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Compressive Sensing Based 2-D DOA Estimation by a Sparse L-Shaped Co-prime Array

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Abstract

Herein, two-dimensional direction-of-arrival (DOA) estimation of impinging signals on a L-shaped coprime array structure, in compressive sensing paradigm is explored. The received signals are compressed generating appropriate kernel in the L-shaped coprime array model. Suitable reconstruction algorithms based on single snapshot are then employed on the compressed signals to develop high resolution DOA estimation in two dimensions. Simulation results in terms of probability of resolution exhibits much better performance as compared with the standard two-dimensional compressive sensing model of DOA estimation using an L-shaped array antenna.

1. Introduction

The estimation problem of Direction-of-Arrival (DOA) of the received signals in two-dimensions (both elevation and azimuth angles) have attracted lot of interest among researchers over the last decade. Out of the several planar array structures, the L-shaped array which consists of two linear arrays of antennas placed orthogonally head-to-head has become a favored configuration as it provides lowest Cramer-Rao lower bound (CRLB) of the estimated wave directions [1-5]. Moreover, the L-shaped structure of antenna array offers more array aperture and better accuracy for 2-D DOA estimation [5].

In recent years, DOA estimation based in compressive sensing paradigm have found astounding considerations and thought [6-7]. It offers an alternative approach to acquire a compressed form of signals with sparse behavior at the sensing stage and enables a high reconstruction class at the receiver stage [8-11]. In DOA estimation or source localization method, the sparsity or compressed representation of the impinging signals is assured using the appropriate angular transformation [12-15]. Compressive sensing model provides distinctive advantages over conventional methods of DOA estimation in increased degrees of freedom and reconstruction by single snapshot instance [12, 14].

Concurrent with the progress in compressive sensing models of DOA estimation, research and development on non-uniform and sparse sensor arrays for signal acquisition grew exhaustively. The co-prime array [13, 14] and the nested array [15] developed as models of sparse array structures found suitable implementations and applications in DOA estimation problems. However, the mutual coupling effect is unavoidable in densely packed nested subarrays which considerably degrades the resolution characteristics [15]. Co-prime arrays [13, 14] are shaped as a co-prime synthesis of two uniform linear subarrays, with one subarray having M elements and inter-element spacing as $N.\lambda/_2$, and the other subarray consisting of N elements with inter-element spacing as $M.\lambda/_2$. Here, M and N are mutual co-prime numbers. If the first sensor or antenna element is common to both the subarrays, the total number of elements of the co-prime array would be $M + N - 1$, offering $M \times N$ degrees of freedom.

In this paper, a single snapshot based 2-D DOA estimation with a co-prime structured L-shaped array antenna, is studied in the compressive sensing paradigm. Although in recent years, use of coprime array and compressive sensing framework in DOA estimation has become very popular among the researchers [14-18], very few work has been reported in 2D, using an L-shaped array. This is mainly due to increased computational complexity and problems in pair matching issues [16]. This paper represents extended research of [17-18], in two dimensions, using a co-prime structured L-shaped array antenna. The reconstruction optimization is performed in compressive sensing paradigm by a single snapshot instance. The simulation results are compared with respect to probability of resolution by varying the signal-to-noise ratio (SNR) and computational complexity of a linearly structured L-shaped array antenna in compressive sensing domain [16].

The rest of the paper is organized as follows. Section-2 is dedicated in modelling a coprime structured L-shaped array of sensors for DOA estimation in two dimensions. In section-3, formation of array pattern reconstruction in compressive sensing paradigm is established. The proposed algorithm is developed in this section. In section-4, simulation results for 2D DOA estimation for co-prime structured L-shaped array and linearly structure L-shaped array in compressive sensing paradigm at a fixed signal-to-noise ratio (SNR) are represented. Performance comparison is accomplished in this section with respect to probability of resolution by varying the signal-to-noise ratio (SNR). Also, a brief comparison on computational complexity analysis is shown in this section.

