Forced KdV equation in magnetorotating electron-positron-ion plasmas

1st Geetika Slathia
Department of Physics
Guru Nanak Dev University
Amritsar-143005, India
gslathia93@gmail.com

2nd Rajneet Kaur
Department of Physics
Guru Nanak Dev University
Amritsar-143005, India
rajneetkaur909@gmail.com

3rd Sunidhi Singla
Department of Physics
Guru Nanak Dev University
Amritsar-143005, India
singla.sunidhi94@gmail.com

4th Manveet Kaur
Department of Physics
Guru Nanak Dev University
Amritsar-143005, India
manveetsandhu7@gmail.com

5th N. S. Saini
Department of Physics
Guru Nanak Dev University
Amritsar-143005, India
nssaini@yahoo.com

***Abstract*—In this investigation, the study of electron acoustic solitary waves (EASWs) in an electron-positron-ion (e-p-i) magnetoplasma comprising of inertial cold electrons, superthermally distributed hot positrons and inertialess electrons have been illustrated in the presence of stationary positive ions. The forced Korteweg-de Vries (fKdV) equation is derived by employing the reductive perturbation method to study the characteristic properties of electron acoustic solitary structures. The effect of various physical plasma parameters namely, the temperature ratio, the strength of the magnetic field, superthermality and density of charged particles have been quantitatively analysed. The combined impact of these parameters greatly enhance the characteristics of the solitary structures. It is remarked that the findings of the present investigation maybe of great importance to explore the underlying properties of various types of nonlinear structures in auroral regions, pulsars, Van Allen radiation belts etc.**

***Index Terms*—Electron acoustic, Superthermal, Forced KdV equation, E-p-i plasma**

I. Introduction

Over the last many years, the electron acoustic waves have been an intriguing topic of research due to its wide range of applications in both space and astrophysical environments and laboratory generated plasma [1]. The occurrence of electron acoustic waves have been vividly observed and reported in various regions such as the Van Allen belts [2], higher altitude polar regions, the region surrounding the auroras etc. These waves mainly occur due to the temperature difference between the electron components. Moreover, the presence of positrons significantly enhances the generation of electron acoustic waves. Many studies have been put forward from time to time by numerous researchers which confirms the existence of electron acoustic waves in various space plasmas [3]–[6]. The study of electron acoustic solitary waves in a superthermal dissipative plasma comprising of both hot and cold electrons and stationary ions has been examined by [5]. They studied the effect of damping on the dynamics of electron acoustic solitons and also the configuration of the multisoliton structures under specific conditions. Singh and Saini [6] investigated the nonlinear properties of both monotonic and oscillatory electron acoustic shocks in a superthermal magnetoplasma. They observed that the fluid viscosity and the anisotropic pressure have emphatic effect on the characteristics of shock waves. Various onboard satellite observations confirms the existence of high energy tail particles in various space plasma such as the solar wind, the magnetospheres of various planets etc [7], [8]. Numerous studies both theoretical as well as experimental have been reported from time to time in view of the non-Maxwellian distribution to witness the formation of different kinds of nonlinear wave structures in various kinds of plasma environments [6], [9], [10]. It is observed that with the inclusion of an external periodic perturbation, different kinds of nonlinear wave phenomena occur in the given system. Various studies have been reported by the authors which illustrate the formation of nonlinear structures in the presence of an external force [11]–[13]. Rustam et al. [11] studied the propagation properties of electron acoustic solitary waves in an unmagnetised superthermal plasma in the presence of periodic force. They obtained an analytical solution in the presence of an external perturbation for the electron acoustic solitary waves. The study of ion-acoustic waves in an electron-pair-ion plasma was examined by Roy and Sahu [13]. They carried out the numerical simulation to study the formation of advancing solitons. They also studied the quasiperiodicity of the system and the significant effects of the quantum diffraction on the system. According to our knowledge, no such investigation of EASWs has been examined in an e-p-i magnetoplasma which motivates us to carry out this investigation and study its characteristic dynamics. The main aim of our present investigation is to study the dynamics of electron acoustic solitary waves in the four component plasma consisting of cold electrons as fluid, Kappa distributed hot electrons and hot positrons and uniform stationary ions in the presence of an external force. The layout of this paper is structured as follows: The fluid model equations governing the dynamics of electron acoustic solitary waves are presented in Section II.

