

# IMPROVING THE FREQUENCY STABILITY OF MICROWAVE OSCILLATORS BY UTILIZING THE TEMPERATURE COMPENSATED DIFFERENCE FREQUENCY OF A DUAL-MODE SAPPHIRE LOADED CAVITY RESONATOR

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## ABSTRACT

A new difference frequency technique to obtain frequency-temperature compensation using high quality anisotropic dielectric resonators is presented (patent pending). Compensation was demonstrated near room temperature, and between solid and liquid nitrogen temperature (50 to 77 K) by frequency locking two microwave oscillators (Dual-Mode configuration) to orthogonally polarized whispering gallery modes in the same sapphire resonator.

## INTRODUCTION

It is well known that anisotropic single crystals also have anisotropic Temperature Coefficients of Permittivity (TCP)[1-3]. Also, it is well known that single crystals (such as synthetically grown sapphires) can be configured as the highest quality-factor (Q-factor) microwave dielectric resonators across a broad temperature range. The high quality-factor is desirable for the realization of precision oscillators, as the precision of the frequency and phase depends directly on this parameter. Also, the precision of an oscillator depends on the frequency independence to external influence such as temperature. Single crystals have a large TCP, and thus the frequency is influenced by fluctuations in temperature.

Standard techniques developed for frequency stabilization of classical microwave oscillators involve either composite dielectric resonators of opposite signs of TCP [4], or monolithic sapphire resonators doped with paramagnetic materials [5]. The drawbacks of standard techniques include enhanced sensitivity to vibration and temperature gradients (composite resonator), unmatched time constant of the compensation material, and reduced Q-factor. An alternative approach to the problem of oscillator frequency stabilization relies on the excitation of two electromagnetic modes in the same resonator, tuned to nearly the same microwave frequency. The two modes must have different Temperature Coefficient of Frequency (TCF), and can be used to determine the relative temperature with nano-Kelvin accuracy by measuring the difference frequency (a few tens of MHz) between the two modes [6, 7]. The resonator temperature stabilization requires a sophisticated feedback control system for adjusting microwave power dissipated in the resonator. Design of such a resonator becomes quite complicated if cryogenic applications are to be considered [8].

In this work we propose a new method of oscillator frequency stabilization utilizing the temperature independence of the difference frequency between two widely spaced (a few GHz) modes excited in the same anisotropic dielectric resonator. Orthogonally polarized Whispering Gallery (WG) modes, such as Transverse Electric (WGE) and Transverse Magnetic (WGH) modes, exhibit different fractional TCF in units  $K^{-1}$ . By selecting modes with the same TCF in units Hz/K, one can achieve the temperature compensation of the difference frequency,

$$\Delta f(T) = f_{WGE} - f_{WGH} \quad (1),$$

at the required temperature. Standard dual-mode techniques [7] may then be adapted to create a temperature compensated oscillator based on the difference frequency of the modes, with the implementation of standard temperature control techniques. The new principle was verified using Whispering Gallery modes in a 5cm diameter Sapphire Loaded Cavity resonator.

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## THEORY

In general the resonator fractional TCF can be expressed as (ignoring dimensional effects, which are an order of magnitude smaller);

$$\frac{1}{f(T)} \frac{\partial f(T)}{\partial T} \Big| \approx -\frac{1}{2} \left[ p_{e\perp} \alpha_{e\perp}^*(T) + p_{e\parallel} \alpha_{e\parallel}^*(T) \right] \quad (2)$$

Here,  $p_{e\perp}, p_{e\parallel}$  are the electric energy filling factors perpendicular and parallel to anisotropy axis respectively, and  $\alpha_{e\parallel}^*$  and  $\alpha_{e\perp}^*$  are the temperature dependent TCP's parallel and perpendicular respectively. WGE modes have  $p_{e\perp} \sim 1$  and  $p_{e\parallel} \sim 0$  and WGH modes have  $p_{e\perp} \sim 0$  and  $p_{e\parallel} \sim 1$ . Thus, it is evident that modes of the same type have similar fractional frequency dependence, and modes of different type have dissimilar fractional frequency dependence. The fractional TCF of the frequency difference,  $\Delta f$ , can be determined by combining (1) and (2), and is given by:

