



## A Linear Decision Feedback Detector For SOQPSK

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### Abstract

The pulse amplitude modulation (PAM) decomposition of continuous phase modulation (CPM) signals has been recently revamped to give another view of ternary shaped offset quadrature phase shift keying (SOQPSK) signals as duobinary CPMs. Taking advantage of this approach, we propose a new linear detector that operates using a decision feedback mechanism. We show that the proposed solution offers a very acceptable bit error rate (BER) performance without the need to employ a Viterbi algorithm.

### 1 Introduction

Shaped offset quadrature phase shift keying (SOQPSK) is a bandwidth efficient single carrier continuous phase modulation (CPM) widely used in aeronautical telemetry and deep space communications [1]. It is distinguished by its ternary alphabet that is generated using a precoder described in [2], and it was developed as a variant of shaped binary phase-shift keying (SBPSK) [3]. The first version of SOQPSK was first adopted as a part of a military standard [4], and was referred to as MIL-STD SOQPSK or SOQPSK-MIL. Then, more bandwidth efficient variants of SOQPSK were introduced by Hill [5] and were referred to as SOQPSK-A and SOQPSK-B. The good spectral efficiency of these versions compared to SOQPSK-MIL is achieved thanks to the partial response nature of the frequency pulse. This modulation is non-proprietary, and its partial response versions can lead to similar bit error rate (BER) performance to the proprietary Feher QPSK [6]. These attractive characteristics led to adopting a SOQPSK variant, namely SOQPSK-TG (less known as SOQPSK-A\*) in the inter range instrumentation group (IRIG) recommendations [7] as a Tier 1 telemetry modulation along with FQPSK.

The CPM nature of SOQPSK makes this modulation very attractive for long distance wireless transmissions as power amplifiers can be used near their maximal throughput without distorting the signal. However, its ternary alphabet and its long frequency pulse make demodulating the signal using the CPM definition complex especially for SOQPSK-TG. The optimal detector for the latter requires a Viterbi algorithm composed of 512 states as highlighted in [8]. Several representations and approximations have been studied in the literature to get reduced complexity detectors for SO-

QPSK such as the cross-correlated trellis-coded quadrature modulation (XTCQM) representation [9] and the pulse amplitude modulation (PAM) decomposition [10, 11]. In this paper, we focus on the PAM decomposition described in [11], where it has been shown that SOQPSK can accurately be approximated as a single PAM because most of the signal energy is contained in one pulse. We take advantage of this consequence, and we propose a linear detector with a decision feedback mechanism using the Ungerboeck approach [12] that offers very attractive BER performance.

### 2 SOQPSK Signal Model

The complex envelope of SOQPSK is expressed as follows [13]

$$s(t; \bar{\alpha}) = \sqrt{\frac{E_s}{T}} \exp \left\{ j \sum_n \alpha_n q(t - nT) \right\}, \quad (1)$$

where  $E_s$  is the energy per information symbol,  $T$  the symbol time duration and  $\bar{\alpha} = \{\alpha_n\}_{n \in \mathbb{Z}}$  are ternary symbols from the alphabet  $\{-1, 0, +1\}$  and are generated according to the following mapping:

$$\alpha_n = (-1)^{n+1} \frac{b_{n-1}(b_n - b_{n-2})}{2}, \quad (2)$$

where  $\{b_n\}_{n \in \mathbb{Z}} \in \{-1, +1\}$ . The Function  $q(t)$  is the phase pulse and represents the time integral of the frequency pulse  $g(t)$  whose time support is equal to  $LT$ . If  $L = 1$ , the signal has a full response frequency pulse. Otherwise, the signal has a partial response one. The phase pulse  $q(t)$  is defined as

$$q(t) = \begin{cases} 0, & t \leq 0 \\ 2h\pi \int_0^t g(\tau) d\tau, & 0 < t < LT \\ h\pi, & t \geq LT, \end{cases} \quad (3)$$

where  $h = \frac{1}{2}$  is the modulation index for SOQPSK and  $\int g(t) dt = \frac{1}{2}$ .

