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Ion acoustic dressed shocks in Earth's magnetosphere

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Abstract

The study of ion acoustic shocks (IAShs) in a plasma with warm inertial ions, Cairns-Tsallis (CT) distributed hot electrons, and an electron beam is presented in this work. The Burgers equation is obtained using the reductive perturbation approach. To arrive at a solution, the Tanh approach is used to examine the ion acoustic shock structures. Additionally, the inhomogeneous Burgers-type equation is obtained by taking higher order effects into account, and its solution leads to the creation of humped-type IA shocks. The combined effects of electron beam and other plasma parameters on the properties of different kinds of IAShs have been analyzed. The findings of this investigation may be useful for studying dynamics of IAShs in space as well as astrophysical plasma environments.

1. Introduction

Different investigations in various types of environments have been carried out to study diverse kinds of nonlinear structures by theoretical as well as experimental methods. The well known solitary structures are emerged as a result of balance between nonlinear and dispersion effects. In addition, due to presence of high density particles, supernovae, change in magnetic field, high energy particles distribution in the medium and abrupt variation in properties of medium, the shock structures are formed by dominating the dissipation effects. Thus these three effects viz., nonlinearity, dispersion and dissipation play an vital role in formation of different types of nonlinear structures. The higher order contribution of these effects helps to eradicate inconsistency between theoretical and experimental studies to understand the clear picture of these nonlinear structures. New sorts of structures, referred to as “dressed solitons” and “dressed shocks,” are seen with significant contributions from higher order effects. In order to understand the relevance of such contributions for the evolution of various types of nonlinear structures in plasmas, it is necessary to additionally take into account the higher-order contributions of nonlinear, dispersion, and dissipation effects. To understand the characteristics of shocks with more precision numerous researchers [1]–[5] have introduced the role of higher order of these effects in different kinds of plasmas. El-Taibany et al. [1] have studied the role of higher order corrections to study the dust ion acoustic shock structures in dense plasma. The properties of humped shocks by including the higher order effects examined by El-Borie and Atteya [3] in a strongly coupled plasma. Kaur et al. [5] studied the heavy and light nucleus acoustic dressed shock structures in white dwarfs. The satellite observations of various regions [6]–[9] explore that due to presence of energetic particles viz.electron/ion beams, the characteristics of electrostatic structures are significantly modified. The collective influence of wave particle interactions and other forces present in environments are responsible for the production of these non-Maxwellian energetic particles. Number of investigations confirmed that these energetic particles are well explained with help of hybrid form of the non-Maxwellian distributions. The q-nonextensive [10] and Cairns nonthermal [11] distributions are collectively in hybrid form are proposed by Tribeche et al. [12]. This hybrid form is well known as Cairns-Tsallis (CT) distribution [12]. It asserts to provide more parametric versatility while modelling nonthermal plasmas. The CT distribution is used to the nonlinear structures when nonthermal nonextensive electrons are present in given plasma atmosphere [13]–[16]. Farooq et. al [14] analyzed the CT distribution for solitary waves in magneto rotating epi plasma. Wang and Du [16] studied the effect of nonextensive and nonthermal parameters on ion acoustic solitary waves in a four component CT distributed plasma. Owing to have importance of higher order contribution and CT distribution in plasma embedded with electron beam, we have studied the characteristics of IAShs in a multicomponent plasma. The manuscript is organized as follows: Section 2 describes the basic set of fluid equations. Section 3 depicts the derivation of Burgers and inhomogeneous Burgers-type equation. Section 4 is devoted to the stationary solutions of Burgers and inhomogeneous Burgers-type equations. Section 5 illustrates the parametric analysis and conclusions are highlighted in section 6.

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2. Fluid Model

To examine the influence of various plasma parameters such as CT distributed electrons, electron beam on the propagation characteristics of first order and higher order IA shocks has been considered. The following set of the normalized equations (continuity and momentum) governs the dynamics of the IAShs:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x} - 3\sigma_i n_i \frac{\partial n_i}{\partial x} + \eta \frac{\partial^2 v_i}{\partial x^2}, \quad (2)$$

$$\frac{\partial n_b}{\partial t} + \frac{\partial (n_b v_b)}{\partial x} = 0, \quad (3)$$

$$\frac{\partial v_b}{\partial t} + v_b \frac{\partial v_b}{\partial x} = \alpha\sigma_b \frac{\partial \phi}{\partial x} - 3\alpha\sigma_b {\delta_b}^2 n_b \frac{\partial n_b}{\partial x}, \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + (1-\delta) n_b - n_i. \quad (5)$$

