been presented. The simplest structure among considered ones is unloaded wire media. It is known in the microwave engineering for a long time as an artificial dielectric with a negative permittivity at low frequencies. We have analyzed in detail the properties of reactively loaded wire media. We have found that crystals with capacitively loaded wires are ordinary artificial dielectrics at low frequencies without any changes of high frequency properties with respect to unloaded wire media. Inductive loading effectively reduces the radius of wires and makes the low-frequency stop band thinner. This can help to position the upper edge of this stop band (where the crystal exhibits some interesting reflective properties) at the frequency as low as required. Resonant *LC*-circuit loadings allow to design crystals with properties which are very sensitive to the circuit resonance. All these loads can be electrically controlled, which opens a way to create smart microwave filters and antenna reflectors.

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