TOWARDS A MODEL OF INCOHERENT SCATTER SIGNAL SPECTRA WITHOUT AVERAGING OVER SOUNDING RUNS

O. I. Berngardt

Department of Upper Atmosphere Physics and Radiowave Propagation,
Institute of Solar-Terrestrial Physics,126, Lermontova Str., POBox 4026, Irkutsk, Russia, 664033.
E-mail:berng@iszf.irk.ru

ABSTRACT

This paper offers a model for incoherent scatter signal spectra without averaging the received signal over sounding runs (realizations) for the case of monostatic sounding. The model is based on the existent theory of radio waves single scattering from the medium dielectric permittivity irregularities, and on the existing kinetic theory of the plasma thermal irregularities. The model explains the width and form of the signal spectrum by macroscopic characteristics of the medium, and its fine 'peaked' structure by characteristics of the ions number phase density.

INTRODUCTION

One of the remote probing techniques for the ionosphere is the method of radio waves incoherent scatter. The method is based on the scattering of radio waves from ionospheric plasma dielectric permittivity irregularities [1]. Furthermore, two different experimental configurations are involved: monostatic (where the receive and transmit antennas are combined) and bistatic (where these antennas are spaced). In actual practice, it is customary to use the monostatic configuration. Ionospheric plasma parameters (ion composition, drift velocity, electron and ion temperatures, and electron density) in this case are determined from the scattered signal received after completion of the radiated pulse. The spectral power of the received signal, averaged over sounding runs ('realizations'), is related (assuming that such an averaging is equivalent to statistical averaging) to the mean spectral density of dielectric permittivity irregularities by the radar equation [2]. The connection of the dielectric permittivity irregularities spectral density with mean statistical parameters of the medium is usually determined in terms of kinetic theory [3-5].

The location and size of the ionospheric region that makes a contribution to the scattered signal (sounding volume) is determined by the antenna beam shape, the sounding radio pulse, and by the time window of spectral processing [6]. The shape of the sounding volume determines also the method's spectral resolution, the accuracy to which the mean spectral density of dielectric permittivity is determined (which, in turn, affects the determination accuracy of macroscopic ionospheric parameters: electron and ion temperatures, the drift velocity, and electron density). The number of realizations, over which the received signal spectral power is averaged, determines the method's time resolution, i.e. its ability to keep track (based on measurements) of fast changes of macroscopic parameters of ionospheric plasma.

Currently most incoherent scatter radars have accumulated extensive sets of the scattered signal individual realizations (private communications of P.Erickson (Millstone Hill), V.Lysenko (Kharkov IS radar), and G.Wannberg (EISCAT)). Therefore, attempts are made to analyze the realizations from different methods which differ from a standard averaging by their sounding runs. Basically, these methods imply looking for small scatterers making the main contribution to the scattered signal. This method is good for analyzing signals scattered from meteors and their traces [7]; however, it is insufficiently substantiated for describing the scattering in the ionosphere.

In the work there were used the experimental data obtained with Irkutsk Incoherent Scatter radar. The radar is located at $52^0 N,104^0 E$, it has sounding frequency 152-160 MHz and peak power 3MW. High signal-to-noise ratio during the experiments under investigation (S/N > 10) allows us to neglect the noice effects when analyzing the signal received. The technique of the incoherent scatter signal processing in Irkutsk IS radar is the following. For each single realization of received signal we calculate spectrum in time window with width equal to the sounding signal duration and with delay corresponding to the radar range to the sounding volume investigated. The sounding signal we use in this experiment is a radiopulse with duration 800 mks. The repeating frequency approximately 25 Hz. Averaging over the 1000 realizations corresponds to 3 % dispersium of the averaged spectrum relative to its mathematical expectation. The reason of using such a simple pulse is to investigate the fine structure if the single (unaveraged) spectrums in this simplest case.

Fig.1 exemplifies the mean spectral power of the scattered signal and its separate realizations, based on the data from the Irkutsk Incoherent Scatter radar. It is evident from the figure that the spectral power of the scattered signal in an individual realization (Fig.1(A-C)) differs drastically from that averaged over realizations (Fig.1(D)); therefore, existing model of the incoherently scattered signal, based on averaging over sounding runs, are inapplicable for its interpretation. For that reason, development of new models of the scattered signal for analyzing its separate realizations without averaging them is important from the theoretical and practical standpoint.