2. Signal Model of an L-Shaped Array for DOA Estimation with Co-prime Subarrays

Fig. 1 shows a formation of sparse co-prime array structure. Fig. 1(a), (b) and (c) represents dense uniform linear arrays (ULA) with R, N and M elements, with inter-element spacing as d, Md and Nd respectively. M and N are co-prime numbers and $d = \lambda/_2$, chosen to optimize the mutual coupling effect and spectral aliasing. Fig. 1(d)

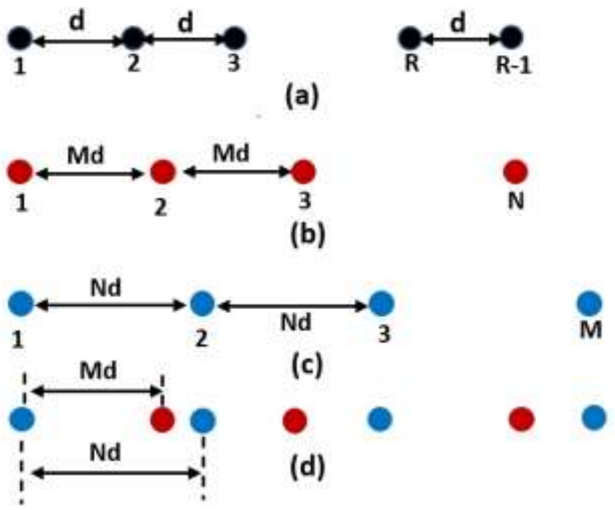

Fig 1. Coprime array geometry: (a) dense uniform linear array (ULA) with R elements with an inter-element spacing of d = λ/2. (b) dense ULA with N elements with an inter-element spacing Md. (c) dense ULA with M elements with an inter-element spacing of Nd. (d) the sparse coprime array. M and N are coprime numbers.

represents the sparse co-prime array structure formed with fig. 1(b) and fig. 1(c) ULAs. It is observed from fig. 1(d) that there are total $M + N - 1$ physical sensors, offering an array aperture of $((N-1)M.\lambda/_2, (M-1)N.\lambda/_2)$. The co-prime array structure of fig. 1(d) is formed by keeping the first elements of figs. 1(b) and 1(c) fixed. Fig. 2 depicts the difference and the symmetric difference co-array structures. For DOA estimation problem, the difference co-array structure is preferred over the symmetric difference counterpart due to reduced computational complexity of the former.

Fig. 3 illustrates an L-shaped sensor array structure for 2-D DOA estimation, placed along the $y - z$ axis of the co-ordinate system. The perpendicular subarrays along $y - axis$ and $z - axis$ consists of $M + N - 1$ number of

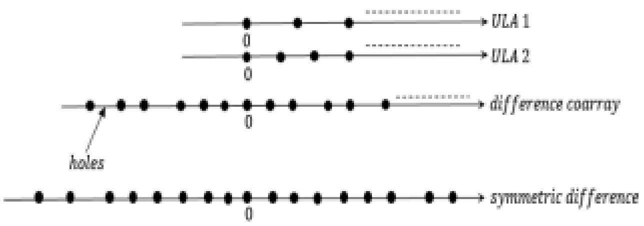

Fig 2. The difference and the symmetric difference co-array

sensors, where M and N are co-prime numbers (similar to the arrangement of fig. 1(b) and 1(c)). The sensor elements are assumed to be isotropic in nature. Narrowband signals from K quasi-stationary targets, located at the far-field region, travel through a non-dispersive, homogenous medium, impinge on the array from $(\theta_i, \varphi_i)_{i=1,2...K}$ directions, where θ_i and φ_i are the elevation and azimuth angles of the targets.

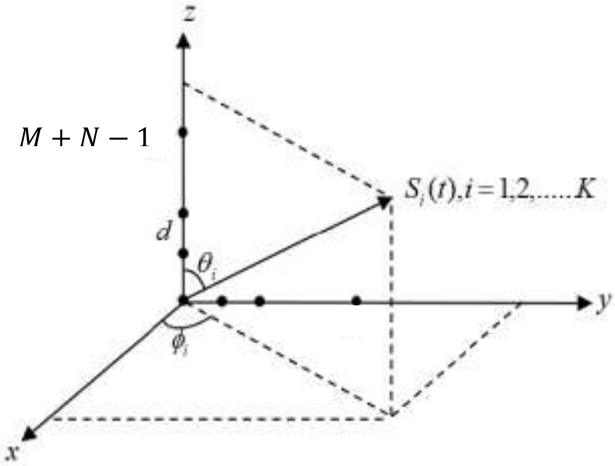

Fig 3. An L-shaped array antenna with co-prime subarrays placed along the $y - z$ axis for DOA estimation