Here $\alpha = \frac{m_i}{m_b}, \sigma_i = \frac{T_e}{T_i}, \sigma_b = \frac{T_b}{T_i}, \delta = \frac{n_{e0}}{n_{i0}} and \delta_b = \frac{n_{b0}}{n_{i0}}$

The electrons obeying CT distribution are described as $n_e = n_{e0}\left(1 + (q-1)\phi\right)^{\frac{1}{q-1}+\frac{1}{2}}\left(1 + H_1\phi + H_2\phi^2\right)$. (6)

$$with H_1 = -\frac{16qe\alpha_e}{(5qe-3)(3qe-1)+12\alpha_e}, H_2 = \frac{16q_e\alpha_e(2q_e-1)}{(5q_e-3)(3q_e-1)+12\alpha_e}.$$

The expressions for the normalized number density of electrons is given as:

$$n_e = 1 + C_1\phi + C_2\phi^2 + \dots\dots\dots,$$

where

$$C_1 = \left(H_1 + \frac{(q_e+1)}{2}\right), C_2 = \left(H_2 + H_1\left(\frac{q_e+1}{2} - \frac{(q_e+1)(q_i-3)}{8}\right)\right)$$

The charge neutrality condition at equilibrium is $n_{i0}=n_{e0}+n_{b0}$ which becomes $\delta+\delta_b=1$with $\delta = \frac{n_{e0}}{n_{i0}}$ and $\delta_b = \frac{n_{b0}}{n_{i0}}$ where n_{i0}, n_{e0} and n_{b0} are the equilibrium number densities of ions, non-Maxwellian electrons and electron beam respectively. Using Taylor's expansion in Eq. (6) and substituting into Eq. (5), we get

$$\frac{\partial^2 \phi}{\partial x^2} = 1 + C_1\phi + C_2\phi^2 + C_3\phi^3 + (1-\delta)n_b - n_i$$

3. Derivations of the Burgers and inhomogeneous Burgers-type equations

In order to derive the Burgers equation for the IAShs for two fluids beam-plasma, the reductive perturbation method has been employed. The stretching coordinates are used as $\zeta = \varepsilon(x - V_p t)$ and $\tau = \varepsilon^2 t$ where ε is a small parameter measuring the weakness of the dispersion and V_p is the phase velocity of IAShs. The perturbations in the dependent variables is described as $F_j = F_{j0} + \sum_{m=1}^{\infty} \varepsilon^{m+1} F_{jm}$ where $F_{j0} = 1$, for $n_{i;b}$ and $F_{j0} = 0$; (v_{b0}); 0 for v_i; (v_b);Φ. Using stretching coordinates and perturbations in Eqs. (1-5) and comparing the coefficients of different orders of , we get the following first order equations as

$$n_{i1} = R\phi_1, v_{i1} = RV_p\phi_1, n_{b1} =- Q\alpha\phi_1, v_{b1} =- \alpha QP\phi_1,$$
$$C_1\phi_1 + (1-\delta)n_{b1} - n_{i1} = 0$$

where, the coefficients P, Q and R are given as

$P = (v_p-v_{b0})$;$Q = \frac{1}{(V_p - v_{b0})^2 - 3\alpha\sigma_b\delta_b^2}$ and $R = \frac{1}{(V_p^2 - 3\sigma_i)}$

The dispersion relation for IAShs is derived as

$$R - (1-\delta)\alpha Q - C_1 = 0$$

Eliminating the second order perturbed quantities, we derive the Burgers equation which governs the dynamics of IAShs

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \zeta} + C \frac{\partial^2 \phi_1}{\partial \zeta^2} = 0. \quad (7)$$

The nonlinear coefficient A and dissipation coefficient C are given $A = \frac{A'_1}{A'_2}$ and $C = \frac{A'_3}{A'_2}$

$$A'_1 = (1-\delta)\frac{3}{2}(\alpha^2(V_p - v_{b0})^2 + \alpha^3\sigma_b\delta_b^2)Q^3 - \frac{3}{2}(V_p^2 + \sigma_i)R^3 + 2C_2.$$
$$A'_2 = -2\left(\alpha(1-\delta)PQ^2 + R^2 v_p\right), \qquad A'_3 = \eta R^2 v_p. \quad (8,9)$$

In order to analyse the role of corrections of higher order effects in the form of dissipation as well as nonlinearity in the given plasma system, the inhomogeneous Burgers-type equation has been derived in the following part. The second order perturbed quantities n_{i2}, n_{b2}, v_{i2} and v in the form $(\phi_1)^2$ and ϕ_1 are re illustrated in [17]:

