REDUCED-ORDER MACROMODELING

OF COMPLEX MULTIPORT INTERCONNECTS

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ABSTRACT

This paper illustrates a technique for the generation of reduced-order lumped macromodels of linear distributed interconnect structures. The method is based on a robust state-space identification algorithm using as raw data discrete-time sequences of input/output waveforms at the accessible ports of the structure. Such sequences can be the result of a full-wave transient electromagnetic simulation using finite differences or finite elements. The poles describing the dominant dynamics of the structure are determined very accurately, allowing for the estimation of a state-space representation of the macromodel. The latter can be easily transformed into equivalent circuits for reduced-order system-level simulations.

INTRODUCTION

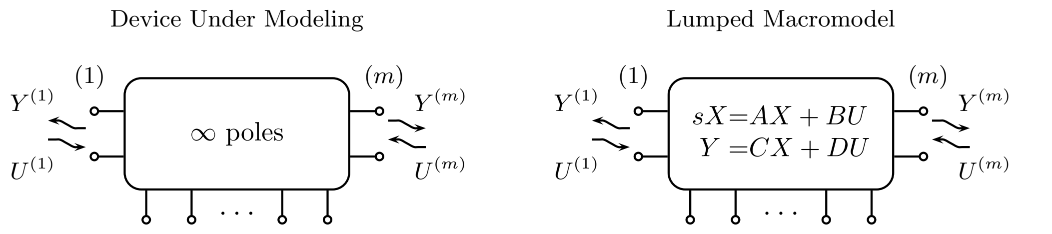
The high complexity of modern electronic systems calls for simplified modeling tools in order to perform system-level simulations for Signal Integrity and Electromagnetic Compatibility applications. Indeed, it is widely recognized that a system-level 3D electromagnetic simulation, including the effects of nonlinear drivers/receivers, is non-feasible even with today's powerful computing tools.

Macromodeling techniques tackle the modeling problem under a different perspective. Various subparts of the system are characterized separately at their accessible ports, either via numerical simulation or direct measurement. This procedure can be applied, e.g., to linear interconnects, junctions, packages, or connectors. Macromodeling techniques allow for the generation of simple equivalents starting from such characterizations. The macromodels mimic the port behavior of the structure and can be easily synthesized as SPICE-like subcircuits for system-level simulations, in order to include nonlinear effects of drivers/receivers.

Several approaches have been presented in the very recent literature for the generation of macromodels. Such methods usually aim at the identification of a set of dominant poles of the Device Under Modeling, henceforth DUM, allowing for reduced-order approximations that are closely related to its actual dynamics. Some methods can be applied to process frequency-domain measurements/characterizations through rational functions approximations, usually performed via nonlinear least squares algorithms or by vector fitting [4]. Some other methods process the large matrices stemming from a full-wave discretization of Maxwell's equations aiming at the construction of a smaller (reduced-order) system capable of preserving the DUT behavior over a prescribed bandwidth [2]. A third class of methods processes measured or simulated time-domain waveforms of input/output port responses. The subject of this paper is related to this latter class of techniques.

Herewith we focus on the reduced-order macromodeling of complex interconnects and packaging structures from either measured or simulated transient port scattering waveforms. In particular, we address and compare several different algorithms trying to identify which are most suitable for the characterization of real-world structures having a very large number of ports. The two main algorithms that are considered in this work are the Block Complex Frequency Hopping (BCFH) algorithm [3] and the Subspace-based State-Space System Identification (4SID) techniques [6]. The BCFH method is a well-known technique, which has been extensively used for several applications. Therefore, we will use BCFH mainly for comparisons, pointing the reader to the vast literature on the subject, and we will focus on 4SID techniques

The 4SID methods determine a state-space representation of the DUM via direct identification of the state matrices $\{A, B, C, D\}$. The poles distribution is a byproduct of this one-step procedure, since such state-space representation can be directly used to synthesize equivalent circuits. The identification algorithm makes use of highly reliable numerical tools (QR and SVD decompositions), and proves less sensitive to time-domain truncation of the port signals with respect to BCFH. Next section outlines the main 4SID-based algorithm that we propose for the specific macromodeling application. Some validations and numerical examples will follow.



$$Y(s) = H(s)U(s) \qquad Y(s) = \{C(sI - A)^{-1}B + D\}\,U(s) = \widehat{H}(s)U(s)$$

Figure 1: State-space based macromodeling. Arrays Y and U collect input and output variables at all ports.

