



## Physical conditions for full control of transmission through non-reflecting metasurfaces

Do-Hoon Kwon<sup>(1)</sup>, Grigori Ptitsyn<sup>(2)</sup>, Ana Díaz-Rubio<sup>(2)</sup>, and Sergei A. Tretyakov<sup>\*(2)</sup>

(1) Department of Electrical and Computer Engineering, University of Massachusetts Amherst, USA

(2) Department of Electronics and Nanoengineering, Aalto University, Finland

### Abstract

Electrically thin composite layers (metasurfaces) can be used to shape transmitted fields. It is usually desirable that reflections from the metasurface are absent, while the phase of the transmission coefficient can be fully controlled by tuning the metasurface structure. More generally, it is desirable to shape both the phase and amplitude of transmitted waves, still allowing no reflections. In this presentation we discuss necessary physical conditions for realizing these properties, both for normal and oblique illuminations, and present examples of new topologies of metasurfaces for control of transmitted waves.

### 1 Introduction

Reflectionless metasurfaces with a complete  $2\pi$  transmission phase coverage are important for multiple applications, such as transmitarrays or holograms. Conventional realizations are based on Huygens' meta-atoms [1, 2, 3], which are designed such that an orthogonal set of tangential electric and magnetic dipole moments are induced upon external plane-wave excitation. When the two induced dipoles satisfy a balanced condition, full transmission and zero reflection can be obtained for lossless cases with the transmission phase that can be designed to have any value within a complete  $2\pi$  range.

For illumination of lossless metasurfaces at normal incidence, it is easy to show that by tuning the electric and magnetic polarizabilities of meta-atoms it is possible to realize any desired phase shift in transmission without compromising reflection; see for example eqs. (9-10) of paper [4]. In the same paper it is shown also that full absorption (zero transmission amplitude) is also realizable within that scenario. Here we first look at possibilities to arbitrarily tune not only the transmission phase but also the transmission amplitude. Next, we consider the necessary conditions for full transmission control using nonreflecting metasurfaces and introduce novel topologies for transmission control devices.

### 2 Normal incidence

Let us assume that each unit cell of the metasurface can be polarized both electrically and magnetically and introduce

collective polarizabilities of unit cells accordingly to

$$p = \widehat{\alpha}_{ee} E_{inc}, \quad m = \widehat{\alpha}_{mm} H_{inc}, \quad (1)$$

where  $p$  and  $m$  are the induced electric and magnetic moments and  $E_{inc}$  and  $H_{inc}$  are the complex amplitudes of the incident fields. Let us further assume the Lorentzian dispersion model for both polarizabilities, defined by the resonance frequency  $\omega_e$ , the amplitude factor  $A_e$ , and the loss factor  $\Gamma$ . For example, balanced spirals obey this law close to the fundamental resonance [4]. Requiring absence of reflection, we need to ensure that the polarizabilities are balanced [3]:

$$\frac{1}{\eta_0 \widehat{\alpha}_{ee}} = \frac{1}{\widehat{\alpha}_{mm}/\eta_0} = \frac{\omega_e^2 - \omega^2 + j\omega\Gamma}{A_e} + \frac{j\omega}{2S}. \quad (2)$$

Here, parameter  $\Gamma$  measures the dissipation in the unit cells, and the last term is due to interactions between the unit cells in the infinite periodic lattice [5] ( $S$  is the unit-cell area). The polarizabilities are normalized to the free-space impedance  $\eta_0$ . This model properly accounts for scattering and dissipation losses and ensures energy conservation in the scattering and absorption processes.

The transmission coefficient (as can be found from eq. (1) in [4], for example) reads

$$t = \frac{\frac{\omega_e^2 - \omega^2 + j\omega\Gamma}{A_e} - \frac{j\omega}{2S}}{\frac{\omega_e^2 - \omega^2 + j\omega\Gamma}{A_e} + \frac{j\omega}{2S}} = \frac{\frac{\omega_e^2 - \omega^2}{A_e} - j\omega \left( \frac{1}{2S} - \frac{\Gamma}{A_e} \right)}{\frac{\omega_e^2 - \omega^2}{A_e} + j\omega \left( \frac{1}{2S} + \frac{\Gamma}{A_e} \right)}. \quad (3)$$

Let us check the known limiting cases: First, if  $\Gamma = 0$  (lossless particles),  $|t| = 1$  and we have full phase coverage in tuning the transmission phase (eq. (10) in [4]). Second, at the resonance frequency and if  $\Gamma = A_e/(2S)$  we have full absorption [4].