### 3 PAM decomposition of SOQPSK

#### 3.1 Duobinary PAM decomposition (DBD)

The duobinary PAM decomposition exploits the mapping described in Section 2. If we expand (2), the symbol  $\alpha_n$  can be expressed as follows:

$$\alpha_n = \frac{1}{2}(\gamma_n + \gamma_{n-1}), \quad (4)$$

where

$$\gamma_n = (-1)^{n+1} b_n b_{n-1}. \quad (5)$$

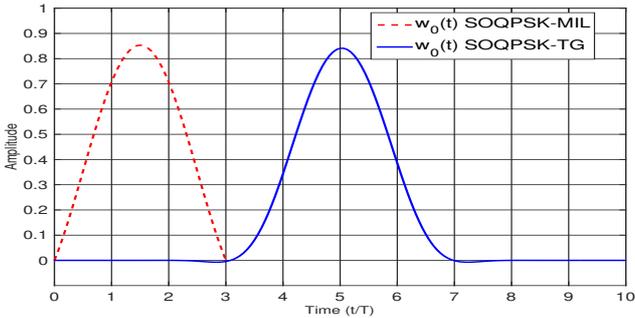
From (4), it is seen that  $\alpha_n$  is the output of a duobinary encoder applied on  $\gamma_n$ . Therefore, SOQPSK can be interpreted as a binary CPM with an equivalent frequency pulse  $g_{eq}(t) = \frac{1}{2}(g(t) + g(t-T))$  of length  $L_d = L + 1$ . As a result, SOQPSK can be written as in [11]

$$s(t; \bar{\alpha}) = \sum_{k=0}^{M-1} \sum_i \rho_{k,i} w_k(t - iT), \quad (6)$$

where  $M = 2^{L_d-1}$ , and the expressions of  $\rho_{k,i}$  and  $w_k(t)$  are detailed in [11]. SOQPSK-MIL can be decomposed into a sum of  $M = 2$  PAM pulses. As for SOQPSK-TG, an exact PAM representation requires  $M = 256$  pulses. In both cases, it has been shown in [11] that the first pulse  $w_0$  contains more than 97,5% of the total signal energy. Therefore, both versions of SOQPSK can be approximated using a single pulse  $w_0$  as follows

$$s(t; \bar{\alpha}) \approx \sum_i b_{2i} w_0(t - 2iT) + j \sum_i b_{2i+1} w_0(t - (2i+1)T). \quad (7)$$

The pulses of SOQPSK-MIL and SOQPSK-TG are plotted in Fig. 1. Equation (7) shows that the odd bit sequence is time shifted by one bit period. Therefore, if we consider a symbol composed of one even bit and one odd bit, we go back to the definition of the classical OQPSK with  $w_0(t)$  as a shaping pulse of length  $3T$  for SOQPSK-MIL and  $10T$  for SOQPSK-TG.



**Figure 1.** PAM representation of SOQPSK-MIL & SOQPSK-TG using DBD

#### 4 Linear decision feedback detector

One of the consequences of the PAM decomposition is to develop reduced complexity detectors. If we conceive a Viterbi detector based on (1) for instance, it would be operating with 1024 states for SOQPSK-TG since the bit rate sampled version of this signal depends on 11 bits. In this section, we exploit the knowledge of the main waveform that links SOQPSK to OQPSK to develop a linear decision

feedback detector. To do so, we consider that SOQPSK can be approximated by using only the main pulse (see (7)) and that  $N$  bits are transmitted. The received signal is then expressed as

$$r(t) \approx \sum_{m=0}^{N-1} \rho_{0,m} w_0(t - mT) + n(t), \quad (8)$$

where  $n(t)$  is an additive complex white Gaussian noise (AWGN) with double-sided spectral density  $N_0$ . The log-likelihood function for  $\bar{b}$  is

$$\Lambda(\bar{b}) = \frac{-1}{2N_0} \int_0^{NT} \left| r(t) - \sum_{m=0}^{N-1} \rho_{0,m} w_0(t - mT) \right|^2 dt. \quad (9)$$

The mathematical development of (9) leads to the following expression

$$\Lambda(\bar{b}) = 2Re \left[ \sum_{m=0}^{N-1} \rho_{0,m}^* y(m) \right] - \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \rho_{0,m}^* R_w(m-n) \rho_{0,n} \quad (10)$$

where

$$R_w(k) = \int w_0(\tau) w_0(\tau - kT) d\tau, \quad (11)$$

and

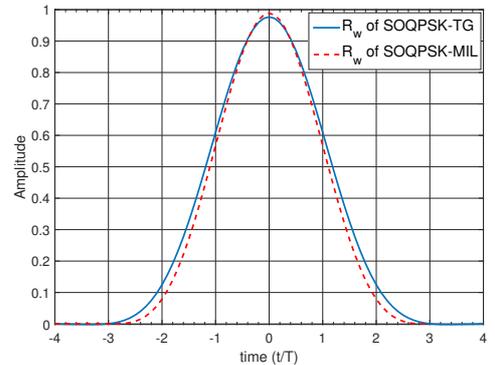
$$y(m) = \int r(t) w_0(t - mT) dt. \quad (12)$$

Ungerboeck showed in [12] that (10) can be simplified as

$$\Lambda(\bar{b}) = \sum_m Re \left[ \rho_{0,m}^* \left( y(m) - \rho_{0,m} \frac{R_w(0)}{2} - \sum_{n \leq m-1} \rho_{0,n} R_w(m-n) \right) \right], \quad (13)$$

$$= \sum_m \lambda(m). \quad (14)$$

The simplification of (13) depends on the properties of the auto-correlation function  $R_w$ . Depending on the used frequency pulse,  $R_w$  can approximately be zero after several bit periods. Fig. 2 displays  $R_w$  for the SOQPSK-TG and SOQPSK-MIL.



