ON THE USE OF PML TERMINATED WAVEGUIDES AS A FAST MEANS TO CALCULATE THE GREEN'S FUNCTIONS OF A 2D MICROSTRIP SUBSTRATE.

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Abstract— A detailed analysis is presented for the formalism that involves using Perfectly Matched Layers (PML's) to terminate open microstrip substrates in order to obtain fast modal series for the 2D Green's functions. It is shown that the modal series composing the Green's functions for the magnetic vector potential and the scalar potential can be partitioned into a series of leaky modes and a series of Berenger modes. It is then demonstrated that, although all series converge exponentially for sufficiently large distances between excitation and observation point, each series has a distinct and clearly less convergent behavior when the excitation point approaches the observation point. Closed form expressions can be found, describing the singularity of the modal series when excitation and observation points are lying inside or on top of the substrate. Finally, the consequences of the presented analysis on the efficiency of the PML formalism are discussed.

1 PML FORMALISM

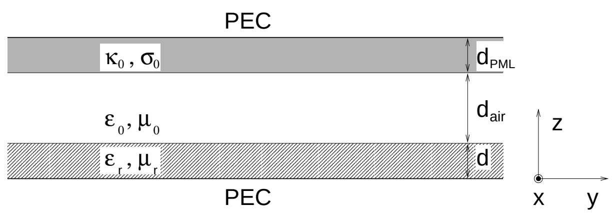

Figure 1: Microstrip configuration.

The rigorous analysis of the Green's functions for 2D open microstrip substrates requires the evaluation of an inverse Fourier transform to convert the closed-form spectral domain solution to the spatial domain. Accurate numerical procedures for this transformation tend to be very complex and time-consuming. To speed up calculations, a number of alternative approaches have been and are being investigated, such as the complex image method and the fast Hankel transform. Quite recently, a new powerful formalism was presented involving the use of PML's to terminate open microstrip substrates, resulting in the configuration shown in Fig. 1. For further analysis, the PML with thickness d_{PML} and with material parameters κ_0 and σ_0 can be combined with the air region to form a single air layer with complex thickness $\tilde{d} = d_{\mathrm{air}} + d_{\mathrm{PML}}(\kappa_0 - j\frac{\sigma_0}{\omega\epsilon_0})$. On the one hand, a fast series of discrete modes can be obtained for the Green's functions of this closed PML-microstrip waveguiding structure. The Green's function $G_{xx}(y, z; y', z')$ inside or on top of the substrate, e.g. , can be written as [1]:

$$G_{xx}(y,z;y',z') = -\sum_{n=1}^{\infty} \frac{\omega\mu_0}{\beta} \frac{\sin(\gamma_r z)\sin(\gamma_r z')\; e^{-j\beta|y-y'|}}{\frac{d}{\mu_r} + \tilde{d}\frac{Y_0^2+Y_r^2}{2Y_0^2} - \frac{\gamma_0^2-\gamma_r^2}{\gamma_0 Y_0 \mu_r}\frac{\sin(2\gamma_r d)}{2\gamma_r} - \frac{Y_0^2-Y_r^2}{Y_0^2}\frac{\tilde{d}}{2}\cos(2\gamma_r d)} \qquad 0 < z, z' \leq d \qquad (1)$$

with $\gamma_0^2 = k_0^2 - \beta^2$, $\gamma_r^2 = k_0^2\epsilon_r\mu_r - \beta^2$, $k_0^2 = \omega^2\epsilon_0\mu_0$, and with $Y_r = \frac{\gamma_r}{j\omega\mu_0\mu_r}$ and $Y_0 = \frac{\gamma_0}{j\omega\mu_0}$. On the other hand, the PML efficiently mimics an open configuration, so that (1) represent a good approximation for the Green's function of the open microstrip substrate. This formalism performs very well, provided that distances between excitation and observation points are sufficiently large. In that case, the modal series converge exponentially, so that only a small number of modes are required to obtain an accurate evaluation of the Green's functions. This exponential convergence, however, disappears when the excitation point approaches the observation point and the efficiency of the method decreases rapidly. The aim of this contribution is to provide a detailed analysis of the modal series for the Green's functions of the magnetic vector potential and the scalar potential for small distances between excitation and observation points. First, this analysis allows us to identify the cases where the formalism breaks down for theoretical reasons. Second, our study provides the means to accelerate the modal series in those cases where the method breaks down for numerical reasons.