$$\frac{1}{\Delta f(T)} \frac{\partial \Delta f(T)}{\partial T} \Big| \approx -\frac{1}{2} \frac{1}{\Gamma - 1} \left[ \Gamma \left( p_{e\perp}^{WGE} \alpha_{e\perp}^* + p_{e\parallel}^{WGE} \alpha_{e\parallel}^* \right) - \left( p_{e\perp}^{WGH} \alpha_{e\perp}^* + p_{e\parallel}^{WGH} \alpha_{e\parallel}^* \right) \right] \quad (3)$$

Here,  $\Gamma(T) = f_{WGE} / f_{WGH}$ , is the frequency ratio of the two modes and in general is temperature dependent.

For a material such as sapphire the permittivity is nearly independent of frequency and the TCP's are well known to be anisotropic ( $\alpha_{e\parallel}^* \neq \alpha_{e\perp}^*$ ) [2]. Thus, for a pair of orthogonally polarized modes, there exists a unique frequency separation that will result in the annulment of the TCF of the difference frequency given by (3) at the required temperature of operation. For a pair of pure Transverse Electric and Transverse Magnetic modes with electrical energy filling factors of order unity, this occurs when the frequency ratio is given by (setting (3) to zero);

$$\Gamma(T) \approx \frac{\alpha_{e\parallel}^*}{\alpha_{e\perp}^*} \quad (4)$$

Thus, the required frequency ratio for compensation is mainly determined by the degree of anisotropy of the TCP's at the required operation temperature. All WG modes are in fact hybrid and have a small amount of the non-dominant field as suggested by equation (3). This must be taken into account for precise calculations of the frequency-temperature compensation point, along with expansion and contraction effects.

## DUAL-MODE TEMPERATURE COMPENSATED RESONATOR-OSCILLATORS

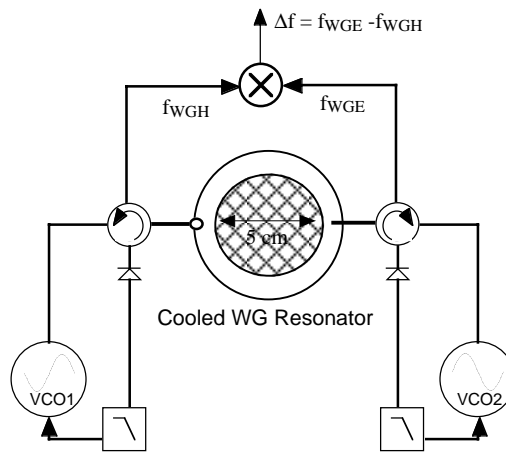


Fig. 1. Dual-mode circuit to generate a stable frequency from the difference frequency of a WGE and WGH mode. The circuit requires two Pound frequency stabilization circuits.

Both solid nitrogen cooled and room temperature Peltier cooled dual-mode microwave oscillators were developed using the same Sapphire Loaded Cavity Resonator. A dual mode oscillator was constructed as shown in fig. 1. Two independent synthesizers were Pound locked to WGE and WGH resonance frequencies, in the same sapphire resonator. The experiment was undertaken near room temperature using a calibrated thermistor to measure the temperature. The resonator was placed in a vacuum chamber and temperature controlled using a thermistor as the temperature sensor to control a thermoelectric Peltier element. The frequency-temperature characteristic was measured by controlling the temperature to specific fixed values and waiting until the resonator came into equilibrium. The same resonator was later cooled with solid nitrogen by placing the vacuum-sealed resonator in a liquid nitrogen cryostat and pumping on the liquid. The experiment was undertaken using a calibrated platinum thermometer to measure the temperature. The resonator frequency-temperature characteristic was measured as the solid melted over a period of two days, which was slow enough for the resonator to remain in equilibrium with the surrounding temperature.