SUBSPACE-BASED SYSTEM IDENTIFICATION

The DUM is represented in Fig. 1. Some linear interconnect structure is considered at a limited number m of accessible ports. The structure is of distributed nature, therefore an infinite number of poles would be required for an accurate representation of the input/output relations $Y(s) = H(s)U(s)$, where U and V are vectors collecting all port variables. The goal is to identify a reduced-order linear lumped system, characterized by n poles and represented by the state matrices $\{A, B, C, D\}$, such that all input/output transfer functions $\widehat{H}(j\omega)$ approximate the true ones over a specified bandwidth $|\omega| < \Omega$. The starting point is a set of transient waveforms of outputs y_k, where k denotes the time index with a suitable sampling time T_s, subject to some combination of input waveforms u_k. Such waveforms can be obtained, e.g., from a full-wave transient simulation based on Finite-Differences or Finite Elements spatial discretization of electromagnetic fields. In such case, $\{u, y\}$ sequences may be identified with ingoing and outgoing transient scattering waves $\{a, b\}$ at the DUM ports.

Since the input/output sequences are sampled, it is convenient to focus our attention on the discrete-time system,

$$\left\{ \begin{array}{lcl} x_{k+1} & = & \widetilde{A}x_k + \widetilde{B}u_k \\ y_k & = & \widetilde{C}x_k + \widetilde{D}u_k \end{array} \right. \tag{1}$$

where x denotes the set of internal discrete-time states. This formulation allows us to use the several theoretical results available for system identification of discrete-time systems [5]. In order to simplify the presentation of the main algorithm, we first consider a Single-Input Single/Output (SISO) system. Generalization to a large number of ports will follow.

First, we note that the above system may be rewritten in matrix form as

$$\mathcal{Y} = \Gamma\mathcal{X} + \Phi\mathcal{U}\,, \tag{2}$$

where $\mathcal{U}, \mathcal{Y}, \mathcal{X}$ denote block Hankel matrices of input, output, and state time sequences, respectively. We recall that given some sequence z_k, the corresponding Hankel matrix is defined as $(\mathcal{Z})_{ij} = z_{i+j-1}$. In the above expression Γ represents the observability matrix

$$\Gamma = \left[\begin{array}{c} \widetilde{C} \\ \widetilde{C}\widetilde{A} \\ \widetilde{C}\widetilde{A}^2 \\ \vdots \end{array} \right], \tag{3}$$

while Φ is a Toepliz matrix of impulse responses. A remarkable feature of Eq. (2) is that the output matrix is expressed as a linear combination of state matrix and input matrix. Therefore, a suitable projection of $\mathcal{Y}$ onto some linear space orthogonal to $\mathcal{U}$ leads to an estimate of the observability matrix, which in turn can be used to estimate the state matrix $\widetilde{A}$. Some details follow, whereas a more complete formulation can be found in [6].

