A GENETIC INVERSE METHOD OF IONOSPHERIC PARAMETER INVERSION FROM BACKSCATTER IONOGRAMS

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ABSTRACT

The oblique backscatter sounding is a powerful tool for detecting the ionosphere. This paper discusses the inverse problem of the ionospheric parameters by applying the inverse theory, and develops a new inverse method for this problem, trying to get over the difficulties of instability and non-uniqueness. The Genetic Algorithms (GAs) are introduced to established a 2-dimensional ionospheric Genetic Inverse Method. This method is tested against the model data with noise. The results in this paper demonstrate that the Genetic Inverse Method is available and efficient for overcoming the instability and non-uniqueness for the inverse problem of backscatter ionograms.

INTRODUCTION

HF radio backscatter detection has the advantages of detecting the ionosphere in large range at long distance[1]; it is a powerful tool of remote sensing the ionosphere and has the potential applications to the management of radio systems[2]. To extract this information from backscatter ionograms is an inverse problem. The leading edge of a backscatter ionogram can be obtained easily and more accurately because the backscattered signal has a steep forward front, we can use it as input data of this inverse problem. Several inversion methods have been developed using the leading edge of a backscatter ionogram. For example, Rao (1974) uses three points on the leading edge of the backscatter ionogram and applies an iterative technique to obtain the parameters of a corresponding QP layer[3], this method can be unstable and the solutions are not always unique. Dyson (1991) was devoted to some improvement of this method[4]. Fridman and Fridman (1994) have developed a technique to determine horizontal structure of the ionosphere from backscatter ionograms and achieved more success[5][6].

From the view of practical application, the robustness and accuracy of an inversion algorithm are most important. The inversion of backscatter ionogram is a nonlinear problem, and its solution is not unique when noise is added to the data. In present methods, in the case of using noisy data, this is not true, the result of inversion can deviate largely from the real one and can be not unique.

Noises exist everywhere in the real world, so we must reduce the influences of noise in the backscatter ionogram inversion. It is important in means of determining the structure of ionosphere and in means of application of the backscatter ionograms.The Genetic Algorithms (GAs) is an optimization method based on the biological evolution principle, and is realized by using the bionic mechanism of natural selection and heredity. As an optimization method, the GAs is itself an inversion method, a robust and real nonlinear inversion method[10][9][11].

This paper discusses the inverse problem of the ionospheric parameters by applying the inverse theory, and develops a new inverse method for this problem, trying to get over the difficulties of instability and non-uniqueness. The Genetic Algorithms (GAs) are introduced to established a 2-dimensional ionospheric Genetic Inverse Method. This method is tested against the model data with noise. The results in this paper demonstrate that the Genetic Inverse Method is available and efficient for overcoming the instability and non-uniqueness for the inverse problem of backscatter ionograms.

Mathematical model of the ionospheric inverse problem

For a certain model of an ionosphere with the electron density distribution, we can construct the group-frequency characteristic, that is the leading edge of a backscatter ionogram. The minimum group path delay $P'(f)$ can be calculated by

$$P'(f) = \int_{S^m} \mu_g(\mathbf{r}, f)\, ds \tag{1}$$

where $\mathbf{r}$ is a radius vector, S^m is the minimum group path, and $\mu_g(\mathbf{r}, f)$ is the plasma group refraction index, which related to the electron density distribution $N(\mathbf{r})$ and radio frequency f by

$$\mu_g(\mathbf{r}, f) = (1 - \frac{\beta N(\mathbf{r})}{f^2})^{-\frac{1}{2}} \tag{2}$$

The magnetic influence is neglected here and later in this paper.

We can rewrite equation (1) using a calculator G,

$$P'(f) = G[f, N_e(\mathbf{r})] \tag{3}$$

and the inverse problem can be written formally as

$$N(\mathbf{r}) = G^{-1}[P'(f)] \tag{4}$$

This is obviously a non-linear inverse problem. If such a problem is treated as an optimization problem with objective function

$$\Phi(N(\mathbf{r})) = \int_{f_1}^{f_2} [P'(f) - G(f, N(\mathbf{r}))]^2\, df \tag{5}$$

it will encounter serious problems such as non-uniqueness and instability. The reasons for this can be explained as the insufficiency of data and the influence of noise according to the inverse theory[12][8]. For overcoming these problems, we reconstruct the objective function as the Tikhonov regularization functional

$$\Phi_\alpha[N(\mathbf{r})] = \int_{f_1}^{f_2} [P'(f) - G(f, N(\mathbf{r}))]^2\, df + \alpha\Omega(N(\mathbf{r})) \tag{6}$$

where

$$\Omega[N(\mathbf{r})] = \int \left[N^2(\mathbf{r}) + q(\nabla N(\mathbf{r}))^2\right] d\mathbf{r} \tag{7}$$

is the Tikhonov stabilizing functional. Here constant $\alpha > 0$ is called the regularization parameter, and constant $q \geqslant 0$. It is proved that the problem of minimizing functional (6) has the solution which is a unique solution at same time.[7].

