APPLICATION OF THE COUNTER-PROPAGATION DEDUCTION (CPD) METHOD IN THE ANALYSIS OF THE EM FIELD IN SPHERICALLY STRATIFIED MEDIA

Qing-yi Zhang(1, 2) Jian-ming Zhu(1)

(1) Department of Electrical Engineering, China University of Mining and Technology at Beijing

Ding 11, Xueyuan Road, Beijing, 100083, China

(2) Department of Mathematics, University of Glasgow, Glasgow G12 8QW, UK

zq@maths.gla.ac.uk zhang_qingyi@hotmail.com

ABSTRACT

A new method — the CPD method is applied to deducing the Debye potentials π_e and π_m, which are introduced to express the EM field in a spherically stratified medium. By introducing a new concept — the Boundary Originated Set (BOS), the method can directly construct the analytic solutions, which, due to the deducing procedure and the expressions being very straightforward, can be easily evaluated numerically, irrespective of the number of the layers.

INTRODUCTION

The multi-layer stratified medium is one of the classical configurations in Electromagnetic (EM) Theory. Analysis of the EM field in it has been carried out systematically by many researchers and for many years [1-5]. However, due to involving iterative calculations for the reflection coefficients, the main theoretical methods, which are analogous to analysis of the voltage and current on a tandem transmission line, become impractical when the layer number is large.

By introducing a new concept named Boundary Originated Set (BOS) of waves, the Counter – Propagation Deduction (CPD) Method is characterized by direct construction of the solution [6]. Due to the constructing procedure and the expression configuration being quite straightforward, the rigorous numerical solution can be easily obtained by programming. As far as the horizontally stratified medium is concerned, the CPD method can be applied not only to a uniform wave case [6] but also to the case of a vertical and horizontal electrical dipole [7], [8]. The efficiency of the method has been confirmed by the numerical analysis of the field excited by an arbitrary shape of a loop antenna [9]. This paper investigates its application to the case that the medium is spherically stratified. Based on [5], this paper focuses on constructing Debye potentials π_e and π_m, which satisfy the homogeneous wave equation. They are introduced to express $\vec{E}$ and $\vec{H}$ in the following way:

$$\vec{E} = \nabla\times\vec{a}_r r\pi_m - \frac{1}{i\omega\varepsilon}\nabla\times\nabla\times\vec{a}_r r\pi_e \tag{1}$$

$$\vec{H} = \nabla\times\vec{a}_r r\pi_e + \frac{1}{i\omega\mu}\nabla\times\nabla\times\vec{a}_r r\pi_m \tag{2}$$

where $\vec{a}_r$ is a unit vector along the r - axis, r is the radial distance of the field point from the origin, and ε, μ is the permittivity, permeability of the medium, respectively.

THE CPD METHOD AND THE BOS IN THE CASE OF A SPHERICALLY STRATIFIED MEDIUM

The basic unit of the CPD method to construct the EM field in a stratified medium is the BOS. The BOS is defined as an elementary set of waves on opposite sides of an interface. They satisfy the boundary conditions at the interface, and can be obtained by analyzing the EM field in a two-layer medium produced by the corresponding source. The components in the BOS are characterized such that once the transmitted component in the back layer is assumed or determined, the others in the front layer, in which the incident component travels, are also uniquely determined by it. The relationship between the transmitted component and the component to be deduced is described by the so-called corresponding deduction coefficient of the BOS.

Thus, the field component in each layer can be directly constructed from the last layer (L_1), where the final transmitted wave propagates, in a counter-propagation direction. Due to the last layer being semi-infinite, the expression of the corresponding field component in it can be assumed. Taking it as the transmitted component in the BOS at the interface I_1, then the other components in the front-layer (L_2) in the BOS can be deduced by the corresponding coefficients; then, taking the deduced components in L_2 one by one as a transmitted component of the corresponding BOS at I_2, the other components of the BOSs in the front-layer (L_3) can be similarly determined. The procedure goes on until all the components in L_n, in which the source or the original incident wave lies, are deduced. Due to all the components being deduced from the assumed component, only one parameter remains unknown. By means of the known source, the unknown parameter can be obtained, and consequently, the resultant field in each layer can also be found.