Section III presents the derivation of the fKdV equation and its analytical solution. In Section IV, numerical analysis and discussion is illustrated. The overall results and conclusions are summarized in Section V.

II. BASIC FLUID EQUATIONS

We consider an electron-positron-ion (e-p-i) magnetoplasma comprising of cold electrons, superthermally distributed hot positrons and inertialess electrons in the presence of stationary positive ions to study the characteristics of nonlinear electron acoustic solitary waves. The wave propagates in the x-z plane and the magnetic field is along the z- axis. The plasma is rotating with frequency Ω about the axis of rotation making an angle θ with the magnetic field direction. The dynamics of the EASWs is governed by the following normalised set of the fluid equations (continuity, momentum and Poisson equations) [14]:

$$\frac{\partial N_c}{\partial T} + \frac{\partial (N_c U_{cx})}{\partial X} + \frac{\partial (N_c U_{cz})}{\partial Z} = 0 \tag{1}$$

$$\frac{\partial U_{cx}}{\partial T} + U_{cx}\frac{\partial U_{cx}}{\partial X} + U_{cz}\frac{\partial U_{cx}}{\partial Z} = \frac{\partial \phi}{\partial X} - \Omega U_{cy} + 2\Omega_0 cos\theta U_{cy} - \frac{\sigma_\perp}{N_c}\frac{\partial N_c}{\partial X} = 0 \tag{2}$$

$$\frac{\partial U_{cy}}{\partial T} + U_{cx}\frac{\partial U_{cy}}{\partial X} + U_{cz}\frac{\partial U_{cy}}{\partial Z} = \Omega U_{cx} + 2\Omega_0 sin\theta U_{cz} - 2\Omega_0 cos\theta U_{cx} \tag{3}$$

$$\frac{\partial U_{cz}}{\partial T} + U_{cx}\frac{\partial U_{cz}}{\partial X} + U_{cz}\frac{\partial U_{cz}}{\partial Z} = \frac{\partial \phi}{\partial Z} - 2\Omega_0 sin\theta U_{cy} - 3\sigma_\parallel N_c\frac{\partial N_c}{\partial Z} = 0 \tag{4}$$

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Z^2}\right)\phi = 1 - \mu + N_c\sigma - \delta + \phi(c_1 + \mu\nu d_1) + \frac{\phi^2}{2}(c_2 - \mu\nu^2 d_2) + S(X,T) \tag{5}$$

At equilibrium, the charge neutrality is given as

$$N_{eh0} + N_{c0} = N_{i0} + N_{p0}, \tag{6}$$

where N_{j0}, (for $j = eh, c, i, p$) are the unperturbed number densities of hot electrons, cold electrons, ions and hot positrons, respectively. For the normalisation of the above equations see [6]. The physical parameters in the Poisson's equation are stated as $\mu = \frac{N_{p0}}{N_{eh0}}$, $\sigma = \frac{N_{c0}}{N_{eh0}}$, $\delta = \frac{N_{i0}}{N_{eh0}}$, $\nu = \frac{T_{eh}}{T_p}$. Also, $\sigma_\perp = \frac{T_{c\perp}}{T_{eh}}$ and $\sigma_\parallel = \frac{T_{c\parallel}}{T_{eh}}$.
The normalised expressions of densities of superthermal electrons and hot positrons are given as,

$$N_{eh} = 1 + c_1\phi + C_2\frac{\phi^2}{2} + ... \tag{7}$$

where, $c_1 = \frac{(\kappa_{eh}-\frac{1}{2})}{(\kappa_{eh}-\frac{3}{2})}$ and $c_2 = \frac{(\kappa_{eh}^2-\frac{1}{4})}{(\kappa_{eh}-\frac{3}{2})^2}$.

$$N_p = 1 - \nu d_1\phi + \nu^2 d_2\frac{\phi^2}{2} + ... \tag{8}$$

where, $d_1 = \frac{(\kappa_p-\frac{1}{2})}{(\kappa_p-\frac{3}{2})}$ and $d_2 = \frac{(\kappa_p^2-\frac{1}{4})}{(\kappa_p-\frac{3}{2})^2}$.

