Optimal Terminating Impedances for Maximizing the Gains of a Four-coil WPT Link

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Abstract

In this paper, a Wireless Power Transfer (WPT) link consisting of four magnetically coupled resonators (a transmitting and a receiving resonator coupled through two relay elements) is analyzed. Standard gain definitions are used for describing the performance of the link and the analytical expressions of the terminating impedances providing the optimal operating conditions are derived.

1 Introduction

Resonant inductive Wireless Power Transfer (WPT) is an attractive technology enabling the wireless recharge of electronic systems [1–6]. The basic configuration uses just two magnetically coupled resonant loops [7, 8]: a transmitting (TX) and a receiving (RX) resonator. However, improved performance can be obtained by using additional resonators (relay resonators) not directly connected to the source and the load [9–16]. In this regard, two possible schemes of interest are the one using three resonators (i.e., a TX, an RX and one relay resonator) [13, 14] and the one using four resonators (i.e., a TX, an RX and two relay resonators) [15].

In particular, in [17] the scheme with a single relay element has been solved referring to the problem of maximizing the gains of the link. It is demonstrated that, for a given link, the three power gains (i.e., the power gain, the available gain and the transducer gain) can be simultaneously maximized by setting the terminating impedances equal to the conjugate image impedances of the network, $Z_{ci}$ ($i = 1, 2$), which, in general, are complex quantities. For the analyzed WPT link, the following results can be obtained for the gains:

\[ G_P = \frac{\chi_{14}^2 \chi_{43}^2 \chi_{32T}^2}{(\chi_{14}^2 + 1)(\chi_{32T}^2 + 1) + \chi_{43}^2} \frac{Q_{2T}}{Q_L}, \]  
\[ G_A = \frac{Q_{1T}}{Q_G} \frac{\chi_{1T4}^2 \chi_{43}^2 \chi_{32T}^2}{(\chi_{1T4}^2 + 1)(\chi_{32T}^2 + 1) + \chi_{43}^2 (\chi_{1T4}^2 + \rho)}, \]  
\[ G_T = \frac{Q_{1T}}{Q_G} \frac{4 \chi_{1T4}^2 \chi_{43}^2 \chi_{32T}^2}{(\chi_{1T4}^2 + 1)(\chi_{32T}^2 + 1) + \chi_{43}^2} \frac{Q_{2T}}{Q_L}. \]

According to the analysis reported in [18], for a generic two–port network the terminating impedances which allow to simultaneously maximize the three power gains are the conjugate image impedances of the network, $Z_{ci}$ ($i = 1, 2$), which, in general, are complex quantities. For the analyzed

2 Analyzed problem

The problem analyzed in this paper is schematized in Fig. 1. A WPT link consisting of four resonators is considered; it is assumed that each resonator consists of a distributed inductor $L_i$ loaded by an appropriate compensating capacitor $C_i$ realizing the resonance condition at the angular resonant frequency $\phi_0$. The resistors $R_i$ model the resonator losses. The parameters summarized in Table 1 are used for the analysis and the power gains (Power Gain, $G_P$, Available Gain, $G_A$, and Transducer Gain, $G_T$) as defined in the context of active two–port network are adopted for describing the performance of the link. The definitions and the expressions in terms of impedance matrix representation of the network are summarized in Table 2.
Figure 1. Equivalent representation of a WPT link using two relay elements.

Table 1. Definitions.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_0$</td>
<td>$\alpha_0 L_2$</td>
</tr>
<tr>
<td>$n_{ij}$</td>
<td>$\sqrt{\frac{M_{i}}{T_{ij}}}$</td>
</tr>
<tr>
<td>$k_{ij}$</td>
<td>$\frac{M_{i}}{\sqrt{T_{ij}k_{ij}}}$</td>
</tr>
<tr>
<td>$Q_i$</td>
<td>$\frac{\alpha_i L_i}{R_i}$</td>
</tr>
<tr>
<td>$Q_G$</td>
<td>$\frac{\alpha G}{R_G}$</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>$\frac{\alpha L_2}{R_L}$</td>
</tr>
</tbody>
</table>

Table 2. Gains definitions and expressions.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_P$</td>
<td>$\frac{P_o}{P_{in}} \frac{Z_{11}}{Z_{22} + Z_{21}}$</td>
</tr>
<tr>
<td>$G_A$</td>
<td>$\frac{P_o}{P_{in}} \frac{Z_{11}}{Z_{22} + Z_{21}}$</td>
</tr>
<tr>
<td>$G_T$</td>
<td>$\frac{P_o}{P_{in}} \frac{4z_{12}^{2}R_{21}R_{22}}{(z_{11} + Z_{0})(Z_{22} + Z_{21}) - z_{12}z_{21}Z_{0}}$</td>
</tr>
</tbody>
</table>

problem, the following expressions can be derived for the conjugate image impedances:

$$Z_{11} = \frac{x_{0n}r_{12}}{Q_{1}} \sqrt{\frac{(x_{14}^{2} + 1)(x_{32}^{2} + 1)}{\rho(x_{13}^{2} + \rho)}}, \quad (4)$$

$$Z_{21} = \frac{x_{0}r_{21}}{Q_{2}} \sqrt{\frac{(x_{14}^{2} + 1)(x_{32}^{2} + 1)}{\rho(x_{13}^{2} + \rho)}}, \quad (5)$$

It can be noticed that, as for the case of a three–resonator link, the conjugate image impedances are purely resistive quantities (i.e., $Z_{11} = R_{11}$ and $Z_{21} = R_{21}$), so that they can be referred as image resistances. By setting $Z_{0} = R_{1}$ and $Z_{L} = R_{2}$, the three gains assume their maximum realizable value, the so called ultimate gain: $G_P = G_A = G_T = G_a$. In particular, for the present case $G_a$ assumes the following expression:

$$G_a = \sqrt{\frac{(x_{14}^{2} + \rho)(x_{32}^{2} + \rho)}{x_{14}x_{13}x_{32}} \rho \left(\frac{x_{14}^{2} + 1)(x_{32}^{2} + 1)}{\rho(x_{13}^{2} + \rho)}\right)^2}.$$

2.2 Validation

In order to verify the theory, circuit simulations have been performed with the commercial tool NI AWR design environment. The parameters of the analyzed resonators are summarized in Table 3 and have been taken from [15] (see

Figure 2. Results obtained from circuital simulations for $G_T$, $G_P$ and $G_A$ by varying: a) the value of $R_L$ for $R_G = R_{11}$, b) the value of $R_G$ for $R_L = R_{21}$. The values assumed for the couplings are: $k_{14} = 0.2$, $k_{43} = 0.1$ and $k_{32} = 0.22$. 

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References


