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QUADRUPOLE TERMS IN MAGNETIC SINGULARITY IDENTIFICATION

Carl E. Baum

Air Force Research Laboratory, Directed Energy Directorate
3550 Aberdeen Avenue SE, Kirtland AFB, NM 87110-5776, USA
magdalena.lopez@kirtland.af.mil

ABSTRACT

In magnetic singularity identification (MSI) of coonducting and permeable scatterers one considers the low-frequency poles with real natural modes and frequencies to represent the magnetic-polarizability dyadic. This is an approximation neglecting the higher-order multipoles. This paper considers the magnetic quadrupole terms as a correction to the dipole-only representation. This leads to the concept of the effective center of a natural mode to minimize the quadrupole contribution. In the case of scatterers with certain symmetries there can also exist natural modes with no magnetic-dipole contribution, but with a quadrupole contribution. One potential use of a quadrupole information is in removing orientation ambiguities of the visually obscured scatterer.

INTRODUCTION

Summarizing from [1, 2] we have an induced magnetic-dipole moment in a highly conducting permeable target of the form

$$
\begin{aligned}
&\tilde{\vec{m}}(s) = \tilde{\overleftrightarrow{M}}(s) \cdot \tilde{\vec{H}}^{(inc)}(\vec{0},s) \\
&\tilde{\vec{H}}^{(inc)}(\vec{r},s) \equiv \text{incident magnetic field} \\
&\sim \; \equiv \; \text{two-sided Laplace transform over time } t \\
&s \equiv \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency}
\end{aligned}
\tag{1}
$$

The magnetic-polarizability dyadic takes various SEM (singularity expansion method) representations as

$$
\begin{aligned}
&\tilde{\overleftrightarrow{M}}(s) = \tilde{\overleftrightarrow{M}}(\infty) + \sum_{\alpha} M_\alpha \vec{M}_\alpha \vec{M}_\alpha \left[s - s_\alpha\right]^{-1} \\
&\frac{1}{s}\tilde{\overleftrightarrow{M}}(s) = \frac{1}{s}\overleftrightarrow{M}(0) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha \left[s - s_\alpha\right]^{-1} \\
&\quad s_\alpha < 0 \quad \text{(real)} \\
&\mathrm{M}_\alpha ,\ \vec{M}_\alpha \quad \text{real}
\end{aligned}
\tag{2}
$$

with equally simple forms in time domain for delta and step responses. The various terms can be computed in various ways [3, 4]. For present purposes we assume that the magnetic polarization (or magnetic-moment density) is known as

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$$\vec{\tilde{\chi}}(\vec{r},s) = \frac{1}{2}\vec{r}\times\left[\overleftrightarrow{\sigma}(\vec{r}) \cdot \vec{\tilde{E}}(\vec{r},s)\right]+\left[\overleftrightarrow{\mu}-\mu_0 \overleftrightarrow{1}\right] \cdot \vec{\tilde{H}}(\vec{r},s) \tag{3}$$

which in turn gives natural modes $\vec{\chi}_\alpha(\vec{r})$ at the natural frequencies s_α. The natural-mode magnetic-dipole moment is then

$$\vec{m}_\alpha = \int_V \vec{\chi}_\alpha(\vec{r})\, dV \ , \quad \vec{\chi}_\alpha \ \text{ real} \tag{4}$$

giving a scattered magnetic-dipole field

$$\vec{H}_\alpha^{(d)}(\vec{r}) = \overleftrightarrow{h}^{(d)}(\vec{r}) \cdot \vec{m}_\alpha \quad , \quad \overleftrightarrow{h}^{(d)}(\vec{r}) = \frac{1}{4\pi r^3}\left[3\,\vec{1}_r\,\vec{1}_r - \overleftrightarrow{1}\right]$$

$$\vec{1}_r = \frac{\vec{r}}{r} \ , \quad r = |\,\vec{r}\,| \tag{5}$$

MAGNETIC QUADRUPOLES

The scattered magnetic field is more generally expanded in a multipole series as

$$\vec{H}_\alpha(\vec{r}) = \vec{H}_\alpha^{(d)}(\vec{r}) + \vec{H}_\alpha^{(q)}(\vec{r}) + O(r^{-5}) \ \text{ as } \ r \to \infty$$