III. Derivation of the Forced KdV equation and its solution

In order to study the dynamics of the electron acoustic solitary waves, the forced KdV equation is derived in the framework of reductive perturbation method. The independent stretching coordinates ξ and τ are used as:

$$\xi = \epsilon^{\frac{1}{2}}(l_x X + l_z - \lambda T), \tag{9}$$

$$\tau = \epsilon^{\frac{3}{2}} T \tag{10}$$

where l_x and l_z are the direction cosines of the wave vector and λ is the phase velocity of the EASWs and ϵ is a smallness parameter ($0 \ll \epsilon \ll 1$) which signifies weakness in the nonlinearity of the system. The dependent variables are expanded as:

$$N_c = 1 + \epsilon N_c^{(1)} + \epsilon^2 N_c^{(2)} + ... \tag{11}$$

$$U_{(cx)} = \epsilon^2 U_{cx}^{(1)} + \epsilon^3 U_{cx}^{(2)} + ... \tag{12}$$

$$U_{(cy)} = \epsilon^{\frac{3}{2}} U_{cy}^{(1)} + \epsilon^{\frac{5}{2}} U_{cy}^{(2)} + ... \tag{13}$$

$$U_{(cz)} = \epsilon U_{cz}^{(1)} + \epsilon^2 U_{cz}^{(2)} + ... \tag{14}$$

$$\phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + ... \tag{15}$$

$$S(x,t) = \epsilon^2 S_2(x,t) + ... \tag{16}$$

We obtain the following set of first order equations and the charge neutrality condition is $1 + \sigma = \delta + \mu$

$$N_c^{(1)} = -\phi^{(1)}\left(\frac{c_1 + \mu\nu d_1}{\sigma}\right) \tag{17}$$

$$U_{cy}^{(1)} = \left(\frac{l_x}{\Omega - 2\Omega_0 cos\theta}\right)\frac{\partial \phi^{(1)}}{\partial \xi} - \left(\frac{\sigma_\perp l_x}{\Omega - 2\Omega_0 cos\theta}\right)\frac{\partial N_c^{(1)}}{\partial \xi} \tag{18}$$

$$\frac{\partial U_{cz}^{(1)}}{\partial \xi} = \left(\frac{-l_z}{\lambda}\right)\frac{\partial \phi^{(1)}}{\partial \xi} + \frac{2\Omega_0 sin\theta}{\lambda}U_{cy}^{(1)} + 3\sigma_\parallel\frac{-l_z}{\lambda}\frac{\partial N_c^{(1)}}{\partial \xi} \tag{19}$$

From the first order equations, we obtain the following dispersion relation as

$$\lambda^2 = \lambda_c^2 + \left(3\sigma_\parallel l_z^2 - \frac{2\sigma_\perp l_x l_z \Omega_0 sin\theta}{\Omega - 2\Omega_0 cos\theta}\right) \tag{20}$$

where, $\lambda_c^2 = \frac{1}{H^2}\left(l_z^2 - \frac{2 l_x l_z \Omega_0 sin\theta}{\Omega - 2\Omega_0 cos\theta}\right)$ and $H = \frac{c_1 + \mu\nu d_1}{\sigma}$.
However, it is evitable from the above equation that the phase velocity λ depends on the various physical plasma parameters, viz., Ω, $\sigma_\perp$ and $\sigma_\parallel$. So, it is important to study their variation numerically.

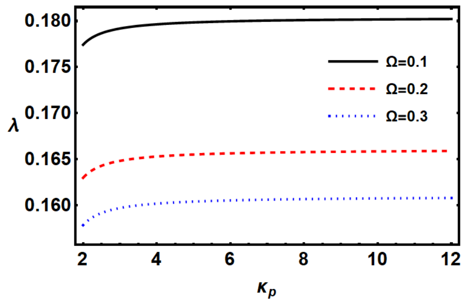

Fig. 1. The variation of the phase velocity (λ) of EASWs versus superthermality index of hot positrons (κ_p) for different values of magnetic field strength (Ω).