One of the most convenient ways to perform this projection is through the following RQ factorization

$$\left[\begin{array}{c} \mathcal{U} \\ \mathcal{Y} \end{array} \right] = \left[\begin{array}{cc} R_{11} & 0 \\ R_{21} & R_{22} \end{array} \right] \left[\begin{array}{c} Q_1 \\ Q_2 \end{array} \right] = RQ\,, \tag{4}$$

where $Q^TQ = I$. If we post-multiply the various terms in Eq. (2) by Q_2^T, we get

$$\mathcal{Y}Q_2^T = R_{22} = \Gamma\mathcal{X}Q_2^T\,, \tag{5}$$

where we used the orthogonality relations $Q_2 Q_2^T = I$ and $Q_1 Q_2^T = 0$. A consistent estimate for the observability matrix Γ may be obtained by performing the SVD decomposition

$$R_{22} = \widehat{\Gamma}\, \Sigma\, V^T . \tag{6}$$

The number of relevant singular values in Σ gives an estimate of the effective rank of the observability matrix, and consequently the order n of the state-space system to be identified. The corresponding columns of $\widehat{\Gamma}$ form the reduced-order observability matrix. Using now Eq. (3), both matrices $\widetilde{C}$ and $\widetilde{A}$ are easily found.

Several approaches [6] can now be used to estimate the remaining matrices $\widetilde{B}$ and $\widetilde{D}$ and convert the macromodel to continuous-time. We found the best results for present application through the following procedure, which uses only the above estimate for the matrix $\widetilde{A}$. First, its eigenvalues λ_i are determined. Then, a straightforward discrete-to-continuous time conversion is performed to obtain the poles of the continuous-time macromodel represented in Fig. 1, or equivalently the eigenvalues of A,

$$p_i = \frac{1}{T_s} \ln(\lambda_i) . \tag{7}$$

Each transfer function of the reduced-order macromodel will have poles taken from the above set.

In case of a DUM with many ports, the best numerical accuracy was obtained by repeating m times the poles estimation with excitation at a single port at the time. All output sequences resulting at the DUM ports are used altogether in a block-matrix form. The obtained m sets of estimated poles are then post-processed by a suitable clustering algorithm to extract the global poles set for the entire multiport macromodel. It should be noted that when real-world packages are considered, often each set of independently estimated poles can actually be used directly for the entire macromodel, thereby skipping the clustering step. This will be illustrated by the examples in the forthcoming section.

Once the poles of the DUM macromodel are known, the partial fraction expansion

$$\widehat{H}(s) = \sum_i H_i \frac{1}{s - p_i} + H_\infty \tag{8}$$

is considered, where H_i are unknown matrices of residues and H_∞ is a matrix collecting the direct coupling constants between all DUM ports. These matrices are easily obtained by a linear least squares fit from original input/output data sequences. This fit can be performed either on time-domain sequences or using frequency-domain data. In this work we use time-domain fitting.

The final step in the algorithm is the identification of the state matrices $\{A, B, C, D\}$ from poles and residues. Once such matrices are found, they can be used to synthesize SPICE-like equivalent circuits through well known techniques. The particular form of the state matrices depends on some arbitrary choice, since several realizations are possible. We can use one of the simplest possibilities, namely a realization in Jordan canonical form. This amounts to setting $A = \mathrm{diag}\{p_i\}$. Therefore, from the state-space equations,

$$\widehat{H}(s) = C\,(sI - A)^{-1} B + D = \sum_i \frac{c_i b_i^T}{s - p_i} + D \tag{9}$$

where c_i and b_i^T are the columns of C and the rows of B, respectively. This indicates that matrices H_i have a theoretical unitary rank. However, since the actual estimates come from a least squares fit, the numerical rank will be generally larger than one. Therefore, some matrix decomposition can be applied to H_i to find suitable minimal realizations that retain the accuracy of the performed fit. In particular, we adopt here the procedure in [1], which is based again on the SVD decomposition.

The following remarks are important. First, there is no theoretical limit in the number of DUM ports. The algorithm has been developed in such a way that the critical operations related to the poles estimation are performed without using all ports information at once, thus leading to very accurate poles estimates. It is well known that this feature is the key factor for an accurate least-squares approximation and for the subsequent steps of the macromodeling algorithm. Second, since the above algorithm is very accurate, the enforcement of the passivity for the DUM macromodel can be performed a posteriori by applying small corrections if some passivity test fails [4].