$$\vec{H}_\alpha^{(q)}(\vec{r}) = \left\langle \overleftrightarrow{h}^{(q)}(\vec{r},\vec{r}\,'); \vec{\chi}_\alpha(\vec{r}\,') \right\rangle \tag{6}$$

$$\overleftrightarrow{h}^{(q)} = \frac{3}{4\pi r^2}\left[\left[\vec{1}_r \cdot \vec{r}\,'\right]\left[5\,\vec{1}_r\,\vec{1}_r - \overleftrightarrow{1}\right] - \vec{1}_r\,\vec{r}\,' - \vec{r}\,'\,\vec{1}_r\right]$$

where the symmetric product indicates integration over $\vec{r}\,'$ over V (the domain of the target). We would like to minimize the quadrupole contribution so as to make the dipole term more accurately represent the scattered field, or alternately use the quadrupole information to aid in target identification.

EFFECTIVE CENTER OF NATURAL MODE

In order to give a scalar measure to the quadrupole term, as in [5] we can define a norm, in this case the 2-norm, over all angles (the unit sphere). After much analytic computation we arrive at a term proportional to the square of the 2-norm as

$$U_\alpha = \frac{4\pi}{3}\left[-2q_\alpha^2 + 3\,Y_\alpha\right] \geq 0$$

$$q_\alpha \equiv \int_V \vec{r} \cdot \vec{\chi}_\alpha(\vec{r})\, dV$$

$$\overleftrightarrow{Q}_\alpha \equiv \int_V \vec{r}\,\vec{\chi}_\alpha(\vec{r})\, dV$$

$$Y_\alpha \equiv \int_V \vec{r} \bullet \left[\overleftrightarrow{Q}_\alpha + \overleftrightarrow{Q}_\alpha^T \right] \bullet \vec{\chi}_\alpha(\vec{r})\, dV \tag{7}$$

One can then define an effective center $\vec{r}_0$ of a natural mode as that which minimizes U_α. Rewriting in terms of shifted coordinates $\vec{r} - \vec{r}_0$ and setting the gradient with respect to $\vec{r}_0$ equal to zero gives.

$$\vec{r}_0 = \left[\overleftrightarrow{1} - \frac{1}{4} \vec{1}_\alpha \vec{1}_\alpha \right] \bullet \left[m_\alpha^{-1} \left[\overleftrightarrow{Q}_\alpha^{(0)} + \overleftrightarrow{Q}_\alpha^{(0)T} \right] \bullet \vec{1}_\alpha \right] - \frac{1}{2} m_\alpha^{-1} q_\alpha^{(0)} \vec{1}_\alpha \tag{8}$$

$$m_\alpha = |\vec{m}|, \quad \vec{1}_\alpha = \frac{\vec{m}_\alpha}{m_\alpha}$$

where a superscript 0 refers to quantities evaluated in the original coordinate system. Note that this result applies for nonzero $\vec{m}_\alpha$.

While this result minimizes U_α in the new coordinate system centered on $\vec{r}_0$, it is not necessarily zero. However, for different natural modes the value of $\vec{r}_0$ may be a function of α, i.e., $\vec{r}_{0,\alpha}$. This can be used to aid in determination of the target orientation.

CONDITIONS FOR ZERO QUADRUPOLE TERM

From [5] we have the conditions for zero quadrupole term, derivable from (7) as

$$\begin{aligned} 0 &= 3\, \vec{1}_r \bullet \overleftrightarrow{Q}_\alpha \bullet \vec{1}_r - q_\alpha \\ \vec{0} &= \overleftrightarrow{1}_r \bullet \left[\overleftrightarrow{Q}_\alpha + \overleftrightarrow{Q}_\alpha^T \right] \bullet \vec{1}_r \\ \overleftrightarrow{1}_r &\equiv \overleftrightarrow{1} - \vec{1}_r \vec{1}_r, \quad \overleftrightarrow{1} = \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z \end{aligned} \tag{9}$$

which must hold for all $\vec{1}_r$. There it is also shown that for this constraint

$$\begin{aligned} \overleftrightarrow{Q}_\alpha &= a \overleftrightarrow{1} + \overleftrightarrow{C}, \quad \overleftrightarrow{C}^T = \overleftrightarrow{C} = \text{real} \\ \vec{1}_r \bullet \overleftrightarrow{Q}_\alpha \bullet \vec{1}_r &= a = \text{real constant} \\ q_\alpha &= 3a \end{aligned} \tag{10}$$

These give us constraints on the form of $\overleftrightarrow{\chi}_\alpha(\vec{r})$ for zero resulting quadrupole term ($U_\alpha = 0$), with our coordinates now centered on $\vec{r}_0$. See [5] for cases that meet these requirements.

