A Real-Time Spectrum Sensing Method by Averaging Spectra of Finite-Time Fourier Transform

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1 Extended Abstract

Spectrum observation/sensing is one of the important issues in radio communication systems. Those systems normally use DFT (discrete Fourier transform) where the input signal is extracted from the original one with a finite time window. However, the observed spectrum is to have false frequency components, what is said spectrum leakage, due to its finite time window. Some countermeasures are known, overlapped FFT, extending the window length, for example [1]; increase of computation complexity or degrade of frequency resolution becomes a problem. The authors of this paper proposed a new spectrum measurement method to obtain a high frequency resolution with a low computation load and the basic idea and principles are shown in [2] with computer simulations. In this paper, the principle of the method why the leakage is suppressed and the basic properties are shown with theoretical analysis.

Figure 1 illustrates the proposed processing of Fourier transform. It averages each the DFT or finite-time Fourier-transformed spectrum after the phase rotation corresponding to the time difference of each the observation window.

Let $g(t)$ and $G_n(f)$ respectively be the input time domain signal and the spectrum normally obtained from the $n$-th observation window assuming $t_n = 0$, the averaged spectrum can be expressed with the phase rotation as

$$\hat{G}_N(f) = \sum_{n=0}^{N-1} e^{-j2\pi ft_n} \hat{G}_n(f),$$

where $\hat{G}_n(f) = \int_{-T/2}^{T/2} g(t_n + \tau)e^{-j2\pi f\tau} d\tau = e^{j2\pi ft_n} \int_{t_n-T/2}^{t_n+T/2} g(t)e^{-j2\pi f t} dt$, (1)

and $T$ is the window length. In case $t_n - t_{n-1} = T$, (1) is written as $\hat{G}_N(f) = \int_{t_{N-1}-T/2}^{t_{N-1}+T/2} g(t)e^{-j2\pi f t} dt$. Note that the spectra $\hat{G}_N(f)$ and $\hat{G}_n(f)$ can be obtained only at the discrete $M$ frequency points $f = m/T$, $-M/2 \leq m \leq M/2 - 1$ where $M$ is the number of DFT points and even. It can be seen that $\hat{G}_N(f)$ gives a good approximation of the spectrum of $g(t)$, that the spectrum leakage can be suppressed, and that the frequency resolution becomes improved at each the frequency point as long as the most of the energy of $g(t)$ is within the extended interval $(t_n - T/2, t_{n-1} + T/2)$. However, the number of frequency points is still $M$ and does not becomes $NM$.

References