2 LEAKY MODES AND BERENGER MODES

In order to be able to investigate the modal expansions for small distances between excitation and observation points, we must concentrate on the behavior of the modes for increasing mode numbers, in which case $\gamma_r \approx \gamma_0 \approx +j\beta$ holds. In [2], we have shown that the modes in the waveguide, formed by the PML combined with the microstrip substrate, can be partitioned into leaky modes and Berenger modes, provided the mode number is sufficiently high and provided the PML acts as a good absorber. In this case, closed-form approximations can be derived for the propation constants for both kinds of modes. The leaky modes of the configuration depend only on the material properties and on the thickness of the microstrip substrate. Considering (1) and assuming $\mu_r = 1$, e.g. , results in following closed-form approximation:

$$+j\beta \approx \gamma_r \approx \frac{(2n+1)\pi}{2d} + j\frac{1}{d}\log\frac{(2n+1)\pi}{k_0 d\sqrt{\epsilon_r - 1}} \tag{2}$$

These modes clearly correspond to the leaky modes of the open microstrip substrate. The Berenger modes, on the other hand, are mainly concentrated in the PML and depend on the material properties and the thickness of the matched layer. For (1) and assuming $\mu_r = 1$, e.g. , following the closed-form approximation is found:

$$+j\beta \approx \gamma_0 \approx \frac{n\pi}{\tilde{d}} - j\frac{1}{\tilde{d}}\log\frac{2n\pi}{k_0 \tilde{d}\sqrt{\epsilon_r - 1}} \tag{3}$$

3 CONVERGENCE BEHAVIOR OF THE MODAL SERIES

Starting from the closed-form approximations for the propagation constants of the leaky and Berenger modes, we are able to derive asymptotic series expansions that describe the behavior of the Green's function's modal series for high mode numbers [1]. During the process and concentrating e.g. on the Green's function (1), it is shown that the series of leaky modes $G_{xx}^{(1)}$ and the series of Berenger modes $G_{xx}^{(2)}$ exhibit a different asymptotic behavior. For the important practical case of a microstrip substrate with no magnetic contrast and the series of leaky modes composing (1) behaves as:

$$G_{xx}^{(1)}(y,z;y',z') \sim \sum_{n=1}^{\infty} -e^{-j\frac{(2n+1)\pi(z+z'-2d)}{2d}} \left(\frac{2\gamma_r}{k_0\sqrt{\epsilon_r - 1}}\right)^{\frac{z+z'-2d}{d}} \frac{j\omega\mu_0\gamma_r}{k_0^2(\epsilon_r - 1)(d + \frac{1}{j\gamma_0})} e^{-j\beta|y-y'|} \tag{4}$$

whereas the series of Berenger modes composing (1) behaves as:

$$G_{A}^{(2)}(y,z;y',z') \sim \sum_{n=1}^{\infty} e^{j\gamma_0(z+z'-2d)} \frac{j\omega\mu_0\gamma_0}{k_0^2(\epsilon_r - 1)(\tilde{d} - \frac{1}{j\gamma_r})} e^{-j\beta|y-y'|} \tag{5}$$

Both are highly divergent when source and observation points are located on the interface. When either source or observation point are located inside the microstrip substrate, the series of Berenger modes becomes convergent, whereas the series of leaky modes diverges when source and observation points approach each other very closely.

4 SINGULAR BEHAVIOR OF THE MODAL SERIES

The loss of convergence for close distances between source and observation points is not surprising, since the Green's functions of the open microstrip problem exhibit singular behavior when the source point approaches the excitation point. In [3], we were able to show that, when the excitation point approaches the source point, and both are lying inside the microstrip substrate, the modal series composing the complete Green's dyadic exhibit the correct singular behavior related to the open structure. However, when we consider both points lying on the interface and in absence of magnetic contrast, a higher-order singularity occurs in the modal series expansion of the Green's function G_{xx}. This singularity is related to the parameters of the PML and, hence, does not correspond to the singularity of the open problem.

5 CONSEQUENCES ON THE EFFICIENCY OF THE FORMALISM - SERIES ACCELERATION

The detailed analysis presented in this contribution can be used to improve the efficiency of the PML formalism for the Green's functions by accelerating the series in the neighborhood of the singularity. To that end, we can use two techniques: numerical series acceleration by means of the Shanks transform and analytical treatment of the singularity. We are able to show that, by applying Shanks transform, the PML formalism stays accurate for separations between source and observation points up to 1/10 of a wavelength. To speed up the series evaluation, and to remain accurate for closer distances, extraction and analytical treatment of the singularity is useful, provided the singularity of the modal series corresponds to the exact singularity of the open problem. The theoretical results obtained in Section 4 show that it is useless to try to use the PML algorithm to evaluate the Green's function G_{xx} (also corresponding to the Green's function of the magnetic vector potential G_A) when there is no magnetic contrast and for excitation-observation separations less than 1/10 of a wavelength.

6 EXAMPLE

Consider a microstrip-PML configuration with $d = 9$mm, $\epsilon_r = 3$ and $\mu_r = 1$, $d_{\mathrm{air}} = 5$mm, $d_{\mathrm{PML}} = 3.5$mm at 12GHz. An open structure can be approximated by choosing a strongly absorbing PML with parameters $\kappa_0 = 10$ and $\frac{\sigma_0}{\omega\epsilon_0} = 8$. In Fig. 2 the exact propagation constants of leaky and Berenger modes are shown and they are compared with the quasi-static eigenvalues, following the analytic formulas (2) and (3). The branches of leaky and Berenger modes can clearly be distinguished and moreover, a good agreement is seen between the exact locations of the eigenvalues and the β values obtained with the analytic expressions (2) and (3), whenever β becomes large as compared to k_0.