The signal vectors, as collected along the perpendicular sub-arrays of y and $z-$ axis can be written as:

$$\boldsymbol{Y} = \boldsymbol{A}_Y(\theta, \varphi)\boldsymbol{S} + \boldsymbol{n}_Y$$
$$\boldsymbol{Z} = \boldsymbol{A}_Z(\theta, \varphi)\boldsymbol{S} + \boldsymbol{n}_Z \tag{1}$$

where

$$\boldsymbol{Y} = [y_1, y_2, \dots \dots y_{M+N-1}]^T$$
$$\boldsymbol{Z} = [z_1, z_2, \dots \dots z_{M+N-1}]^T \tag{2}$$

are the received signal vectors of dimension $(M + N - 1) \times 1$ and $[.]^T$ is the vector transpose. $\boldsymbol{A}_Y(\theta, \varphi)$ and $\boldsymbol{A}_Z(\theta, \varphi)$ are the Vandermonde array steering matrices of dimensions $(M + N - 1) \times K$ where each column vector $\boldsymbol{a}_Y(\theta_i, \varphi_i)_{i=1,2,...K}$ and $\boldsymbol{a}_Z(\theta_i, \varphi_i)_{i=1,2,...K}$ are the steering vectors corresponding to the K^{th} target signal. The source signal vector is $\boldsymbol{S} = [s_1, s_2, \dots \dots s_K]^T$, which is the $K \times 1$ vector of the complex monochromatic impinging signals. $\boldsymbol{n}_Y$ and $\boldsymbol{n}_Z$ are the complex additive white Gaussian noise with zero mean and variance σ^2.

3. Single Snapshot 2-D DOA Estimation based on Compressive Sensing

CS acquires the data at the co-prime sensor array in a compressed form and reconstructs back the original data efficiently from fewer samples at the receiver. It checks for a domain where the signal can be distinguished sparsely. In our DOA estimation problem, the sparsity is certain as the number of impinging signals K is less than the number of sensor elements in each subarray. The observation area is discretized by N_S angles and with the use of the steering matrices, constructive interferences in all angular directions are established. This results generation of a scan angle or sparsity basis matrix of dimension $(M+N-1)\times N_S$, as:

$$\begin{aligned}&\boldsymbol{\Psi}_n(\theta,\varphi)\\&=\left[\boldsymbol{a}_n(\theta_1,\varphi_1),\boldsymbol{a}_n(\theta_2,\varphi_2),\ldots\ldots\boldsymbol{a}_n(\theta_{N_S},\varphi_{N_S})\right]\end{aligned}\qquad(3)$$

Thus, (1) and (2) transforms to:

$$\begin{aligned}\boldsymbol{Y}&=\boldsymbol{\Psi}_Y(\theta,\varphi)\boldsymbol{S}+\boldsymbol{n}_Y\\ \boldsymbol{Z}&=\boldsymbol{\Psi}_Z(\theta,\varphi)\boldsymbol{S}+\boldsymbol{n}_Y\end{aligned}\qquad(4)$$

The observation or sensing matrix $\boldsymbol{\Phi}$ senses the most useful information in the signal and transforms the received signal into a measurement vector $\boldsymbol{y}$ in encrypted and compressed form. For the co-prime array model, the sensing matrix can be represented of the form:

$$\begin{bmatrix}1 & 0 & \cdots & 0\\ \vdots & & \ddots & \vdots\\ 0 & 0 & \cdots & 1\end{bmatrix}_{K\times(N+M-1)}\qquad(5)$$

where 1 denotes the selection of the array element.

For proper reconstruction of the original signal, the observation matrix $\boldsymbol{\Phi}$ should be uncorrelated with the sparsity basis matrix $\boldsymbol{\Psi}$, and should follow the restricted isometric property [14].

For the sparse reconstruction solution, the greedy approach (somewhat similar to $\|\ell\|_0-norm$) of optimization is the most popular, for its simplicity and lower complexity of computation. The $\|\ell\|_0-norm$ counts the number of non-zero entries $\left(y_0=\sum_{i=1}^{N}1_{,y_1\neq 0}\right)$, leading to the optimization problem,

$$\min_{\boldsymbol{y}[i]\in\mathbb{C}^Q}\|\boldsymbol{y}\|_0\quad subject\ to\quad \boldsymbol{y}[i]=\boldsymbol{\Phi}\boldsymbol{x}[i]\qquad(6)$$

As the signal sources are assumed a priori, the Orthogonal Matching Pursuit (OMP) suit the most appropriate solution for sparse signal reconstruction. This is a non-convex optimization in which the reconstruction is based on $\|\ell\|_0-norm$. The overwhelming accuracy and low failure rate of OMP [16] together with reasonable computational complexity makes it an appropriate sparse reconstruction algorithm in a single snapshot instance.