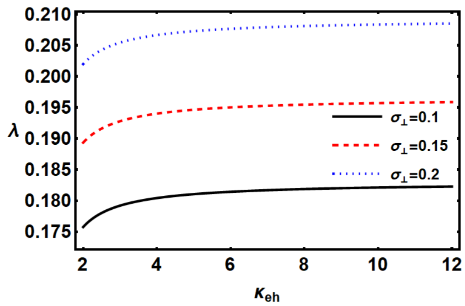

Fig. 2. The variation of the phase velocity (λ) of EASWs versus superthermality index of hot electrons (κ_{eh}) for different values of perpendicular temperature ratio (via $\sigma_\perp = \frac{T_{c\perp}}{T_{eh}}$).

After eliminating the higher order quantities from the second order equations, we obtain the following fKdV equation for the EASWs in an e-p-i plasma:

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = C\frac{\partial S_2(X,T)}{\partial \xi}. \tag{21}$$

Where we set $\phi^{(1)} = \phi$ for simplication. The nonlinear coefficient $A = \frac{1}{2\lambda\sigma H}\left[-\lambda_c^2\alpha - \sigma H^2\left(3\lambda^2 + 3\sigma_\parallel l_z^2 + \frac{2\sigma_\perp l_x l_x \Omega_0 sin\theta}{\Omega - 2\Omega_0 cos\theta}\right)\right]$, dispersion coefficient $B = \frac{1}{2\lambda\sigma H}\left[\lambda_c^2 + \sigma\lambda^2 \frac{\lambda^2(1+\sigma_\perp H)}{(\Omega-2\Omega_0 cos\theta)^2}\left(l_x^2 + \frac{2\sigma_\perp l_x l_x \Omega_0 sin\theta}{\Omega - 2\Omega_0 cos\theta}\right)\right]$ and $C = \frac{\lambda_c^2}{2\lambda\sigma H}$.

Now, by employing single variable transformation $\zeta = \xi - \lambda\tau$ in Eq. (21), we obtain the following analytical solution of the fKdV equation [15], [16];

$$\phi = Psech^2\left(\sqrt{\frac{P}{12B}}\left(\zeta - \frac{P - 3ACQ}{3}\tau\right)\right). \tag{22}$$

with the forcing term

$$S = PQsech^2\left(\sqrt{\frac{P}{12B}}\left(\zeta - \frac{P - 3ACQ}{3}\tau\right)\right). \tag{23}$$

where P and Q are two arbitrary constants.

In this section, the numerical analysis has been carried out to study the characteristics of EAWSs in an e-p-i plasma. The dependence of the different plasma parameters such

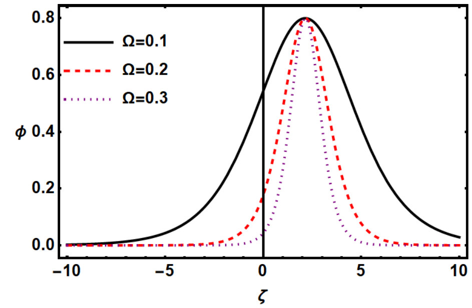

Fig. 3. The variation of the positive polarity EA-solitons profile (ϕ) versus ζ for different values of magnetic field strength (Ω).

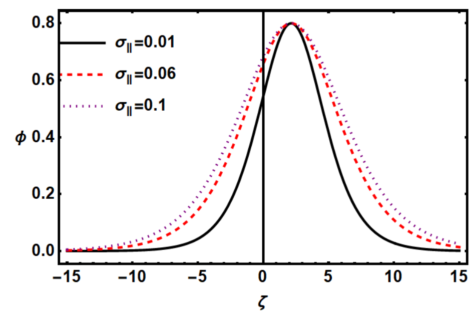

Fig. 4. The variation of the positive polarity EA-solitons profile (ϕ) versus ζ for different values of parallel temperature ratio (via $\sigma_\parallel = \frac{T_{c\parallel}}{T_{eh}}$).

