Robust Beamforming for X-Band Phased Array Weather Radar

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Abstract

Phased array weather radars (PAWRs) are increasingly becoming viable because of their operational advantage in the agile scanning of large precipitation volumes without any mechanical motion and an efficient beam scheduling-and-tracking. Conventional PAWR estimates precipitation profiles through Fourier beamforming. However, this method is not robust to interference, clutter and mismatch in steering vectors. In this work, we offset these disadvantages by using advanced robust and adaptive beamformers in PAWR. The volumetric nature of the precipitation target makes direct application of these techniques to PAWR non-trivial. We validate our methods through data collected from the solid-state low-power X-band PAWR developed at IIT Palakkad.

1 Introduction

Monitoring, prediction, and estimation of severe weather phenomena such as flash floods, tornadoes and microbursts is of critical importance [1]. Traditionally, weather radars employ huge parabolic dish antennas and mechanically scan the precipitation volume [2, 3]. This approach is quite inefficient to track very fast moving storms [4]. Recently, phased array weather radars (PAWRs) have been introduced to bring agility and efficiency in the scanning operations. A phased array radar comprises of several antenna elements, that form a highly directional beam without requiring any mechanical motion, achieves beam-steering electronically by adjusting the relative phase of excitation in its constituent elements [5].

The radar estimates the direction-of-arrival (DoA) along the precipitation profiles by adopting spatial filters called beamformers. Conventional technique is Fourier beamforming which is spatial equivalent of a matched filter. It applies uniform phase shift to steer the beam in a specified direction. This computationally simple method is the most optimal in the presence of white noise. However, its precipitation estimates could be considerably biased because of high sidelobe levels. Recently, adaptive beamforming techniques have been employed in phased-array-based atmospheric and weather radars to mitigate the sidelobes of conventional beamformers [6, 7, 8]. These methods have been previously investigated for point-target surveillance radars in detail [9]. However, their application to volumetric targets such as precipitation remains relatively unexamined.

A few studies [7, 6, 10] discuss improvement in sidelobe levels by applying Capon or minimum variance distortionless response (MVDR) beamformer [11] which is capable in rejecting interfering signals such as clutter from directions other than the direction-of-interest. However, this method requires sufficiently high number of samples to accurately estimation of the sample covariance matrix and is prone to steering vector mismatch. In [8], a minimum-mean-squared-error (MMSE) beamformer is proposed which employs prior information to reduce the required number of pulses in the covariance estimation process. The adaptive beamspace processing suggested in [6] improves upon Capon beamformer by making it robust to steering vector mismatch. However, this method is computationally expensive and not useful for real-time deployment.

The performance of most adaptive beamformers degrades when there is an imprecise knowledge of the steering vector, sample size is small or prior information is unavailable. To mitigate these disadvantages, robust adaptive beamforming (RAB) such as diagonal loading [9, 12] has been proposed in point target radars. Although RAB has not been applied to PAWRs so far, their application in other volumetric target sensors such as wind profilers has been very popular. For example the norm-constrained directionally-constrained minimization-of-power (NC-DCMP) beamformer used in [13, 14, 15] for middle and upper troposphere measurements is similar to the classic diagonal loading beamformer [12] and controls the signal-to-noise ratio degradation. It is dependent on the norm constraints whose values are empirically determined in advance. An optimized version of the NC-DCMP was proposed in [7] to increase signal-to-interference-ratio for enhanced probability of detection. This algorithm evaluates residual clutter and noise powers and is especially useful for weak received signals.

In this paper, we investigate RAB beamformers for PAWR. This method will take into consideration various signal
and hardware imperfections and address the problems using the optimized combination of different beamforming techniques. We verify our methods through experimental data obtained by an X-band PAWR developed in-house at IIT Palakkad. In the next section, we describe the system model and various beamformers.