**Figure 2.** Auto-correlation  $R_w$  of  $w_0(t)$

From Fig. 2, (13) can be simplified by considering that  $R_w(k) = 0$  for  $k \geq 3$ . Moreover, given the fact that the autocorrelation  $R_w$  is real-valued, the metric  $\lambda(m)$  can be expressed as follows

$$\lambda(m) = \begin{cases} b_m [\text{Re}(y(m)) - b_{m-2}R_w(2)], & m \text{ even,} \\ b_m [\text{Im}(y(m)) - b_{m-2}R_w(2)], & m \text{ odd.} \end{cases} \quad (15)$$

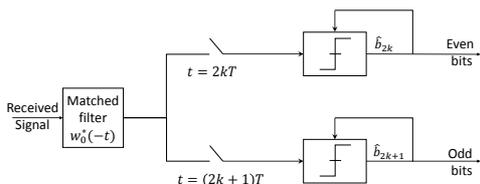
It has been shown in [14] that the detection process can be performed via a Viterbi algorithm. In this paper, we propose instead to keep the integrate & dump detection structure while taking advantage of the estimated bits of the previous epoch by introducing a decision feedback mechanism. If we denote  $\hat{b}_{m-2}$  the estimation of  $b_{m-2}$  at epoch  $m-2$ , then  $\lambda(m)$  becomes

$$\lambda(m) = \begin{cases} b_m [\text{Re}(y(m)) - \hat{b}_{m-2}R_w(2)], & m \text{ even,} \\ b_m [\text{Im}(y(m)) - \hat{b}_{m-2}R_w(2)], & m \text{ odd.} \end{cases} \quad (16)$$

Since we deal with a maximization problem, (16) leads to the following decision feedback (DF) linear detector

$$\hat{b}_m = \begin{cases} \text{sign}(\text{Re}\{y(m)\} - \hat{b}_{m-2}R_w(2)), & m \text{ even,} \\ \text{sign}(\text{Im}\{y(m)\} - \hat{b}_{m-2}R_w(2)), & m \text{ odd.} \end{cases} \quad (17)$$

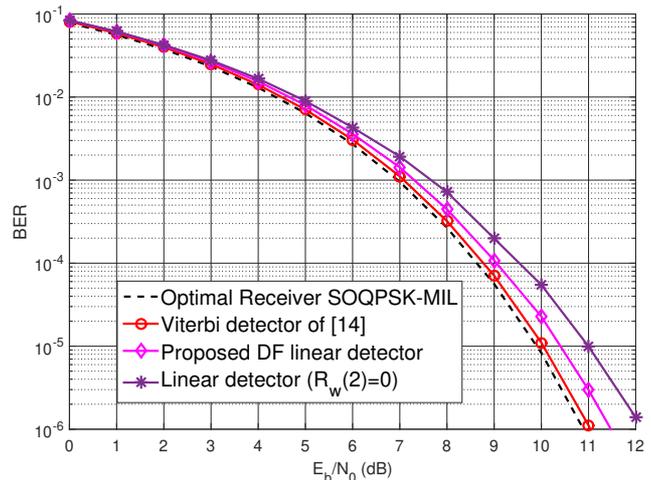
We can also notice that even and odd indexed bits can be processed independently like the other linear OQPSK-type detectors given in [15]. The architecture of this detector is depicted in Fig. 3.



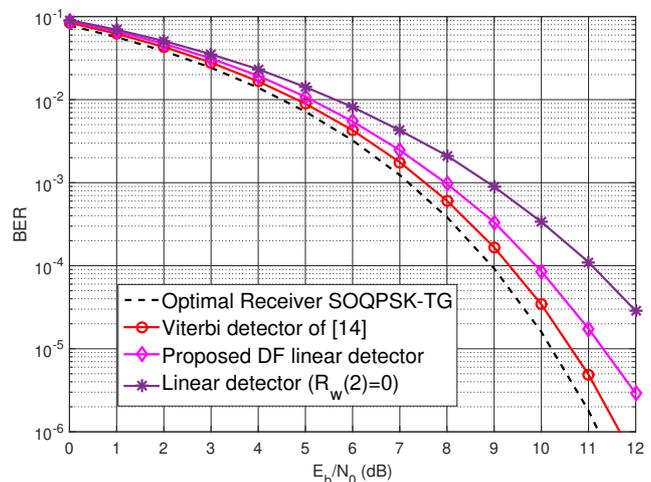
**Figure 3.** Linear decision feedback detector architecture