Sometimes it is useful to suppose that incoherent scattering signal is a random gaussian one (for example, [8-9]). But, it is well known that the signal received is a deterministic function of ionospheric dielectric permittivity e and is fully determined in first approximation by the Born's formula (in one or another its form [10-11]), this relation could be called as a radar equation for signals [11]. The dielectric permittivity irregularities also could be supposed as a random functions, but they are deterministic functional of some other functions (in case of collisionless unmagnetized palsma with one ions type those functions are phase density of the ions and electrons as functions of velocity, location and time, ion composition and temperatures of the ions and electrons, this functional dependence is determined by the Landau's solution[12]). If one could determine all these unknown functions, the received signal shape in single realization will be fully determined, and could be analyzed without using any statistical methods. Such an approach, for example, is used in radioacoustical technique of the atmosphere sounding when the dielectric permittivity irregularities (by which the radiosignal is scattered) are generated by the acoustical wave [13].

At first, it is necessary to understand qualitatively, what information one could obtain from one realization of the IS signal. It is well known, that after any statistical processing of a function a part of the information is loosed irreversibly (for example. when one calculates the first n statistical moments, all the rest moments, starting with n+1 are still unknown). That is why, if the statistical characteristics of the realizations (mean spectral power or correlation function) are depend on the ion and electron temperatures and the ion composition then single realization must depend on all those parameters and on some new 'additional' parameters. It is clear that to determine temperatures and ion composition from averaged signal parameters is much easier than from single realization (because the second one includes additional parameters), and we can use the ones obtained from mean spectral power, with necessary spatial and spectral resolution, using different techniques, for example alternating codes[14]. But the new 'additional' parameters can be determined from single realizations only.

The aim of this paper is to find out the functional dependence of single realization spectrum on all the parameters, including well known (temperatures and ion composition) and the new ones, which could describe the single realizations spectrum properties. For this propose we will use for analysis only signals with high signal to noise ratio (more than 10), because in this case the noise effects could be neglected and the received signal could be supposed as only IS signal without presence any noise.

INITIAL EXPRESSIONS.

To analyze the individual realizations of the scattered signal, it is necessary to have a convenient expression relating the spectrum of the scattered signal to the space-time spectrum of dielectric permittivity irregularities without averaging over realizations. Such an expression for a monostatic experimental configuration was obtained and analyzed in [11], and it could be converted into the form:

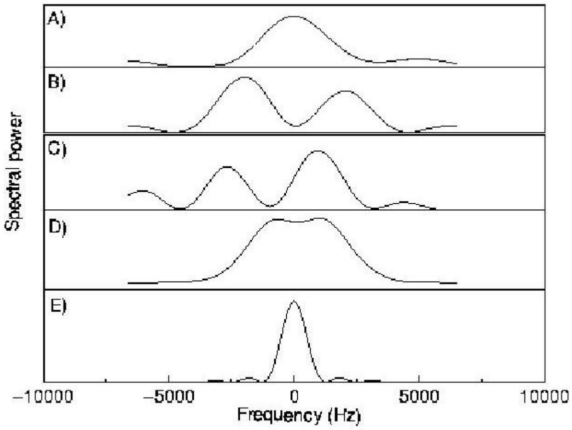

Fig.1. The spectral power of three different realizations (A-C), averaged over 1000 realizations spectral power of the scattered signal (D) and spectral power of the sounder signal envelope (E) as deduced using the data from the Irkutsk incoherent scatter radar.

$$u(\omega)=\int H(\omega-\nu,k-2k_0-\frac{\nu}{2c})\frac{g(-\hat{k})}{k}\varepsilon(\nu,\vec{k},T_1-T_0/c,-\hat{k}T_0c/2)d\nu d\vec{k} \qquad (1)$$

Where $H(\omega,k)\approx o(\omega-kc/2)a(kc/2)$ - a kernel determined by $o(\omega),a(\omega)$ - spectrums of the receiving window and sounding pulse, $g(\hat{k})$ - antenna pattern multiplier, $\varepsilon(\nu,\vec{k},T,\vec{R})=\int\varepsilon(t,\vec{r})e^{i(t-T)\nu}e^{-i\vec{k}(\vec{r}-\vec{R})}d\vec{r}dt$ - spectrum of dielectric permittivity irregularities, $\hat{k}=\vec{k}/k$, k_0,c - wave number of sounding wave and light speed, T_1,T_0 - receiving window start time and delay between sounding signal and receiving window.