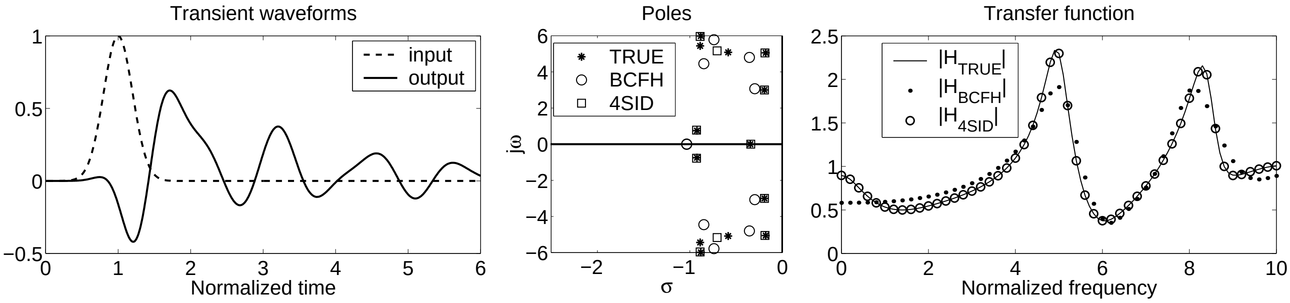


Figure 2: Identification of a synthetic system with randomly generated poles/residues.

NUMERICAL EXAMPLES

The first numerical example shows that the proposed technique appears more robust than BCFH when truncated transient waveforms are processed. Careful positioning of the expansion centers in the complex plane for computations of moments needed by BCFH is necessary in this case. Such feature is illustrated in Fig. 2, depicting the results of the identification of a SISO system with randomly generated poles and residues. The dominant poles are well estimated by 4SID, leading to better accuracy.

The second example (Fig. 3) deals with a 32-port package structure. The transient scattering waves (left panel) obtained through a full-wave FDTD simulation are used to estimate the global set of poles (center panel). These are then used to determine the state-space representation of the device macromodel. Some of the frequency-domain scattering parameters are depicted in the right panel of Fig. 3. The approximated curves (dots) are almost undistinguishable from the original curves (thin continuous lines).

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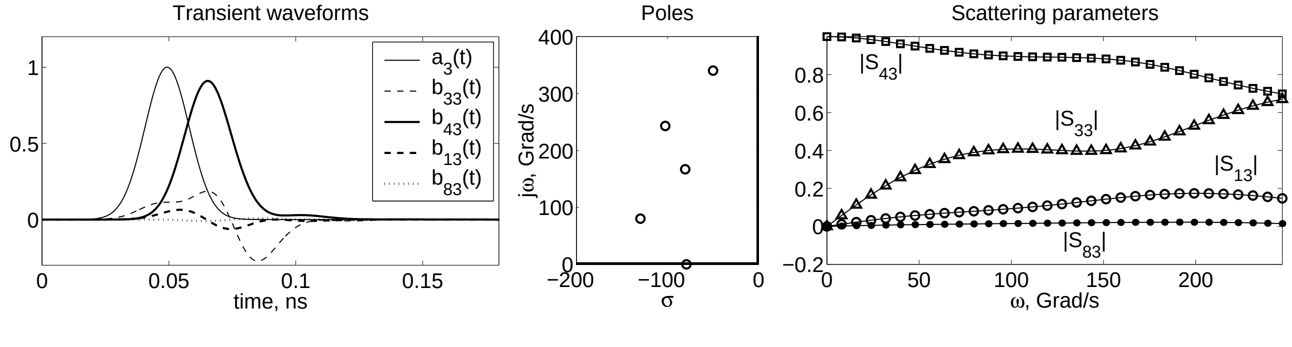

Figure 3: Macromodeling of a 32-port package.