THE GENETIC INVERSE METHOD

Considering only 2-Dimensional case, $N(\mathbf{r})$ varies only in the sector along the detecting direction

$$N(\mathbf{r}) = N(r, \theta) \tag{8}$$

For numerical calculation, we can express $N(\mathbf{r})$ by its values $N_{i,j}$ at the grid nodes of a sector like fig.1. These values of $N_{i,j}$ are the decision variables for the optimization problem that minimize the objective function (6). Now we solve this problem by using the Genetic Algorithms (GAs).

Using a ray tracing method, we can calculate the minimum group path delay $G(f, N(\mathbf{r}))$ and the objective function value for an ionospheric model $N(\mathbf{r})$ represented by $N_{i,j}$. The realization of an inversion using GAs is very simple, which is described brief by following steps:

(1) encoding

The GAs evolve the models by its encoding. Here float encoding of decision variables is adopted, i.e. the values of $N_{i,j}$ are directly treated as genes, which are arranged in certain order to form a chromosome (or an individuals) that represent a model.

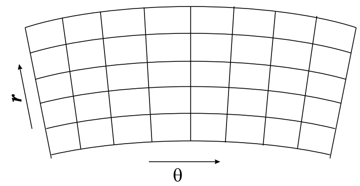

Figure 1: 2-D ionosphere and numerical expression of $N(\mathbf{r})$.

(2) searching range

The searching range for a decision variable must be given.

(3) the genetic operators

The genetic operation is composed by three operators:

1) selection: the linear ranking selection is used, i.e. the models are ranked according to the its objective function values, and related to certain fitness values, respectively. At the same time, the elitist model is used while selection.

2) crossover: the arithmetic crossover is used.

3) mutation: the non-uniform mutation is used.

TESTING THE GENETIC INVERSE METHOD AGAINST SIMULATED DATA

We test the genetic inverse method by using the simulated data. A model of ionosphere is given first, and the minimum group path delays can obtained by ray tracing, then some random noise is added. These 'observed' data are used as the INPUT to begin the inversion.

As an example, the result of a 2-D inversion of spherically stratified ionosphere is presented in fig.2. The input data is calculated from a given model, and a normal distributed noise with standard deviation $\sigma_d = 30\,\mathrm{km}$ is added. It should note that all the initial models are generated randomly. In fig.2(a)-(c), the best searching result of first generation, i.e the best model after one iteration of genetic operation, is illustrated. where, (a) the contour of electron density distribution, solid line—"real value", dashed line—searching result; (b) model parameter deviation from "real value", model parameters at grid nodes are arranged by the order from left to right and from bottom to top. solid line—"real value", stars and dashed line—current searching result; (c) the leading edge of current best model. solid line—"observed data", dashed line with symbols—searching result.

Fig.2(d)-(f) are the results after 200 iterations of genetic operation. From fig.2(e) and (f), we can find that the deviation of the best model from the real model is relatively small, and the calculated leading edge is almost the same as observed data. For the contours in fig.2(d), in the region with height from 100km to about 135km and range from 0km to about 1200km, the inversion result is very close to the given model. But in the other part, there are differences between the inversion result and the real model, this is because there are no or less minimum group path passing that area.

Fig.2(g)-(i) illustrate the convergence of the inversion, which demonstrate the decreasing of objective function value $\bar{\Phi}_\alpha[N(\mathbf{r})]$(average), $\Phi_{\alpha,\min}[N(\mathbf{r})]$(best model) and $\Phi_{\min}[N(\mathbf{r})]$(squared deviation of best model), respectively.

Some other models of ionopshere are also tested, and the results are reasonable and encourageable.

CONCLUSION

This paper discusses the inverse problem of the ionospheric parameters by applying the inverse theory, and develops a new inverse method for this problem, trying to get over the difficulties of instability and non-uniqueness. The Genetic Algorithms (GAs) are introduced to established a 2-dimensional ionospheric Genetic Inverse Method. This method is tested against the model data with noise. The results in this paper demonstrate that the Genetic Inverse Method is available and efficient for overcoming the instability and non-uniqueness for the inverse problem of backscatter ionograms.

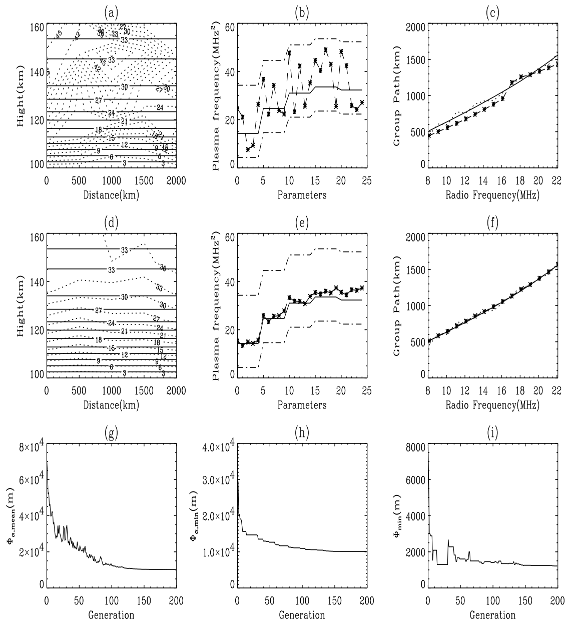

Figure 2: The inversion results of a 2-D inversion of spherically stratified ionosphere.

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