In accordance with [3,4,15], assume that the spectrum of small-scale dielectric permittivity irregularities is determined by the Landau solution [12]. Then the low-frequency (ion-acoustic) part of the irregularities spectrum in a statistically homogeneous, unmagnetized, collisionless ionospheric plasma with one sort of ions is determined by plasma macroscopic parameters (electron and ion temperatures, ion composition, and drift velocity), and by unknown conditions in the moment T related to which this spectrum is calculated - the ions number phase density in a six-dimensional phase space of velocities and positions of particles. It is known that the dielectric permittivity irregularities spectrum at large wave numbers of the sounding wave $k_0>\omega_N/c$ (where ω_N is plasma frequency) , is proportional to the electron density irregularities spectrum (which is given by the well known expression (for example, [4])), and have the form:

$$\varepsilon\left(\omega,\vec{k},T\right)\sim\frac{G_e(\omega,\vec{k})}{\varepsilon_\parallel(\omega,\vec{k})}\int\frac{\exp(i\vec{k}\vec{r})f_{i1}(\vec{r},\vec{v},T)}{\omega-\vec{k}\vec{v}+i0}d\vec{r}d\vec{v} \qquad (2)$$

where $\varepsilon_\parallel(\omega,\vec{k})=1+G_e(\omega,\vec{k})+G_i(\omega,\vec{k})$ - is longitudinal dielectric permittivity; wave number k should be small enought to wave length be smaller than Debye length (Solpiter approximation). Most part of the IS radars have the sounding frequencies (50-1000 MHz) within these limitations; $f_{e,i0}(\vec{v})$ - are equilibrium distribution functions of the electrons and ions velocity and their densities; $m_{e,i},q_{e,i}$ - are the mass and charges of electrons and ions, respectively; $f_{i1}(\vec{v},\vec{k},T)$ - the ions number phase density in a six-dimensional phase space of velocities and positions of particles (the ions number phase density, INPD, at t=T), with the summation made over all ions. Generally equilibrium distribution functions $f_{e,i0}(\vec{v})$ are taken to be Maxwellian, with the temperatures T_e,T_i for electrons and ions, respectively. Then the functions $G_{e,i}(\omega,\vec{k})$ have the well-known analytical expression too [4,12].

To obtain a model of the incoherent scatter signal we substitute the Landau's expression for dielectric permittivity irregularities (2) into the expression for the scattered signal spectrum (1). Using in (2) the spatial spectrum $f_{i1}(\vec{k},\vec{v},T)$ of the ions number phase density $f_{i1}(\vec{r},\vec{v},T)$, and upon interchanging the order of integration, we obtain the one-realization model for the scattered signal spectrum:

$$u(\omega)=\int K(\omega,\vec{k},v_\parallel)F_{i1}(\vec{k},v_\parallel;T_1-T_0/2)d\vec{k}dv_\parallel\ , \qquad (3)$$

where

$$K(\omega,\vec{k},v_\parallel)=\frac{g\left(-\hat{k}\right)}{k^2}\int\frac{G_e(\nu,\vec{k})}{\varepsilon_\parallel(\nu,\vec{k})}\frac{H(\omega-\nu,k-2k_0-\nu/c)}{\nu-kv_\parallel-i0}d\nu \qquad (4)$$

$$F_{i1}(\vec{k},v_\parallel;T)=\int f_{i1}(\vec{k},\vec{v};T)\delta(\vec{k}\vec{v}-kv_\parallel)d\vec{v} \qquad (5)$$

is unknown function we want to determine from experiment and has a form similar to the Radon transform of the function f_{i1}.

Thus the kernel $K(\omega,\vec{k},v_\parallel)$ is completely determined by the sounder signal, the receiving window, and by macroscopic characteristics of ionospheric plasma (4). The expression (3) clearly shows the meaning of the kernel $K(\omega,\vec{k},v_\parallel)$: it determines the selective properties of the model, i.e. the possibilities of determining the unknown function $F_{i1}(\vec{k},v_\parallel;T)$ from the measured $u(\omega)$. Hence it can be termed the weight volume in the space $\vec{k},v_\parallel$, or ambiguity function. Since the function $H(w,k)$ is a narrow-band one, with its carrier concentrated near zero, the function of indefiniteness $K(w,\vec{k},v_{\parallel})$ has also a limited carrier in k near $k=2k_0$.

