URSI - RCRS 2022, IIT (Indore), India, 1 - 4 December, 2022

Phase Controlled Electromagnetically Induced Transparency in a Closed Cyclic Lambda Atomic System

Monika Thakran* [1,2], Yashika Aneja[2], and S. K. Dubey [1,2]
(1) Academy of Scientific and Innovative Research (AcSIR), Ghaziabad-201002, India
(2) CSIR-National Physical Laboratory, Dr. K. S. Krishnan Marg, New Delhi-110012

Abstract

In this work, Electromagnetically Induced Transparency (EIT) has been studied in a microwave driven closed cyclic Lambda atomic system. It is observed that in the absence of microwave field a normal transparency window is obtained, but when a microwave field is also applied between hyperfine levels along with the optical fields an amplification and attenuation in EIT signal is observed depending upon the relative phase between fields driving the atomic system. In fact, a new probe field is generated by microwave and coupling field which interfere constructively or destructively with incident probe field leading to either amplification, attenuation or asymmetricity in probe transmission signal. Also, dependency of phase sensitive amplification on ground state decoherence is also studied. This novel, controllable and amplified EIT signal can be potentially used for the transfer of coherent microwave analog or digital message signal over larger distance using optical fibers where amplification is preliminary requirement.

1. Introduction

Electromagnetically induced transparency (EIT) was first observed by Harris et al. in 1989 in his theoretical work employing aspects of coherent population trapping [1]. It has found applications in transferring quantum features between interacting systems and can be used for creating and storing information. Advancement in quantum information processing using quantum electrodynamical superconducting circuits which operates in microwave domain have sparked a great deal of interest in creating interfaces between optical and microwave frequencies [2], [3]. Most recent application of EIT and its driven phenomenon is included in atom-based sensors, radio over fiber atomic antennas and receivers for the purpose of optical communication between microwave and optical fields [4]–[6].

There have been demonstrations of nonlinear optical wave mixing and amplification also using resonant atomic systems [7], [8] but studies that include microwave and optical field interrogation for optical amplification is very few. Also, utilization of phase dependent amplification of optical signal using microwave is still be done for optical communication, where amplification is preferred important than bandwidth.

In this article a theoretical analysis on interaction of rubidium atoms with microwave and optical energies in a closed cyclic Lambda atomic system is presented. Firstly, the optical Bloch equation are formulated for the atomic system and then solved under steady state approximation to study the dynamics of atomic system. Thereafter, output probe absorption as a function of probe detuning is studied by plotting imaginary of density matrix element corresponding to probe absorption. Further, obtained spectra of probe absorption is observed under various conditions; a) coupling and microwave off, b) coupling on and microwave off, c) coupling and microwave on with relative phase varied from 0 to $3\pi/2$. The effect of ground state decoherence on probe absorption is also studied analytically.

2. Theoretical Consideration of Atomic Model

The closed cyclic Lambda atomic model under consideration is shown in figure 1 where level $|1\rangle$ and $|2\rangle$ are ground state hyperfine levels and level $|3\rangle$ is an excited state fine level. Here a probe beam of Rabi frequency Ω_p is applied between atomic states $|1\rangle$ and $|3\rangle$ and a strong coupling beam of Rabi frequency, Ω_c is applied between atomic states $|2\rangle$ and $|3\rangle$. Both these transitions are electrically allowed dipole transitions. In addition to Lambda (Λ) model [9], a microwave field of Rabi frequency Ω_μ is also applied between atomic states $|1\rangle$ and $|2\rangle$ making the system a closed cyclic Lambda (Δ) atomic model.

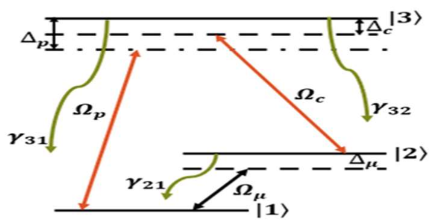

Figure 1. Energy level diagram of Rb^{87} in a closed cyclic Lambda (Δ) atomic model. Ω_i 's are complex Rabi frequencies of interacting fields. γ_i's are the spontaneous decay rates.