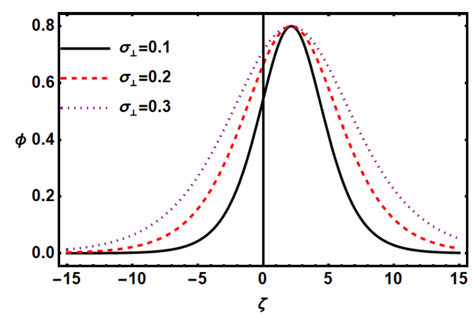

Fig. 5. The variation of the positive polarity EA-solitons profile (ϕ) versus ζ for different values of perpendicular temperature ratio (via $\sigma_\perp = \frac{T_{c\perp}}{T_{eh}}$).

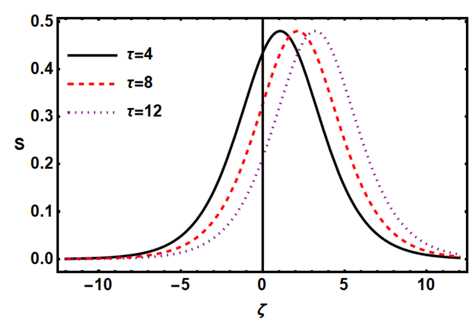

Fig. 6. The variation of the forcing term (S) versus ζ for different values of τ.

as superthermality indices of charged species (κ_{eh}, κ_p), temperature ratios ($\sigma_{\parallel}$, $\sigma_{\perp}$), strength of the magnetic field (Ω) etc. on the EASWs has been illustrated. The relevant plasma parameters required for the numerical analysis are observed from the various data observed in the space and astrophysical environments [14]. It is observed that the dynamics of the EASWs are significantly altered under the influence of different physical plasma parameters. Figure 1 illustrates the variation of phase velocity (λ) of EASWs with superthermal index of hot positrons (via κ_p) for different values of magnetic field strength(via Ω). It is observed that the phase velocity of EASWs increases gradually with increase in the values of κ_p but drecreases as the value of Ω is increased. Figure 2 illustrates the variation of phase velocity (λ) of EASWs with superthermal index of hot electrons (via κ_{eh}) for different values of perpendicular temperature ratio (via$\sigma_{\perp} = \frac{T_{c\perp}}{T_{eh}}$). The phase velocity ($\lambda$) is increased with an increase in the values of κ_{eh} and $\sigma_{\parallel}$. It is observed from the variations that the dispersion properties are significantly altered by the magnetorotating effects which modifies the characteristics of EASWs. Figure 3 depicts the variation of the positive potential EASWs profile ϕ versus ζ for different values of magnetic field strength(via Ω). It is noticed that with increase in the values of Ω the width of the soliton decreases but the amplitude of the soliton remains unchanged. In Figure 4, the variation of the positive potential EASWs profile ϕ versus ζ for different values of parallel temperature ratio (via$\sigma_{\parallel} = \frac{T_{c\parallel}}{T_{eh}}$) is illustrated. It is observed that with increase in the values of $\sigma_{\parallel}$ the amplitude of the soliton remains unchanged but the width of the soliton increases. In Figure 5, the variation of the positive potential EASWs profile ϕ versus ζ for different values of perpendicular temperature ratio (via$\sigma_{\perp} = \frac{T_{c\perp}}{T_{eh}}$) is depicted. It is seen that as the values of $\sigma_{\perp}$ is increased, the amplitude of the EASWs remains unchanged but the width of the solitons is significantly enhanced. Figure 6, shows the variation of the forcing term S versus ζ for different values of τ. It is found that as the values of τ is increased, the amplitude and the width of the EASWs remains unaltered and they propagate slowly with time. Hence, it is stressed that the various plasma parameters have significantly influenced the nonlinearity as well as dispersion effects that subsequently modified the characteristics of EASWs in the given plasma environment.