CONCLUSION

According to the resulting model (3), the form of scattered signal single spectrum is determined both by a determinate component (the weight volume (4)), and by a random (i.e. dependent on time by the unknown way) component (5). A random component is the function (4) determined by the spartial harmonics packet of the INPD $f_{i1}(\vec{k},\vec{v},t)$ with wave numbers k, concentrated near $2k_0$ and calculated relative to the moment $t=T$. The moment T is determined by the moments of the sounding signal transmitting and spectral processing receiving window location, and corresponds to the middle moment between them $T=T_1-T_0/2$. The weight volume (4) determines the parameters of this wave packet, the region of wave vectors and velocities in $F_{i1}(\vec{k},v_{\parallel},T)$ which make the main contribution to the scattered signal at a particular frequency w.

In this paper we have suggested an interpretation of separate realizations of incoherently scattered signal spectra. It is based on the radar equation [11] and kinetic theory of ion-acoustic oscillations of a statistically homogeneous unmagnetized, collisionless ionospheric plasma with one sort of ions[12].

Based on the proposed model in (3) and a Gaussian approximation of the spectra of the sounder signal envelope and the receiving window, a qualitative comparison of the model with experimental data from the Irkutsk incoherent scatter radar was carried out. The comparison showed a quilitative agreement for some simplified model, based on additional assumptions about the properties of the function $F_{i1}(\vec{k},v_{\parallel},T)$.

ACKNOWLEDGEMENTS

I am grateful to B.G.Shpynev for making the data from the Irkutsk IS radar available and to A.P.Potekhin for fruitful discussions. The work has been done under partial support of RFBR grants #00-05-72026 and #00-15-98509.

REFERENCES

[1] Evans, J. V., “Theory and practice of Thompson scatter” in *Proc.IEEE*, vol.57, pp.496-530, 1969

[2] Tatarsky, V. I., *Wave Propagation in a Turbulent Atmosphere*, Moscow, Nauka, 1967. (in Russian)

[3] Clemmow, P. C. and J. P. Dougherty*, Electrodynamics of Particles and Plasmas*, Addison-Wesley, 1969.

[4] Sheffield, J., *Plasma Scattering of Electromagnetic Radiation*, Academic Press, 1975.

[5] Kofman, W., “Plasma instabilities and their observations with the incoherent scatter technique” in *Incoherent scatter: theory, practice and science*, edited by D.Alcayde, pp. 33-65, technical report 97/53, EISCAT scientiffic association, 1997.

[6] Suni, A. L., V. D. Tereshchenko, E. D. Tereshchenko, and B. Z. *Khudukon, Incoherent Scatter of Radio Waves in the High-Latitude Ionosphere*, AN SSSR, Apatity, 1989. (in Russian)

[7] Pellinen-Wannberg, A., A. Westman, G. Wannberg, K.Kaila, “Meteor fluxes and visual magnitudes from EISCAT radar event rates: a comparison with cross-section based magnitude estimates and optical data” in *Ann.Geophysicae*, vol.16, pp.1475-1485, 1998

[8] Farley, D.T., “Incoherent scatter power measurements: a comparison of various techniques” in *Radio Science*, vol.4, pp.139-142 1969

[9] Zhou Q.H., Zhou Q.N. and Mathews J.D., “Arithmetic average, geometric average and ranking: application to incoherent scatter radar data processing” in *Radio Science*, vol.34, pp.1227-1237, 1999

[10] Ishimaru A., *Wave propogation and scattering in Random media*, Academic Press, V.1, 1978.

[11] Berngardt, O. I. and A. P. Potekhin, “Radar equations in the radio wave backscattering problem.” in *Radiophysics and Quantum electronics*, vol.43, pp. 536-544, 2000.

[12] Landau, L. D., “Towards an electron plasma oscillations” in *Zhurnal eksperimentalnoi i teoreticheskoi fiziki*, vol.16, pp. 574-586, 1946 (in Russian)

[13] Kalistratova M.A. and Kon A.I., *Radioacoustical sounding of the atmoshpere*, Moscow, Nauka, 1985.(in Russian)

[14] Lehtinen, M. S., *Statistical theory of incoherent scatter measurements*, Ph.D.thesis, Univ.of Helsenki, Helsenki, Finland, 1986.

[15] Akhiezer, A. I., I. A. Akhiezer, P. V. Polovin, A. G. Sitenko, K. N. Stepanov, *Plasma Electrodynamics*, Moscow, Nauka, 1974. (in Russian)