In our atomic model, the optical probe and coupling fields are assumed to be travelling through the cell along the z axis with wavenumbers k_p, k_c, initial phases ϕ_p, ϕ_c and angular frequencies ω_p, ω_c respectively. The microwave field is considered to be a standing wave inside a microwave cavity therefore no propagation phase is associated with it. The interaction picture Hamiltonian of this closed cyclic Lambda (Δ) atomic model by applying rotating wave approximation and choosing level $|1\rangle$ as reference level is given by:

$$\hat{H}(r) = -\Delta_p|3\rangle\langle 3| - \left(\Delta_p - \Delta_c\right)|2\rangle\langle 2| - \frac{\Omega_\mu(r)}{2}e^{-i\Delta\Phi}|2\rangle\langle 1| - \frac{\Omega_p(r)}{2}|3\rangle\langle 1| - \frac{\Omega_c(r)}{2}|3\rangle\langle 2| + H.C. \quad (1)$$

where, $H.C.$ denotes Hermitian conjugate, Δ_p and Δ_c are detunings of probe beam and coupling beam respectively, $\Delta\Phi(z) = z\left(k_p - k_c\right) + \phi_\mu$ is the relative phase between three interacting electromagnetic fields and ħ is taken to be 1. The terms $\Omega_i(r)$ are complex Rabi frequencies of the interacting fields given by:

$$\Omega_c(r) = d_{23}\, E_c(\mathrm{r})e^{i\phi_c},$$
$$\Omega_p(r) = d_{13}\, E_p(\mathrm{r})e^{i\phi_p},$$
$$\Omega_\mu(r) = d_{12}\, B_c(\mathrm{r})e^{i\phi_\mu + i(\omega_p - \omega_c - \omega_\mu)t + i(k_p - k_c)z} \quad (2)$$

where, $E_{p,c}$ is the electric field associated with probe and coupling beam and, d_{23} and d_{13} are electric dipole moment corresponding to the transition in which probe and coupling beam applied respectively. Similarly, B_c is the magnetic field associated with microwave field and d_{12} is the corresponding magnetic transition dipole moment.

These Rabi frequencies of the probe, coupling, and microwave fields are assumed to be spatially uniform using the dipole approximation, thus yielding the constants $\Omega_p(r) \equiv \Omega_p$, $\Omega_c(r) \equiv \Omega_c$ and $\Omega_\mu(r) \equiv \Omega_\mu$. In order to have nonlinear interaction between the interaction fields a three-photon resonance condition is satisfied by keeping $\Delta_c = 0$ and $\Delta_p = \Delta_\mu$ in all our calculations, thus making the temporal phase factor

$$\omega_p - \omega_c - \omega_\mu = 0 \quad (3)$$

ensuring time independent Rabi frequencies. The dynamical evolution of the rotated density matrix elements of the atomic system, in the interaction picture is given by the master equation [10]

$$\frac{\partial\rho(z,t)}{\partial t} = -i\left[\hat{H}(z), \rho(z,t)\right] + \sum_{k=1}^{5}\mathcal{L}(\hat{c}_k)\,\rho(z,t) \quad (4)$$

with $\mathcal{L}(\hat{c}_k)$ being the Lindblad super operator

$$\mathcal{L}(\hat{c})\,\rho(z,t) = \hat{c}\,\rho(z,t)\hat{c}^\dagger - \frac{1}{2}\{\,\rho(z,t), \hat{c}^\dagger\hat{c}\,\} \quad (5)$$

acting on operators $\hat{c}_1 = \sqrt{(\bar{n}+1)\gamma_{12}}|1\rangle\langle 2|$, $\hat{c}_2 = \sqrt{\bar{n}\gamma_{12}}|2\rangle\langle 1|$, $\hat{c}_3 = \sqrt{\gamma_{13}}|1\rangle\langle 3|$, $\hat{c}_4 = \sqrt{\gamma_{23}}|2\rangle\langle 3|$, and $\hat{c}_5 = \sqrt{\gamma_c}(|1\rangle\langle 1| - |2\rangle\langle 2|)$, where, $\bar{n}$ is the average number of thermal photons in the bath at temperature T, γ_c is the ground state decoherence modelling the finite quality factor of microwave cavity and γ_{12} and $\gamma_{23} = \gamma_{13}$ represents the spontaneous decay rates from levels $|2\rangle$ and $|3\rangle$ respectively. The density matrix elements obtained using equation 4 and equation 5 are