IV. Conclusions

In this investigation, we have examined the salient features of EASWs in an e-p-i magnetoplasma comprising of cold electrons, superthermally distributed hot positrons and inertialess electrons in the presence of stationary positive ions. By employing the reductive perturbation method, the forced Korteweg-de Vries equation is obtained for EASWs. The analytical solution of fKdV equation is obtained to explore the properties of EASWs under the influence of external periodic force. It is observed that combined effects of various plasma parameters namely, the temperature ratio, the strength of the magnetic field, superthermality and density of charged particles significantly alter the characteristic properties of different EASWs. The amplitude of EASWs enervates with the rise in the strength of the magnetic field while the amplitude of the EASWs excels with rise in the value of temperature ratio. The findings of the present investigation may be of great help to understand the basic features of EASWs in various space plasmas such as pulsars, auroras, Van Allen radiation belts etc.

ACKNOWLEGEMENT

Authors gratefully acknowledge the support for this research work from Department of Science and Technology, Govt. of India, New Delhi under DST-SERB project No. CRG/2019/003988.

References

[1] R. L. Mace, S. Baboolal, R. Bharuthram and M. A. Hellberg, “Arbitrary amplitude electron-acoustic solitons in a two-electron-component plasma,” Journal of Plasma Physics, **45**(1), 323-338 (1991).

[2] S. A. Voronov, A. M. Galper, V. G. Kirilov-Ugryumov, S. V. Koldashov and A. V. Popov, “Charge composition of the high-energy electrons and positrons in the Van Allen radiation belts,” JETP Letters, **43**, 306–307 (1986).

[3] J. N. Han, W. S. Duan, J. X. Li, Y. L. He, J. H. Luo, Y. G. Nan et al., “Study of nonlinear electron-acoustic solitary and shock waves in a dissipative, nonplanar space plasma with superthermal hot electrons,” Physics of Plasmas, **21**, 1–10 (1991).

[4] G. S. Lakhina, S. V. Singh and A. Kakad, “Ion- and electron-acoustic solitons and double layers in multi-component space plasmas,” Advances in Space Research, **47**, 1558-1567 (2011).

[5] S. Sultana and I. Kourakis, “Electron-scale dissipative electrostatic solitons in multi-species plasmas,” Physics of Plasmas, **22**, 102302(1-8) (2015).

[6] K. Singh and N. S. Saini, “Effect of anisotropic pressure on electron acoustic oscillatory and monotonic shocks in superthermal Magnetoplasma,” Radio Science Journal, **54** (12), 1192-1203 (2019).

[7] M. P. Leubner and N. Schupfer, J. Geophys. Res. **105**, 27387, (2000).

[8] V. Formisano, G. Moreno, and F. Palmiotto and P. C. Hedgecock, “Solar wind interaction with the Earth’s magnetic field: 1. Magnetosheath,” J. Geophys. Res. **78**, 3714, (1973).

[9] N. S. Saini and I. Kourakis, “Dust-acoustic wave modulation in the presence of superthermal ions,” Phys. Plasmas **15**, 123701 (2008).

[10] G. Slathia, K. Singh and N. S. Saini, “Dust acoustic dressed solitons in Jovian magnetosphere,” IEEE Trans. Plasma Sci., **50** (6), 1723-1731 (2022).

[11] R. Ali, A. Saha, P. Chatterjee, “Analytical electron acoustic solitary wave solution for the forced KdV equation in superthermal plasmas,” Physics of Plasmas, **24**(12), 122106(1–9) (2017).

[12] S. Chandra, “Analytical and Simulation Studies of Forced KdV Solitary Structures in a Two-Component Plasma,” Journal of the Korean Physical Society, **76**(6), 469-478 (2020).

[13] D. Roy and B. Sahu, “Excitation of electrostatic plasma waves by a moving charged source in a quantum plasma,” Advances in Space Research, **67**(3), 1039-1048 (2021).

[14] K. Jilani, A. M. Mirza and T. A. Khan, “Electron acoustic solitons inmagneto-rotating electron-positron-ion plasma with nonthermal electrons and positrons,” Astrophysics and Space Science, **355**, 255–263 (2015).

[15] E. Pelinovsky, A. C. Yalciner et al. (Eds.), “Submarine Landslides and Tsunamis,” Kluwer Academic Publisher, Netherlads (2003).

[16] Z. Jun-Xiao and G. Bo-Ling, “Analytic solutions to forced KdV equation,” Commun. Theor. Phys. **52**, 279 (2009).