After analytic calculations and eliminating the third order perturbed quantities, we have derived the inhomogeneous Burgers-type equation in the following form:

$$\frac{\partial \phi_2}{\partial \tau} + A\frac{\partial(\phi_1\phi_2)}{\partial \zeta} - C\frac{\partial^2 \phi_2}{\partial \zeta^2} = T_1\frac{\partial^3 \phi_1}{\partial \zeta^3} + T_2(\phi_1)^2\frac{\partial \phi_1}{\partial \zeta} + T_3\left(\frac{\partial \phi_1}{\partial \zeta}\right)^2 + T_4\phi_1\frac{\partial^2 \phi_1}{\partial \zeta^2}, \quad (10)$$

where, coefficients T_1; T_2; T_3 and T_4 are illustrated in Appendix. Thus, it is concluded that the basic set of Eqs. (1)–(5) is reduced to the Burgers equation, Eq. (7), for the first-order perturbed potential ϕ_1 and the inhomogeneous Burgers-type equation, Eq. (10), for the second-order perturbed potential ϕ_2.

4. The stationary solutions of burgers and inhomogeneous burgers-type equations

In this section, we have determined the solutions of Burgers and inhomogeneous Burgers-type equations by using single variable transformation $\xi = \zeta - 2C\tau$ subtle balance of nonlinearity and dissipation prompts the formation of shocks. By making use of single variable transformation and the Tanh method [18], the shock-like solution of the Burgers equation (7) is obtained as

$$\phi_1(\xi) = \phi_m\left(1 + \tanh(\xi)\right), \quad (11)$$

where $\phi_m = 2C/A$ represents the peak amplitude of IAShs. The solution of inhomogeneous Burgers-type equation (11) is obtained by following the procedure as used in Ref. [18]. By using variation of parameters method and Abel's Theorem, the complete solution for IAShs with contributions of first and higher (second) order effects is described as $\phi_{final} = \phi_1+\phi_2$, The complete solution is written as

$$\phi_{final} = \phi_m(1+\tanh\xi) + \left(\frac{16}{3}T_2\frac{C^2}{A^2} + \frac{4}{3}T_3\frac{C}{A^2} - \frac{4}{3}T_4\frac{C}{A^2}\right) + \chi_1 sech^2\xi + \chi_2 sech^2\xi \ln(\cosh\xi) - \chi_3 sech^2\xi$$

χ_1; χ_2,χ_3 are given in Appendix. (12)

5. PARAMETRIC ANALYSIS

We have carried out numerical analysis of the solutions of Burgers equation (7) and the inhomogeneous-type Burgers equation (10). The values of nonlinear coefficient (A) and dissipation coefficient (C) are significantly influenced by various plasma parameters such as q-

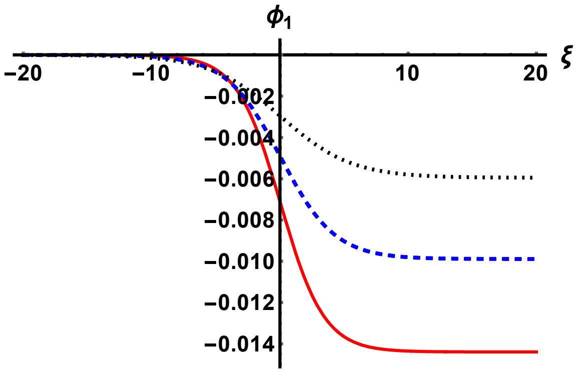

Fig. 1. The variation of negative potential shock profile with ξ for different values of q_e with $\delta_b = 0.0095$ and other fixed parameters are same as given in the discussion. For solid (Red) curve; $q_e = 0.65$, dashed (Blue) curve; $q_e = 0.7$ and dotted (Black) curve; $q_e = 0.8$

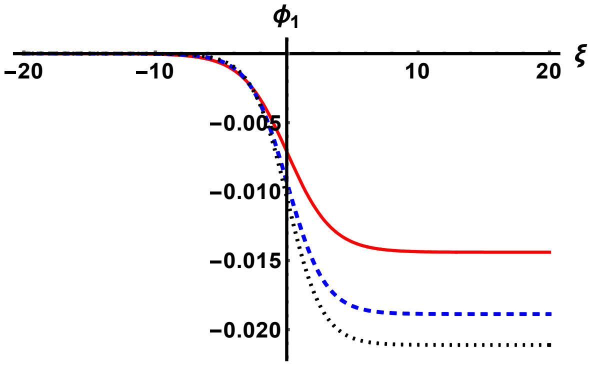

Fig. 2. The variation of negative potential shock profile with ξ for different values of α_e with $\delta_b = 0.0095$ and other fixed parameters are same as given in the discussion. For solid (Red) curve; $\alpha_e = 0.1$, dashed (Blue) curve; $\alpha_e = 0.2$ and dotted (Black) curve; $\alpha_e = 0.3$.