4. Simulation Results

An L-shaped array antenna with head-to-head orthogonal co-prime subarrays is constructed along the $y-z$ axis as shown in fig. 3. It is assumed that $M=5$ and $N=9$, such that the total elements of the co-prime sub-arrays are $M+N-1=13$. A target is assumed in the far-field region, located at $(45^0,52^0)$, the reflected signals of which impinge on the co-prime L-shaped array antenna. The sparse reconstruction and estimation of DOA by the co-prime L-shaped array is performed by MATLAB simulations in the compressive sensing paradigm. Similar estimation is realized by randomly selecting elements (compressive sensing only) of an L-shaped array. The results are compared in terms of probability of resolution and computational complexity.
Fig. 4. depicts the plot of array factor with angle-of-arrival (or DOA) of the impinging signal for both co-prime structured L-shaped array and ULA based L-shaped array in the compressive sensing paradigm at $SNR=0\ dB$.

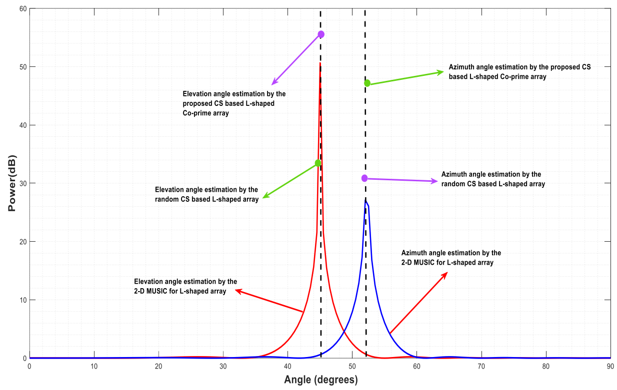

Fig 4. Simulation results of DOA estimation of a single target by an L-shaped array and a co-prime based L-shaped array, both in compressive sensing paradigm.

Fig. 5. Illustrates a comparison plot of probability of resolution by varying the SNR values.

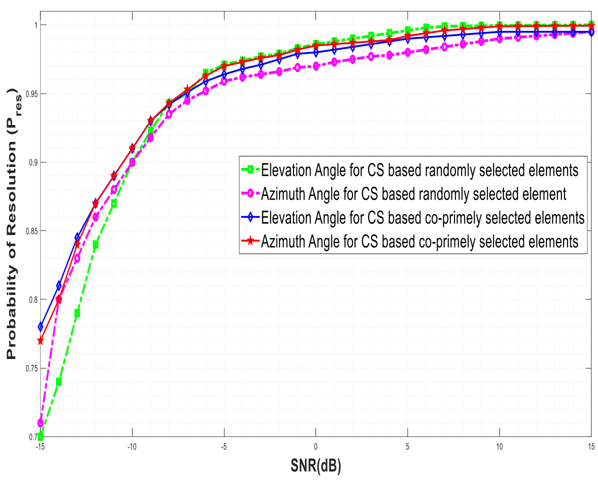

Fig. 5. Variation of P_{res} with SNR values in dB

The probability of resolution is defined as [18]:

$$P_{res} = Prob\left\{\left|\hat{\theta}_i - \theta_i\right| \leq \frac{\Delta\theta}{2}\right\}, i = 1 \dots \dots m \quad (7)$$

where $\Delta\theta = \min\{|\theta_{i1} - \theta_{i2}|, 1 \leq i_1 \leq i_2 \leq m\}$ and $\hat{\theta}_i$, θ_i are the estimated and actual value of the angles respectively.
Comparison of Computational Complexity: Table-1 below describes a comparison based on complexity of computation.

Table 1: Comparison on Computational Complexity

Algorithm	Complexity
Random CS-OMP	$O(K\ M\ N)$
Co-prime based CS-OMP	$O(K^3 + MN^2 - 1)$

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