Next we note that the transmission coefficient expression (3) has the form

$$t = \frac{a - jb}{a + jc}, \quad (4)$$

where the real parameters  $a$ ,  $b$ , and  $c$  can be independently controlled varying the unit cell dimensions, shape, and the array period. Since  $b$  and  $c$  are of the form  $b = d_1 - d_2$ ,  $c = d_1 + d_2$  with some arbitrary positive  $d_1$ ,  $d_2$  ( $d_1$  depends on

the choice of the array period, while  $d_2$  depends on the unit cell structure and materials), the following relation holds:

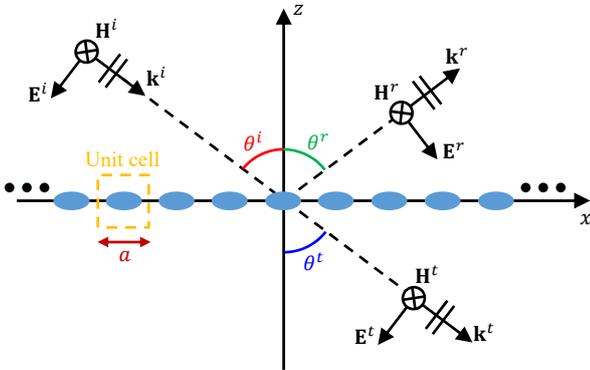
$$|\text{denominator}|^2 = |\text{numerator}|^2 + \frac{2\Gamma}{SA_e}. \quad (5)$$

Hence, the inequality  $|t| \leq 1$  is guaranteed for all possible values of  $1/(2S)$  and  $\Gamma/A_e$ , for all loss factors  $\Gamma \geq 0$ . Then,  $1/2S - \Gamma/A_e$  can take any value from  $-\infty$  to  $+\infty$  for any given value of  $\Gamma \geq 0$ . This proves that it is possible to fully control transmission phase for any *arbitrary* level of absorption in the metasurface, never compromising the absence of reflections.

These considerations also clearly show that the presence of both electric and magnetic polarizations in unit cells is necessary to allow transmission control in zero-reflection scenario. Only in this case we can control transmission and reflection properties independently, since the metasurface creates *different* scattered fields on the two sides of the surface. Next, we will consider the necessary condition in the most general case, allowing arbitrary direction of illumination.

### 3 Oblique incidence

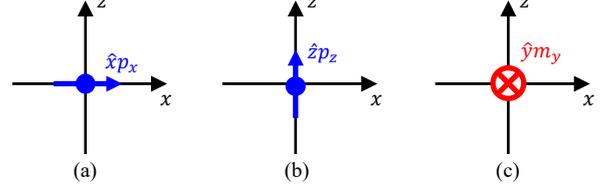
Let us consider a TM-polarized plane-wave illumination of a generic periodical metasurface formed by electrically small unit cells, as illustrated in Fig. 1. The array period is smaller than  $\lambda/2$ , so there are no diffraction lobes in the scattered field.



**Figure 1.** Reflection and transmission of a TM-polarized plane wave by a metasurface in the  $xy$ -plane situated in free space.

In contrast to the normally excited array, under oblique illumination in each unit cell also normally directed dipole moments can be excited. For simplicity, we restrict the discussion to non-bianisotropic topologies, in which case three dipole moments can be induced, as shown in Fig. 2.

Studying the normal incidence case, we saw that full transmission control was enabled by the presence of both electric dipole scatterers, which scatter symmetrically into the upper and lower half spaces, and magnetic dipoles, which



**Figure 2.** Three elemental dipole meta-atoms for TM-polarization reflection and scattering. (a) An  $x$ -directed electric dipole. (b) A  $z$ -directed electric dipole. (c) A  $y$ -directed magnetic dipole.

scatter anti-symmetrically. For this reason, in the case of oblique incidence we write the reflection  $r$  and transmission  $t$  coefficients as the sums of the symmetric and anti-symmetric components:

$$r = \frac{E_{0x}^r}{E_{0x}^i} = r_s + r_a; \quad r_s = \frac{E_{0x}^{s,s}}{E_{0x}^i}, \quad r_a = \frac{E_{0x}^{s,a}}{E_{0x}^i}, \quad (6)$$