$$\dot{\rho}_{11} = -i\frac{\Omega_p(r)}{2}\rho_{13} + i\frac{\Omega_p^*(r)}{2}\rho_{31} - i\frac{\Omega_\mu(r)}{2}e^{-i\Delta\Phi}\rho_{12} + i\frac{\Omega_\mu^*(r)}{2}\rho_{21}\,e^{i\Delta\varphi} + (\bar{n}+1)\gamma_{12}\rho_{22} - \bar{n}\gamma_{12}\rho_{11} + \gamma_{13}\rho_{33} \quad (6)$$

$$\dot{\rho}_{22} = -i\frac{\Omega_c(r)}{2}\rho_{23} + i\frac{\Omega_c^*(r)}{2}\rho_{32} - i\frac{\Omega_\mu(r)}{2}e^{-i\Delta\varphi}\rho_{12} + i\frac{\Omega_\mu^*(r)}{2}\rho_{21}\,e^{i\Delta\varphi} - (\bar{n}+1)\gamma_{12}\rho_{22} + \bar{n}\gamma_{12}\rho_{11} + \gamma_{23}\rho_{33} \quad (7)$$

$$\dot{\rho}_{33} = i\frac{\Omega_p(r)}{2}\rho_{13} - i\frac{\Omega_p^*(r)}{2}\rho_{31} + i\frac{\Omega_c(r)}{2}\rho_{23} - i\frac{\Omega_c^*(r)}{2}\rho_{32} - \gamma_{13}\rho_{33} + \gamma_{23}\rho_{33} \quad (8)$$

$$\dot{\rho}_{31} = i\frac{\Omega_p(r)}{2}(\rho_{11} - \rho_{33}) - i\frac{\Omega_\mu(r)}{2}e^{-i\Delta\varphi}\rho_{32} + i\frac{\Omega_c(r)}{2}\rho_{21} - d_1\rho_{31} \quad (9)$$

$$\dot{\rho}_{32} = i\frac{\Omega_p(r)}{2}\rho_{12} + i\frac{\Omega_c(r)}{2}(\rho_{22} - \rho_{33}) - i\frac{\Omega_\mu(r)}{2}e^{i\Delta\varphi}\rho_{31} - d_3\rho_{32} \quad (10)$$

$$\dot{\rho}_{21} = -i\frac{\Omega_p(r)}{2}\rho_{23} + i\frac{\Omega_c^*(r)}{2}\rho_{31} + i\frac{\Omega_\mu(r)}{2}(\rho_{11} - \rho_{22})\,e^{-i\Delta\varphi} - d_2\rho_{21} \quad (11)$$

$\dot{\rho}_{12}$, $\dot{\rho}_{13}$ and $\dot{\rho}_{23}$ are given by the complex conjugate of $\dot{\rho}_{21}$, $\dot{\rho}_{31}$ and $\dot{\rho}_{32}$ respectively. The steady state solution of Density Matrix element 'ρ_{31}' corresponding to probe absorption under weak probe approximation i.e., $\Omega_c \gg \Omega_p > \Omega_{RF}$ Ω_{RF} is given by

$$\rho_{31} = \frac{i d_2\Omega_p(z)}{4d_1d_2 + +d_3[\Omega_c(z)]^2} - \frac{\Omega_c(z)\Omega_{RF}(z)e^{-i\Delta\varphi}}{4d_1d_2 + [\Omega_c(z)]^2} \quad (12)$$

where
$$d_1 = \frac{\bar{n}}{2}\gamma_{12} + \frac{\gamma_{13}}{2} + \frac{\gamma_{23}}{2} + \frac{\gamma_c}{2} - i\Delta_p$$
$$d_2 = (\bar{n} + \tfrac{1}{2})\gamma_{12} + 2\gamma_c - i(\Delta_p - \Delta_c) \quad (13)$$
$$d_3 = (\bar{n} + \frac{1}{2})\gamma_{12} + \frac{\gamma_{23}}{2} + \frac{\gamma_{13}}{2} + \frac{\gamma_c}{2} - i\Delta_c$$

are complex detunings.