EFFECT OF SYMMETRY (ROTATION AND REFLECTION)

The imposition of point symmetry groups (rotation and reflection) on the targets gives special properties to the natural modes. For the magnetic-dipole moment this is treated in detail for these groups in [6].

The quadrupole moment is treated in [2, 5]. Depending on the symmetries in $\vec{\chi}_\alpha(\vec{r})$ the dipole and quadrupole can have various combinations of zero and nonzero. If the dipole term is zero, the coupling of an approximately uniform incident magnetic field to the quadrupole is quite weak. A nonzero dipole with associated quadrupole couples to the incident field more strongly.

Let us briefly mention some of the results obtained, especially for zero quadrupole moment associated with nonzero dipole moment. A target with inversion symmetry has the group

$$I = \{ \overleftrightarrow{1}, -\overleftrightarrow{1} \} \tag{11}$$

for which $\vec{\chi}_\alpha$ can be split into two

$$\vec{\chi}_{\alpha,\pm}(-\vec{r}) = \pm \vec{\chi}_{\alpha,\pm}(\vec{r}) \tag{12}$$

giving for the magnetic-dipole moments

$$\vec{m}_{\alpha,+} \neq \vec{0} \ , \quad \vec{m}_{\alpha,-} = \vec{0} \tag{13}$$

and for the quadrupole term

$$q_{\alpha,+} = 0, \quad \overleftrightarrow{Q}_{\alpha,+} = \overleftrightarrow{0} \ , \quad q_{\alpha,-} \neq 0, \quad \overleftrightarrow{Q}_{\alpha,-} \neq \overleftrightarrow{0} \tag{14}$$

with the coordinate center taken as the center of symmetry. Here we see zero quadrupole term for + subscript.

There are various results for C_{Nt}, discrete rotation symmetry (N-fold axis) with a transverse symmetry plane. There is a longitudinal (axial) magnetic dipole. The associated $q_\alpha = 0$ while the associated $\overleftrightarrow{Q}_\alpha = \overleftrightarrow{0}$ for $N \geq 2$. The transverse magnetic dipole is doubly degenerate for $N \geq 3$. The associated $q_\alpha = 0$ for $N \geq 2$ while the associated $\overleftrightarrow{Q}_\alpha = \overleftrightarrow{0}$ for $N = 2$ and for $N \geq 4$.

CONCLUDING REMARKS

In some cases, particularly involving symmetry, the quadrupole terms associated with magnetic-dipole natural modes are zero. In any event one can define an effective center of a natural mode to minimize the quadrupole contribution. If different natural modes have different effective centers this can potentially be used to orient the target by appropriate measurements.

REFERENCES

1. C. E. Baum, “Low-Frequency Near-Field Magnetic Scattering from Highly, But Not Perfectly Conducting Bodies”, Ch. 6, pp. 163-218, in C. E. Baum (ed.), *Detection and Identification of Visually Obscured Targets*, Taylor & Francis, 1999.
2. C. E. Baum, “Quadrupole Terms in Magnetic Singularity Identification”, Interaction Note 569, 2001.
3. C. E. Baum, N. Geng, and L. Carin, “Integral Equations and Polarizability for Magnetic Singularity Identification”, Interaction Note 524, 1997.
4. N. Geng, C. E. Baum, and L. Carin, “On the Low-Frequency Natural Response of Conducting and Permeable Targets”, *IEEE Trans. Geoscience and Remote Sensing*, 1999, pp. 347-359.
5. C. E. Baum, “Quadrupole Terms in Magnetic Singularity Identification, Part 2”, Interaction Note 572, 2001.
6. C. E. Baum, “The Magnetic Polarizability Dyadic and Point Symmetry”, Ch. 7, pp. 219-242, in C. E. Baum (ed.), *Detection and Identification of Visually Obscured Targets*, Taylor & Francis, 1999.