Implications of Higher Symmetries in Periodic Structures

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Abstract

In this paper, we explain the concept of higher symmetries, their implications and possible applications—within electromagnetic technology.

1. Definition of Higher Symmetries

There are two types of higher symmetries: glide and twist symmetry. A periodic unit cell possesses twist symmetry if it is repeated after a translation and a rotation [1,2]. Mathematically, this means that for a period $d$ in $z$ direction, a $p$-fold twist-symmetric structure will be invariant under the following transformation:

$$
\begin{align*}
    z &\rightarrow z + d/p \\
    \varphi &\rightarrow \varphi + 2\pi/p \\
    r &\rightarrow r
\end{align*}
$$

with $\varphi$ and $r$ being cylindrical coordinates, and $p$ an integer. On the contrary, a unit cell will be glide-symmetric if it can be repeated after a translation and a mirroring [3,4], which can be expressed as follows:

$$
\begin{align*}
    x &\rightarrow x + d/2 \\
    y &\rightarrow y + d/2 \\
    z &\rightarrow -z
\end{align*}
$$

For example, in Fig. 1, we have represented a two-dimensional glide-symmetric structure. The unit cell is rectangular, with periodicity $d$ in these two dimensions. Additionally, the unit cell is mirrored vertically to produce the top layer.

![Figure 1. Two-dimensional glide-symmetric periodic structure. The unit cell is rectangular, and it is composed by a conical shape inserted in both bottom and top layers.](image)

2. Implications and Applications of Higher Symmetries

Higher symmetries have demonstrated to produce interesting features for a given number of applications. For example, in [5-7], glide symmetry demonstrated to be able to increase the bandwidth of operation, and in [8,9] to increase the density of the materials. These properties can be employed to produce, for example, Luneburg lenses, which find application in high frequency 5G antennas.

Additionally, glide-symmetric holey metallic unit cells were demonstrated to produce broadband bandgaps, in contrast to conventional holey structures. These properties created an opportunity to produce cost-efficient version of gap-waveguide technology [10-12], and contact-less flanges with low leakage [13].

More recently, glide-symmetric unit cells have been employed to enable low-dispersive propagation in co-planar waveguides and slots [14, 15], which are suitable to produce low-dispersive leaky wave antennas.

Finally, more exotic configurations based on twist symmetries have been recently studied [16-18]. Twist-symmetric structures demonstrated to produce low-dispersive responses [16,17] and bandgaps that associated to their symmetries [17,18]. These configurations find application for low-dispersive leaky-wave antennas, and low-loss fully-metallic high frequency technology as reconfigurable filters and phase shifters.

It is remarkable that higher-symmetric structures require a large coupling between sub-elements in order to produce a distinctive behaviour when compared with conventional technology. This has also motivated the recent development of numerical and analytical methods which can characterise these structures. These methods include circuit models as in [19] and mode matching technique as introduced in [20-22].

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4. References