### Peltier Cooled Oscillator

Figure 2, shows the measurement of the difference frequency versus temperature characteristic, for various pairs of WGE and WGH modes near room temperature. A calibrated thermistor was used to read out the temperature and a frequency counter to measure the difference frequency between WGE and WGH modes. In particular we measured compensation points between the  $WGE_{10,0,0}$  (9.120 GHz) and  $WGH_{8,0,0}$  (6.556 GHz) modes at  $-24.6^{\circ}$  Celsius ( $\Gamma = 1.391$ ), and the  $WGE_{11,0,0}$  (9.804 GHz) and  $WGH_{9,0,0}$  (7.166 GHz) modes at  $29.9^{\circ}$  Celsius ( $\Gamma = 1.368$ ). The curvature of both these compensation points was  $8 \times 10^{-8}/K^2$ .

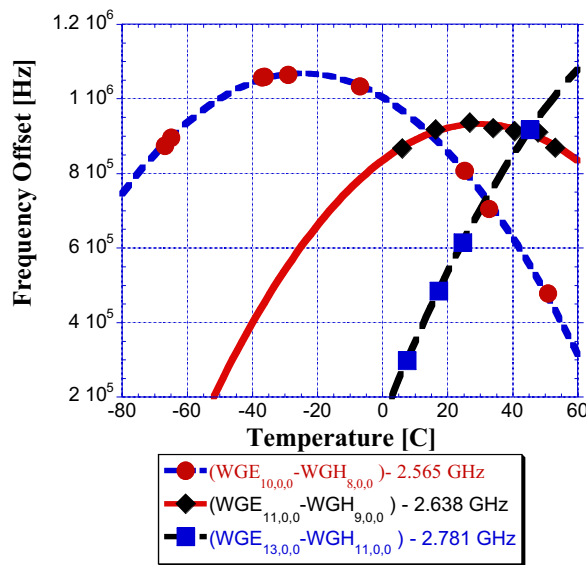


Fig.2. The difference frequency of various WGE and WGH modes measured as a function of temperature.

### Solid Nitrogen Cooled Resonator-Oscillator

Figure 3, shows the measurement of the difference frequency versus temperature characteristic for the  $WGE_{11,0,0}$  and  $WGH_{8,0,0}$  modes near solid nitrogen temperature. A calibrated platinum thermometer was used to read out the temperature and a frequency counter to measure the difference frequency between the modes. The compensation points between the  $WGE_{11,0,0}$  (9.876 GHz) and  $WGH_{8,0,0}$  and  $WGH_{8,0,0}$  (6.624 GHz) mode was measured to be at 58 K, and the curvature was measured to be  $3 \times 10^{-8}/K^2$ .

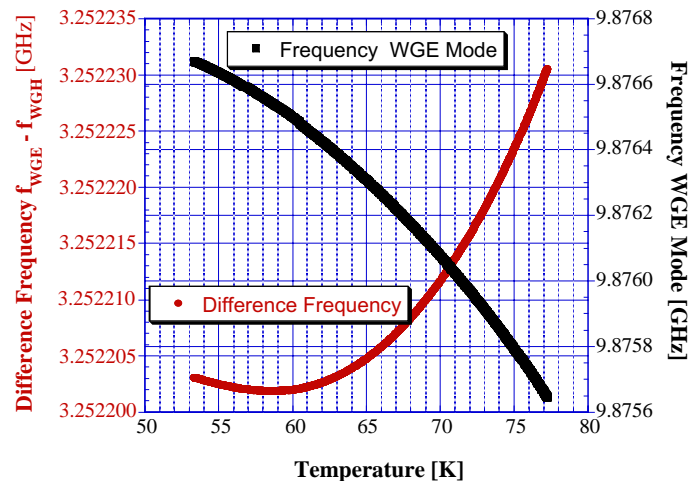


Fig.3. The frequency of the WGE<sub>11,0,0</sub> mode and the difference between the WGE<sub>11,0,0</sub> and WGH<sub>8,0,0</sub> modes measured as a function of temperature.

## CONCLUSION

We have constructed dual mode resonator-oscillator based on a high-Q sapphire resonator that generates a frequency that is first-order invariant to fluctuations in temperature. This was achieved by exploiting the anisotropy of the Temperature Coefficients of Permittivity in a uniaxial sapphire crystal resonator, and was achieved near solid nitrogen and room temperature.

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