3. Results and Discussions

The response of an atomic system to resonant light can be effectively described by its susceptibility to probe field. The imaginary part of susceptibility gives the absorption while the real part gives the dispersion of field in the atomic medium. Since, the optical susceptibility is proportional to the off-diagonal element of density matrix i.e. ρ_{31}, hence we have studied the variation of imaginary part of ρ_{31} as a function probe laser detuning to observe the probe absorption. When a strong resonant coupling beam is applied along with the probe beam (without any microwave field), transparency in the probe absorption spectra is obtained as shown in figure 2. Under this condition, only first term of equation 12 contributes, while second term becomes zero. The results are similar to Electromagnetically Induced Transparency (EIT) observed in Lambda atomic Model [9].

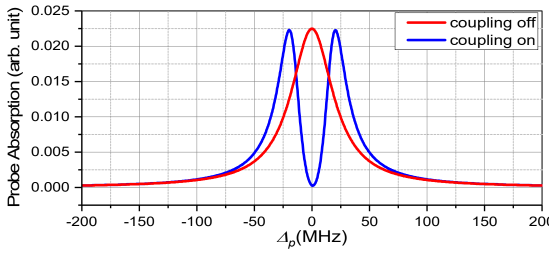

Figure 2. Probe Absorption as a function of probe detuning. Red curve shows the case when is coupling off and Ω_p=1 MHz, $\gamma_{23} = \gamma_{13}$ = 2π.6 MHz, γ_{12} = 3π kHz and γ_c= 0 applied. Blue Curve shows the case when coupling of Ω_c= 40 MHz is also applied.

Next, when microwave field (Ω_μ= 0.8 MHz) is applied, the transitions in-between the hyperfine ground states is enabled. The relative phase between probe, coupling and applied microwave signal is varied and change in absorption & EIT is analyzed. The significant change has been observed at different values of relative phase between interacting electromagnetic fields as depicted in figure 3. Here, the probe rabi frequency, control rabi frequency and other parameters are kept same as before (as in figure 2).

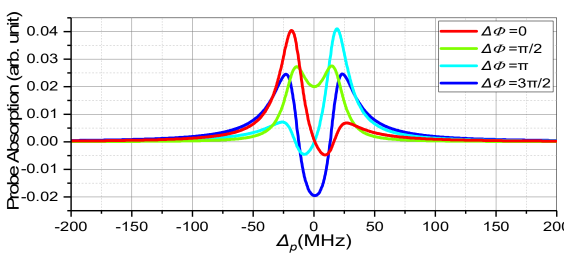

Figure 3. Probe Absorption as a function of probe detuning (Δ_p) at various values of relative phase ($\Delta\Phi$) between probe, coupling and microwave fields. (a) $\Delta\Phi = 0$, shown by red color; (b) $\Delta\Phi = \frac{\pi}{2}$, shown by green color; (c) $\Delta\Phi = \pi$, shown by sky blue color; and (d) $\Delta\Phi = \frac{3\pi}{2}$, shown by blue color.

The absorption peaks obtained at $\Delta\Phi = 0$ and at $\Delta\Phi = \pi$ are mirror images of each other with absorption is enhanced in the red detuned region (i.e., left side of resonance frequency), while reduced in the blue detuned region (i.e., right of resonance frequency) for $\Delta\Phi = 0$ and vice versa in case of $\Delta\Phi = \pi$. The results are similar to the one reported by Manjappa et. al [10].

It is worth noting that the probe transparency characteristic of EIT is further amplified (negative absorption) at $\Delta\Phi = \frac{3\pi}{2}$ and attenuated at $\Delta\Phi = \frac{\pi}{2}$ (figure 3), in comparison to the transparency window when no microwave field applied (figure 2). This establishes that the probe absorption and corresponding transparency window can be controlled by controlling the relative phase between three electromagnetic fields.