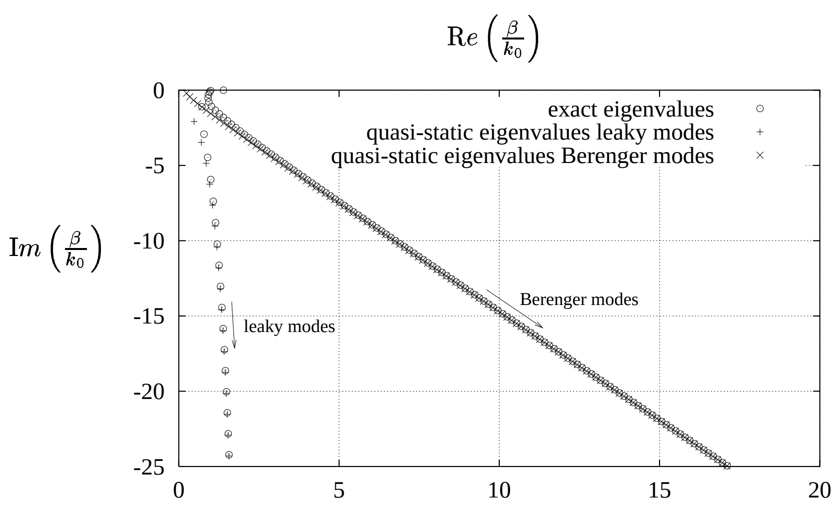

Figure 2: Location of the eigenvalues $\frac{\beta}{k_0}$ of the TE Berenger and leaky modes.

In Fig. 3 we have calculated the Green's function G_{xx} in a number of ways for both excitation and observation points located at the interface. As discussed in [1], up to 359 leaky TE modes and 961 Berenger TE modes were available to calculate G_{xx}, with the aim of obtaining a relative accuracy of 10^{-7}. With this number of modes available, direct computation of the two modal series (curve 2) gives acceptable results for $k_0 y \geq 0.1$, whereas with the use of series acceleration (curve 3), by means of a Shanks transformation, there is a good agreement with the spectral domain approach (curve 1) for values of $k_0 y$ up to 0.01. However, for excitation and observation points on the interface and for small values of $k_0|y - y'|$, the series (1) exhibits divergent behavior. Curve 4 gives the series calculation when the exact singular behavior is extracted in both modal series and treated analytically. This procedure does not lead to the correct result of curve 1. As a matter of fact, not much is gained at all. For this particular case (no magnetic contrast and both source and observation points on the interface), it was shown in [3] that higher-order singularities appearing in the Berenger and leaky mode series DO NOT cancel out and force the series (1) to adopt an altogether different, hence incorrect, behavior than the exact solution for the open microstrip substrate. It is proven in [3], however, that for all other situations (other components of the Green's dyadic, source or observation point inside the substrate, or presence of magnetic contrasts) higher-order singularities appearing in the Berenger and leaky mode series DO cancel out, resulting in the correct singular corresponding to the Green's function of the open microstrip substrate. Moreover, in [4] we have shown that the PML formalism is useful in a Method of Moments technique, provided that for small values of $k_0|y - y'|$ the quasi-static part of the microstrip Green's function is used.

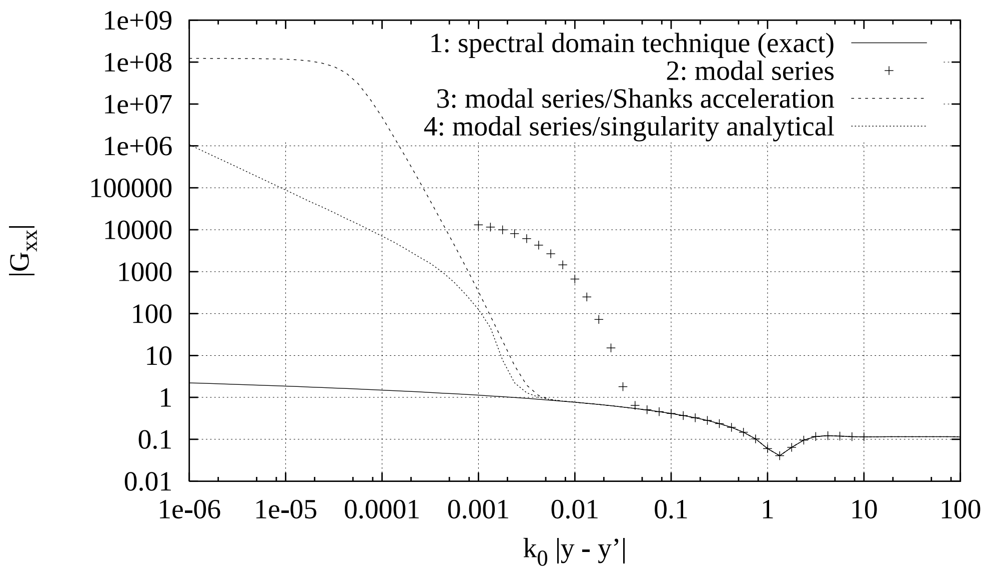

Figure 3: The Green's function $G_{xx}(y, d; y', d)$ for excitation and observation points at the interface.

References

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