The boundary conditions for π_e, π_m in a two-layer spherically stratified medium can be simplified as:

$$\frac{1}{C_2}\frac{\partial}{\partial r} rF_2\bigg|_{r=r_1} = \frac{1}{C_1}\frac{\partial}{\partial r} rF_1\bigg|_{r=r_1} \tag{3}$$

$$F_2\big|_{r=r_1} = F_1\big|_{r=r_1} \tag{4}$$

where, when $F = \pi_e$, C denotes ε, and when $F = \pi_m$, C denotes μ; and ε_p, μ_p (p = 1, 2) is the ε, μ in L_p.

These can be obtained by means of the relationship between the Debye potentials and tangential part of $\vec{E}$ and $\vec{H}$, and the general expressions of the π_e and π_m as shown in [5]. To match (3) and (4), if π_e in the back-layer L_1 is traveling and assumed to be:

$$\pi_{e,1} = h_n^{(1)}(k_1 r) P_n^m(\cos\theta) e^{jm\phi} \tag{5}$$

where k is the wave number in the corresponding medium; $h_n^{(1)}(kr)$ and $j_n(kr)$ in the following expressions are the spherical Bessel functions; and $P_n^m(\cos\theta)$ is the associated Legendre polynomials,

then, π_e in the front-layer L_2 should be:

$$\pi_{e,2} = \Gamma_{e,h,1} h_n^{(1)}(k_2 r) P_n^m(\cos\theta) e^{jm\phi} + \Gamma_{e,j,1} j_n(k_2 r) P_n^m(\cos\theta) e^{jm\phi} \tag{6}$$

where

$$\Gamma_{e,h,1} = \frac{\sqrt{\mu_1\varepsilon_2}\hat{H}_n^{(1)}{}'(k_1 r_1)\hat{J}_n(k_2 r_1) - \sqrt{\mu_2\varepsilon_1}\hat{H}_n^{(1)}(k_1 r_1)\hat{J}_n{}'(k_2 r_1)}{i\varepsilon_1\sqrt{\mu_1/\varepsilon_2}} \tag{7}$$

$$\Gamma_{e,j,1} = \frac{\sqrt{\mu_2\varepsilon_1}\hat{H}_n^{(1)}(k_1 r_1)\hat{H}_n^{(1)}{}'(k_2 r_1) - \sqrt{\mu_1\varepsilon_2}\hat{H}_n^{(1)}(k_2 r_1)\hat{H}_n^{(1)}{}'(k_1 r_1)}{i\varepsilon_1\sqrt{\mu_1/\varepsilon_2}} \tag{8}$$

where $\hat{H}_n^{(1)}(x) = x h_n^{(1)}(x)$, $\hat{J}_n(x) = x j_n(x)$ and r_1 is the radius of the interface.

If π_e in the back-layer is standing and assumed to be:

$$\pi_{e,1}' = j(k_1 r) P_n^m(\cos\theta) e^{jm\phi} \tag{9}$$

then, π_e in the front-layer should be:

$$\pi_{e,2}' = \Gamma_{e,h,1}' h_n^{(1)}(k_2 r) P_n^m(\cos\theta) e^{jm\phi} + \Gamma_{e,j,1}' j_n(k_2 r) P_n^m(\cos\theta) e^{jm\phi} \tag{10}$$

where

$$\Gamma_{e,h,1}' = \frac{\sqrt{\mu_1\varepsilon_2}\,\hat{J}_n(k_2 r_1)\hat{J}_n'(k_1 r_1) - \sqrt{\mu_2\varepsilon_1}\,\hat{J}_n(k_1 r_1)\hat{J}_n'(k_2 r_1)}{i\varepsilon_1\sqrt{\mu_1/\varepsilon_2}} \tag{11}$$

$$\Gamma_{e,j,1}' = \frac{\sqrt{\mu_2\varepsilon_1}\,\hat{H}_n^{(1)}{}'(k_2 r_1)\hat{J}_n(k_1 r_1) - \sqrt{\mu_1\varepsilon_2}\,\hat{H}_n^{(1)}(k_2 r_1)\hat{J}_n'(k_1 r_1)}{i\varepsilon_1\sqrt{\mu_1/\varepsilon_2}} \tag{12}$$

Thus, we have obtained two types of the BOS with each of them consisting of three components: a traveling component and a standing component in the front layer, and a traveling or a standing transmitted component in the back layer. $\Gamma_{e,h,1}, \Gamma_{e,j,1}$, and $\Gamma_{e,h,1}', \Gamma_{e,j,1}'$ are called the deduction coefficients of the BOS in the TM wave case. Here, "*e*" in the subscripts emphasizes the TM wave case; "1" denotes the interface; and "*h*" or "*j*" gives the names to the coefficients. Replacing 2,1 in (7), (8), and (11), (12) with *(k+1), k,* $\Gamma_{e,h,k}, \Gamma_{e,j,k}$, and $\Gamma_{e,h,k}', \Gamma_{e,j,k}'$, the coefficients in the BOS at an arbitrary interface I_k of a n-layer spherically stratified medium can be obtained correspondingly.