2 Signal Model

Consider a uniformly spaced phased array antenna that has \( N \) elements. Then, for the \( i \)th snapshot (or pulse repetition interval), the complex-valued discrete-time received signal \( N \times 1 \) column vector \( \mathbf{x}_i = [x_{i,0}, \ldots, x_{i,N-1}]^T \) are given by

\[
\mathbf{x}_i = \mathbf{A}s_i + \mathbf{n}_i,
\]

where \( s_i = [s_{i,0}, \ldots, s_{i,M-1}]^T \) is the source signal vector of \( M \) samples from a given range bin, \( \mathbf{n}_i \) is additive spatio-temporal white Gaussian noise, \( \mathbf{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_{M-1})] \) is the \( N \times M \) receiving matrix with the \( i \)th column

\[
\mathbf{a}(\theta_i) = \begin{bmatrix}
\frac{1}{\sqrt{2\pi}} \sin \theta_i \\
\vdots \\
\frac{1}{\sqrt{2\pi}} (N-1) \sin \theta_i
\end{bmatrix}
\]

where \( d \) is the channel inter-element spacing, \( \lambda \) is the operating wavelength of the radar, and \((\cdot)^T\) denotes transpose operation.

The received signals \( \mathbf{x}_i \) are combined to form complex estimates \( \{y_{1,m}\}_{m=1}^{M-1} \) of the precipitation profile from the direction \( \theta_m \) as follows

\[
y_{1,m} = \mathbf{w}^H(\theta_m)\mathbf{x}_i,
\]

where \((\cdot)^H\) is the conjugate transpose and \( \mathbf{w}(\theta_m) \), \( m = 1, \ldots, M \) are carefully selected weight vectors. The goal of the beamforming algorithm is to determine \( \mathbf{w}(\theta_m) \).

Once the estimates \( \{y_{1,m}\}_{m=1}^{M-1} \) are available for \( L \) snapshots, standard moments such as the reflectivity (or power), mean Doppler velocity and spectrum width are computed from the autocovariance of \( \{y_{1,m}\}_{m=1}^{M-1} \) through the pulse-pair processing method [16]. We now summarize some of the common beamforming method to determine the weight vector.

Fourier Beamformer: This is equivalent to spatial matched filtering. So, the weight vector is simply a copy of the corresponding steering vector, i.e., \( \mathbf{w}_{FR}(\theta_m) = \mathbf{a}(\theta_m)/N \).

Capon/MVDR Beamformer: This method requires only the knowledge of the desired signal direction of arrival and determines the weight vector by solving the optimization

\[
\begin{align*}
\text{minimize} & \quad \mathbf{w}^H(\theta_m)\mathbf{R}_{xx}(0)\mathbf{w}(\theta_m) \\
\text{subject to} & \quad \mathbf{w}^H(\theta_m)\mathbf{a}(\theta_m) = 1,
\end{align*}
\]

where \( \mathbf{R}_{xx}(0) = \frac{1}{L} \sum \mathbf{x}_i\mathbf{x}_i^H \) is the lag-0 autocorrelation of the received signal. The solution of this problem is [17, 18]

\[
\mathbf{w}_{CP}(\theta_m) = \frac{\mathbf{R}_{xx}^{-1}(0)\mathbf{a}(\theta_m)}{\mathbf{a}^H(\theta_m)\mathbf{R}_{xx}^{-1}(0)\mathbf{a}(\theta_m)}
\]

Diagonal Loading: Since the beamformers involve inversion of a covariance matrix, the numerical stability of the weight vectors is affected while inverting a matrix with small eigen values. The diagonal loading helps with the stability and reduces the spread in weight amplitudes. In this method, a quadratic inequality constraint of bounding the weight coefficients is added to (4) as follows:

\[
\begin{align*}
\text{minimize} & \quad \mathbf{w}^H(\theta_m)\mathbf{R}_{xx}(0)\mathbf{w}(\theta_m) \\
\text{subject to} & \quad \mathbf{w}^H(\theta_m)\mathbf{a}(\theta_m) = 1, \\
& \quad \mathbf{w}^H(\theta_m)\mathbf{w}(\theta_m) \leq T,
\end{align*}
\]

where \( T \) is the constant norm constraint. The solution to this problem gives

\[
\mathbf{w}_{DL}(\theta_m) = \frac{(\mathbf{R}_{xx} + \alpha \mathbf{I})^{-1}\mathbf{a}(\theta_m)}{\mathbf{a}^H(\theta_m)(\mathbf{R}_{xx} + \alpha \mathbf{I})^{-1}\mathbf{a}(\theta_m)}
\]