## 4.1 Simulation results

We plot in Fig. 4 and Fig. 5 the BER performance of DF detector for both SOQPSK versions. We also plot the BER performance of the Viterbi detector given in [14] as well as the case where the linear detector is used without a decision feedback mechanism (i.e., by considering that  $R_w(2) = 0$ ). We can notice from these figures that this mechanism improves the performance of the detector by almost 0.6 dB for SOQPSK-MIL and 1 dB for SOQPSK-TG at  $\text{BER} = 10^{-5}$  without the need to use a Viterbi algorithm. That makes this solution attractive for real-time implementation.



**Figure 4.** BER performance of the proposed DF linear detector - SOQPSK-MIL



**Figure 5.** BER performance of the proposed DF linear detector - SOQPSK-TG

## 5 Conclusion

In this paper, we took advantage of the PAM decomposition of SOQPSK to develop a new linear decision feedback detector. The latter is based on the Ungerboeck approach and offers very acceptable BER performance without using a Viterbi algorithm.

## References

- [1] J. H. Yuen, J. Hamkins, and M. K. Simon, *Autonomous Software-Defined Radio Receivers for Deep Space Applications*. John Wiley & Sons, Oct 2006.
- [2] M. K. Simon, *Bandwidth-Efficient Digital Modulation with Application to Deep-Space Communications*. John Wiley & Sons, Jan 2005.
- [3] M. J. Dapper and T. J. Hill, "SBPSK: A Robust Bandwidth-Efficient Modulation for Hard-Limited Channels," vol. 3, pp. 458–463, Oct 1984.

- [4] D. I. S. Agency, "Department of Defense Interface Standard, Interoperability Standard for Single-Access 5-kHz and 25-kHz UHF Satellite Communications Channels," Mar 1999.
- [5] T. J. Hill, "An Enhanced, Constant Envelope, Interoperable Shaped Offset QPSK (SOQPSK) Waveform For Improved Spectral Efficiency," *International Telemetry Conference Proceedings*, 2000. [Online]. Available: <http://hdl.handle.net/10150/606498>
- [6] P. S. K. Leung and K. Feher, "F-QPSK-a Superior Modulation Technique For Mobile and Personal Communications," *IEEE Transactions on Broadcasting*, vol. 39, no. 2, pp. 288–294, Jun 1993.
- [7] IRIG, "Telemetry Standards, IRIG Standard 106-17 (Part 1), Chapter 2," Aug 2017.
- [8] M. Geoghegan, "Implementation and Performance Results for Trellis Detection of SO-QPSK," *International Telemetry Conference Proceedings*, Oct 2001. [Online]. Available: <http://hdl.handle.net/10150/606382>
- [9] L. Li and M. K. Simon, "Performance of Coded OQPSK and MIL-STD SOQPSK with Iterative Decoding," *IEEE Transactions on Communications*, vol. 52, no. 11, pp. 1890–1900, Nov 2004.
- [10] E. Perrins and M. Rice, "PAM Representation of Ternary CPM," *IEEE Transactions on Communications*, vol. 56, no. 12, pp. 2020–2024, Dec 2008.
- [11] R. Othman, A. Skrzypczak, and Y. Louët, "PAM Decomposition of Ternary CPM With Duobinary Encoding," *IEEE Transactions on Communications*, vol. 65, no. 10, pp. 4274–4284, Oct 2017.
- [12] G. Ungerboeck, "Adaptive Maximum-Likelihood Receiver for Carrier-Modulated Data-Transmission Systems," *IEEE Transactions on Communications*, vol. 22, no. 5, pp. 624–636, May 1974.
- [13] J. B. Anderson, T. Aulin, and C.-E. Sundberg, *Digital Phase Modulation*. New York: Plenum Press, Sep 1986.
- [14] R. Othman, Y. Louët, and A. Skrzypczak, "A Reduced Complexity OQPSK-Type Detector for SOQPSK," in *2018 IEEE 88th Vehicular Technology Conference (VTC-Fall)*, 2018 to be published.
- [15] M. Geoghegan, "Optimal Linear Detection of SOQPSK," *International Telemetry Conference Proceedings*, Oct 2002. [Online]. Available: <http://hdl.handle.net/10150/606382>