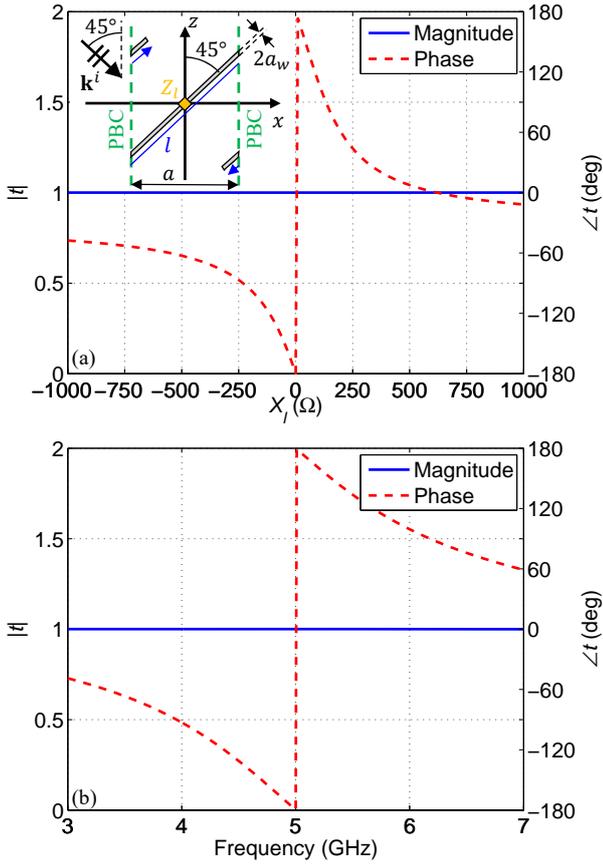
$$t = \frac{E_{0x}^t}{E_{0x}^i} = t_s - r_a, \quad t_s = 1 + r_s. \quad (7)$$

Here  $r_s$  and  $r_a$  represent reflection coefficients for the symmetric and anti-symmetric components, respectively. In (7),  $t_s$  denotes the transmission coefficient in the absence of anti-symmetric scattering.

The physical reason for zero reflection property is cancellation of scattering into the specular direction: the symmetric and anti-symmetric scattering components should interfere destructively in the reflection direction. In contrast to normally illuminated metasurfaces, here we find two dipole moment configurations, the tangential magnetic moment and the normally oriented electric moment, that scatter anti-symmetrically. This brings us to a conclusion that for oblique incidence full control over transmission can be achieved using only electric polarizations.

### 4 Numerical example

Probably the easiest way to create a null in the specular direction is to use an array of tilted electric dipoles, illustrated in Fig. 3. Here, the radiation into the specular direction is forbidden simply because of the proper dipole orientation. As discussed above, all the conditions for full control of both phase and magnitude of transmitted waves are satisfied, although only electric dipoles are induced. In this example, we assume that the dipoles are lossless and show that the transmission phase can be tuned with complete freedom by varying the reactive loads of the dipoles. In fact, both phase and amplitude can be fully controlled by varying both imaginary and real (for lossy loads) parts of the load impedance. In the presentation, we will show other possible realizations, such as arrays of dielectric bars at optical frequencies.



**Figure 3.** Plane wave scattering by a planar array of tilted dipoles. The meta-atom comprises a conducting thin-wire dipole of length  $l$  and wire radius  $a_w$ . The dipole lies in the  $xz$ -plane and the wire is assumed to be perfectly conducting. A point load with a reactive impedance  $Z_l = jX_l$  is connected at the dipole middle point. (a) The transmission magnitude and phase with respect to the load reactance  $X_l$  at 5 GHz. The inset shows a side view of the unit cell together with the illuminating plane wave. (b) The transmission coefficient as a function of frequency when the load port is shorted ( $X_l = 0$ ). The unit-cell dimension is  $a = 20$  mm in both  $x$ - and  $y$ -axis directions. The length and radius of the thin-wire dipole are  $l = 33.9$  mm and  $a_w = 0.25$  mm.

## 5 Conclusion

In conclusion, we have shown that the necessary physical conditions for full control of transmission through non-reflecting metasurfaces is excitation of polarizations which scatter both symmetrically and anti-symmetrically on the two sides of the array. This property enables creation of scattering pattern null in the specular direction and leaves enough degrees of design freedom to tune both the phase and amplitude of transmitted waves. The tilted dipole realization has an important advantage of exceptionally robust elimination of reflections in extremely broad frequency ranges: this property is ensured by the array geometry. New possibilities to realize Huygens' nonreflecting metasurfaces without induced magnetic moments are especially attractive for optical applications, where magnetic response is usually

weak. Moreover, realizations based on only electric dipole resonant modes can offer wider bandwidth of high transmission through the metasurface. Detailed derivations and more examples can be found in paper [6].

## 6 Acknowledgements

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