Diagonally loaded minimum variance (DLMV): If the unity constraint in (4) is changed to \( \mathbf{w}^H(\theta_m)\mathbf{a}(\theta_m) = c(\theta_m) \) where \( c \) is the constant gain in the direction \( \theta_m \), then the resulting spatial filter is called DLMV with weight vectors

\[
\mathbf{w}_{DL}(\theta_m) = \frac{\mathbf{R}_{xx}^{-1}(0)\mathbf{a}(\theta_m)c(\theta_m)}{\mathbf{a}^H(\theta_m)\mathbf{R}_{xx}^{-1}(0)\mathbf{a}(\theta_m)}
\]

3 Radar Specifications

Weather radars at X-band are now being widely used for rain rate measurements because of their low-cost and small size as well as sensitivity to small raindrops sizes [19, 16]. Further, their deployment on mobile platforms [2] and low-power consumption implies that the radars can be deployed in a network. There are also recent efforts toward building solid-state X-band PAWR to allow transmission of low peak powers without losing the radar range resolution.

IIT Palakkad has an ongoing project on designing a solid-state X-band PAWR. The radar transmitter and
Table 1. X-Band PAWR Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency</td>
<td>9.6 GHz</td>
</tr>
<tr>
<td>Peak power</td>
<td>20 W</td>
</tr>
<tr>
<td>Pulse width</td>
<td>0.6 µs</td>
</tr>
<tr>
<td>Polarization</td>
<td>Dual polarization</td>
</tr>
<tr>
<td>Receiver dynamic range</td>
<td>&gt; 90 dB</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>10 dBZ @ 5 km</td>
</tr>
<tr>
<td>Receiver noise figure</td>
<td>&lt; 4 dB</td>
</tr>
<tr>
<td>Minimum Detectable Signal</td>
<td>&lt; -100 dBm</td>
</tr>
<tr>
<td>Antenna type</td>
<td>Microstrip patch array</td>
</tr>
<tr>
<td>Antenna 3 dB lobe</td>
<td>&lt; 5° elevation and azimuth</td>
</tr>
<tr>
<td>Antenna gain</td>
<td>25 dB</td>
</tr>
</tbody>
</table>

Figure 1. Microstrip patch array antenna designed to operate in the frequency range of 9.6 to 10.6 GHz. Antenna pattern shown in Figure 2 and 3

receiver are based on solid-state components. The waveforms will be transmitted and received both in H- and V-polarizations. The frequency of operation is approximately 9.6 GHz and the peak transmitted power of the signal is approximately 20 W. The radar is still being developed and the transmitter as well as receiver beamwidths will be approximately 5° for any elevation angle. The sensitivity is approximately expected to be 10 dBZ at a range of about 5 km. A volume scan would last less than 1 minute. The receiver is designed to have a dynamic range of 90 dB and a noise figure of less than 4 dB.

One of the main requirements of a low-power solid-state X-band radar is an antenna with narrow beamwidth (less than 5°) and low side-lobe (30 dB below the main-lobe) so as to achieve the required resolution at high accuracy [20]. The requirements for low power and small size are being addressed using a microstrip patch array antenna. The antenna is shown in Figure 1 and has physical dimensions of approximately 10 cm on each side. The antenna beam pattern is shown in Figure 2. This antenna has a beamwidth higher than 6° and side lobes are approximately 10 dB below the main lobe [21]. This limitation of the antenna can be mitigated by using phased array that employs robust adaptive beamforming to create a narrow beam and reduce the side lobe levels. The number of antenna elements is 64 and the radar specifications, the antenna voltage-standing-wave-ratio (VSWR) measurements are provided in Table 1, Figure 3, respectively.

This paper reports ongoing efforts to build X-band PAWR and implementation of adaptive beamforming in its signal processor. The radar data analysis will be reported after the deployment of the radar and subsequent field campaigns. The future work involves the development and validation of the radar sub-systems as well as their integration into a single system.

References