By exchanging μ_p for ε_p and ε_p for μ_p ($p = k$ or $k+1$), $\Gamma_{m,h,k}, \Gamma_{m,j,k}$, and $\Gamma_{m,h,k}', \Gamma_{m,j,k}'$, the deduction coefficients of the BOS for π_m in the TE wave case can be easily obtained according to the duality between π_e and π_m.

AN ILLUSIVE EXAMPLE

Assuming a known source is in the core (L_3) of a three-layer spherically stratified medium with the expressions being:

$$\pi_{e,3i} = C_{e0} h_n^{(1)}(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{13}$$

$$\pi_{m,3i} = C_{m0} h_n^{(1)}(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{14}$$

then, the resultant standing part of the EM field in the core can be deduced as follows.

At first, deduce π_e in each layer. Assuming that π_e in the last layer L_1 is:

$$\pi_{e,1} = P_{e1} h_n^{(1)}(k_1 r) P_n^m(\cos\theta) e^{jm\phi} \tag{15}$$

then, by $\Gamma_{e,h,1}, \Gamma_{e,j,1}$, the other two components in the BOS in L_2 can be deduced. They are:

$$\pi_{e,11} = \Gamma_{e,h,1} P_{e1} h_n^{(1)}(k_2 r) P_n^m(\cos\theta) e^{jm\phi} \tag{16}$$

$$\pi_{e,10} = \Gamma_{e,j,1} P_{e1} j_n(k_2 r) P_n^m(\cos\theta) e^{jm\phi} \tag{17}$$

Similarly, taking $\pi_{e,11}$, $\pi_{e,10}$ as the transmitted component of the BOS at I_2, respectively, then, the other components in each BOS in L_3 can be deduced and are:

$$\pi_{e,111} = \Gamma_{e,h,2}\Gamma_{e,h,1} P_{e1} h_n^{(1)}(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{18}$$

$$\pi_{e,110} = \Gamma_{e,j,2}\Gamma_{e,h,1} P_{e1} j_n(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{19}$$

$$\pi_{e,101} = \Gamma_{e,h,2}'\Gamma_{e,j,1} P_{e1} h_n^{(1)}(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{20}$$

$$\pi_{e,100} = \Gamma_{e,j,2}'\Gamma_{e,j,1} P_{e1} j_n(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{21}$$

Superposition of the traveling components in the core (L_3) should be equal to the source expressed in (13), thus, the assumed parameter P_{e1} in (15) can be obtained:

$$P_{e1} = \frac{C_{e0}}{\Gamma_{e,h,2}\Gamma_{e,h,1} + \Gamma_{e,h,2}'\Gamma_{e,j,1}} \tag{22}$$

Consequently, the standing part of π_e in L_3 can be deduced and is:

$$\pi_{e,30} = C_{e0} \frac{\Gamma_{e,j,2}\Gamma_{e,h,1} + \Gamma_{e,j,2}'\Gamma_{e,j,1}}{\Gamma_{e,h,2}\Gamma_{e,h,1} + \Gamma_{e,h,2}'\Gamma_{e,j,1}} j_n(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{23}$$

The standing part of π_m in the core (L_3) can be similarly deduced and is:

$$\pi_{m,30} = C_{m0} \frac{\Gamma_{m,j,2}\Gamma_{m,h,1} + \Gamma_{m,j,2}'\Gamma_{m,j,1}}{\Gamma_{m,h,2}\Gamma_{m,h,1} + \Gamma_{m,h,2}'\Gamma_{m,j,1}} j_n(k_3 r) P_n^m(\cos\theta) e^{jm\phi} \tag{24}$$

CONCLUSION

The Debye potentials π_e and π_m in spherically stratified media are directly constructed by the Counter-Propagation Deduction method, by means of which the EM field can be obtained. After studying the Boundary Originated Set for π_e and π_m, which is the basic unit for the construction, an example is given showing that the form of the constructed expression is straightforward, and thus, the rigorous numerical solution can be easily evaluated by programming.

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