The two terms in equation 12 represent the Linear and Hybrid absorption. Figure 4 shows contribution of both these absorption types to the total absorption as a function of probe detuning (Δ_p) at various values of relative phase of interacting fields. The hybrid absorption arises from the second order nonlinear susceptibility resulting from the combined effect of magnetic and electric dipole transition induced by microwave and optical coupling field that contributes to new optical probe field generation [11].

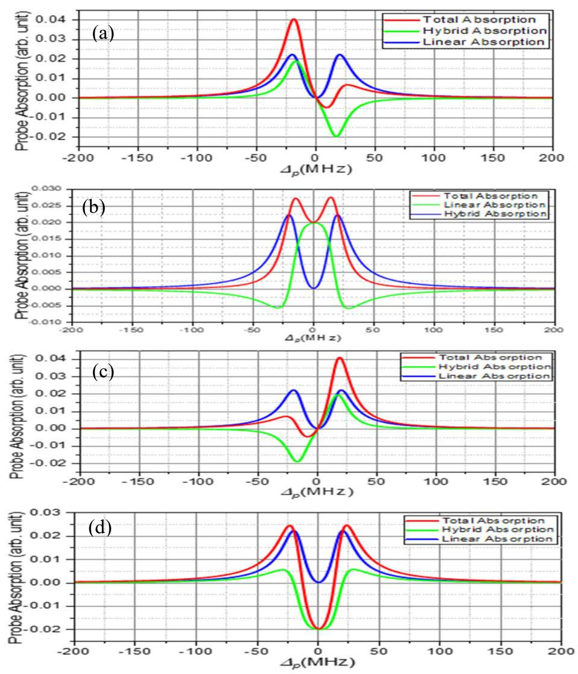

Figure 4. Interference of Linear and Hybrid absorption as a function of probe detuning (Δ_p) at various values of relative phase ($\Delta\Phi$); (a) $\Delta\Phi = 0$; $\Delta\Phi = \frac{\pi}{2}$; (c) $\Delta\Phi = \pi$; and (d) $\Delta\Phi = \frac{3\pi}{2}$.

It can be observed, that at $\Delta\Phi = \frac{\pi}{2}$, complete absorption of generated probe field takes place in the EIT region of incident probe field thereby decreasing the amplitude of output probe transparency signal (figure 4(b)). On the other hand, at $\Delta\Phi = \frac{3\pi}{2}$, generated probe signal and applied probe signal become transparent in same detuning range, resulting in constructive interference and hence amplifying the amplitude of EIT peak (figure 4(d)).
Further, generated probe field shows absorption in the red detuned region and complete transmission in blue detuned region for a particular frequency range near resonance for $\Delta\Phi = 0$ and vice-versa happens for $\Delta\Phi = \pi$ following a total amplification and de-amplification (figure 4a and 4c).

Thus, depending upon the relative phase input probe and generated probe can have constructive or destructive interference resulting in either amplification or de-amplification in the EIT signal.

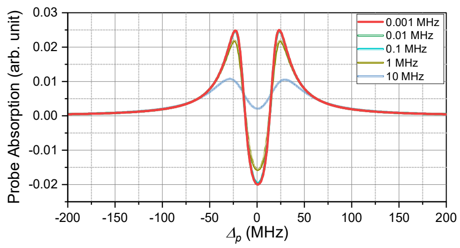

Figure 5. Variation of Probe transparency window with ground state decoherence, γ_c at $\Delta\Phi = \frac{3\pi}{2}$.

Further, we have also shown the effect of ground state decoherence on the probe transparency signal at $\Delta\Phi = \frac{3\pi}{2}$. It can be observed from figure 5, that the amplification of probe transmission signal is maximum for all value of $\gamma_c < \Omega_{\mu,p}$, But for $\gamma_c > \Omega_{\mu,p}$ attenuation takes place.

4. Conclusion

The behavior of EIT to microwave field and its phase with respect to other fields is explored in a closed cyclic Lambda atomic model. We have found that amplification and de-amplification of probe transmission signal can be achieved depending upon the phase difference of the applied fields (probe, coupling and applied RF), when all the three electromagnetic fields are in resonance. Another important criterion to achieve have phase dependent amplification is that γ_c (spontaneous decay for coupling field) should be close to zero. These findings can have potential application in optical communication using radio over fiber employing coherent transfer and amplification of analog and digital microwave signals to optical frequencies.

5. Acknowledgements

The work is carried out under CSIR-FTT scheme sponsored project entitled "Broadband Rydberg Atom Based Quantum Sensing". This work is also supported in part by UGC-NET fellowship by Union Grants Commission. Authors are also thankful to Director, CSIR-NPL for providing a working environment for research and other facilities. The authors would like to thank Mr. Asheesh Kumar Sharma for his kind reading of the manuscript and fruitful technical discussions.

6. References

[1] S. E. Harris, J. E. Field, and A. Imamoğlu, "Nonlinear optical processes using electromagnetically induced transparency," *Phys. Rev. Lett.*, vol. 64, no. 10, pp. 1107–1110, 1990, doi: 10.1103/PhysRevLett.64.1107.

[2] Z. L. Xiang, S. Ashhab, J. Q. You, and F. Nori, "Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems," *Rev. Mod. Phys.*, vol. 85, no. 2, pp. 623–653, 2013, doi: 10.1103/RevModPhys.85.623.

[3] N. J. Lambert, A. Rueda, F. Sedlmeir, and H. G. L. Schwefel, "Coherent Conversion Between Microwave and Optical Photons—An Overview of Physical Implementations," *Adv. Quantum Technol.*, vol. 3, no. 1, pp. 1–15, 2020, doi: 10.1002/qute.201900077.

[4] C. L. Holloway, M. T. Simons, J. A. Gordon, and D. Novotny, "Detecting and receiving phase-modulated signals with a rydberg atom-based receiver," *IEEE Antennas Wirel. Propag. Lett.*, vol. 18, no. 9, pp. 1853–1857, 2019, doi: 10.1109/LAWP.2019.2931450.

[5] A. Tretiakov, C. A. Potts, T. S. Lee, M. J. Thiessen, J. P. Davis, and L. J. Leblanc, "Atomic microwave-to-optical signal transduction via magnetic-field coupling in a resonant microwave cavity," *Appl. Phys. Lett.*, vol. 116, no. 16, pp. 1–6, 2020, doi: 10.1063/1.5144616.

[6] D. H. Meyer, K. C. Cox, F. K. Fatemi, and P. D. Kunz, "Digital communication with Rydberg atoms and amplitude-modulated microwave fields," *Appl. Phys. Lett.*, vol. 112, no. 21, 2018, doi: 10.1063/1.5028357.

[7] F. Wen, H. Zheng, X. Xue, H. Chen, J. Song, and Y. Zhang, "Electromagnetically induced transparency-assisted four-wave mixing process in the diamond-type four-level atomic system," *Opt. Mater. (Amst).*, vol. 37, no. C, pp. 724–726, 2014, doi: 10.1016/j.optmat.2014.08.020.

[8] M. D. Lukin, P. R. Hemmer, M. Löffler, and M. O. Scully, "Resonant enhancement of parametric processes via radiative interference and induced coherence," *Phys. Rev. Lett.*, vol. 81, no. 13, pp. 2675–2678, 1998, doi: 10.1103/PhysRevLett.81.2675.

[9] S. S. Nande, M. Thakran, H. S. Rawat, and S. K. Dubey, "Study of the Electromagnetic-Induced Transparency and its Dependence on Probe Decay for Cascade, Lambda, and Vee Models," *Mapan - J. Metrol. Soc. India*, vol. 37, no. 2, pp. 347–355, 2022, doi: 10.1007/s12647-021-00510-9.

[10] M. Manjappa, S. S. Undurti, A. Karigowda, A. Narayanan, and B. C. Sanders, "Effects of temperature and ground-state coherence decay on enhancement and amplification in a Δ atomic system," *Phys. Rev. A - At. Mol. Opt. Phys.*, vol. 90, no. 4, pp. 1–6, 2014, doi: 10.1103/PhysRevA.90.043859.

[11] K. V. Adwaith, A. Karigowda, C. Manwatkar, F. Bretenaker, and A. Narayanan, "Coherent microwave-to-optical conversion by three-wave mixing in a room temperature atomic system," *Opt. Lett.*, vol. 44, no. 1, p. 33, 2019, doi: 10.1364/ol.44.000033.