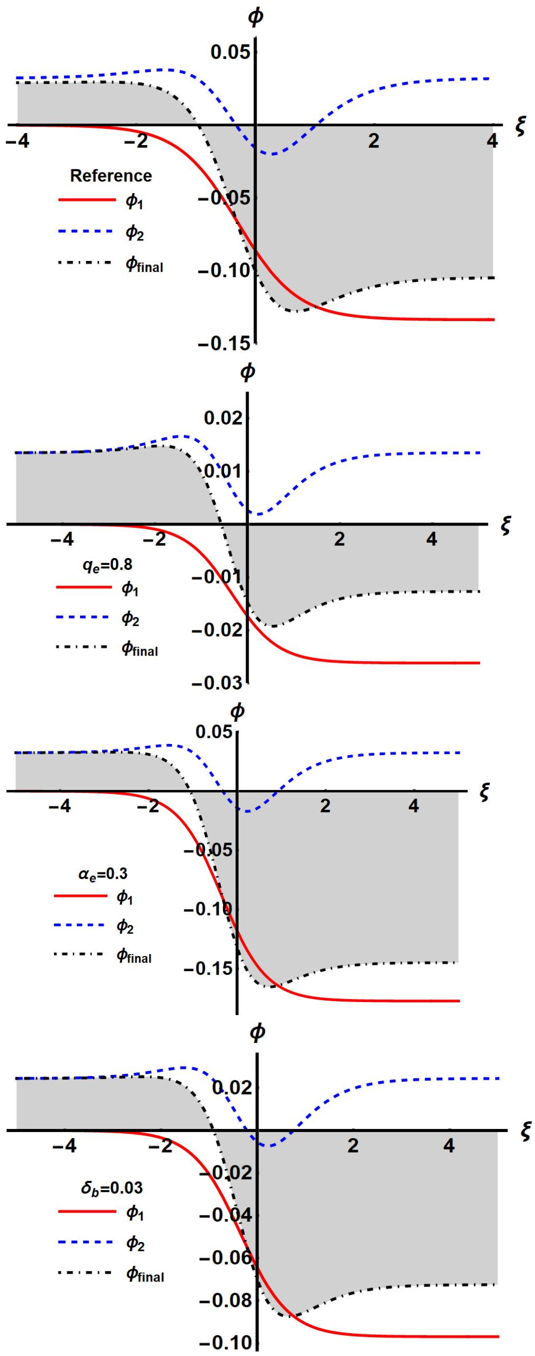

Fig. 3. Plot of ϕ_1 (solid curve), ϕ_2 (dashed curve) and ϕ_{final} (dot-dashed curve) against ξ with fixed parameters (for reference curve (a)) as $\delta = 0.85$, $\delta_b = 0.0095$, $\alpha_e = 0.3$ and $q_e = 0.65$, plot (b) for $q_e = 0.8$, (c) $\alpha_e = 0.3$ and (d) $\delta_b = 0.01$.

nonextensivity (q_e) and nonthermality of electrons (via α_e) and beam to ion number density ratio (via δ_b). For numerical analysis, parameters are considered from the data of Earth's magnetosphere [8] as $n_e \approx n_i \approx 10cm^3$, $n_b \approx 0{:}2cm^3$, $T_b \approx 1$ eV and $T_e \approx 990$ eV . Therefore, it is of foremost interest to comprehend the impacts of different parameters on the properties of IA shock structures for both cases. The effect of q-nonextensive parameter of electrons (via q_e) on the profile of negative potential shock structures is presented in Fig. 1. It is noticed that the amplitude of negative potential IAShs is decreased along negative axis of ϕ_1 with increase in the value of q_e. In other words, it is emphasized that the energy consumption by shocks are suppressed smoothly with the enhancement of q_e. In Fig. (2), the variation of nonthermal parameter α_e with ξ is presented and is observed that with increase in α_e , there is increase in the amplitude of IAShs. Now, we have analyzed the contributions of higher order effects in nonlinearity and dissipation on the characteristics of IAShs in the form of dressed shocks. According to the reductive perturbation technique, the condition $|\phi_1| \geq |\phi_2|$ should be satisfied. [17], [18]. Fig 3 (a) is the reference plot with certain values of plasma parameters with which Figs. 3 (b), (c) and (d) showing the variation with q_e, α_e and δ_b respectively are compared. It is seen that with increase in the value of q_e there is an decrease in the amplitude of the shock waves. On comparing Fig. 3 (a) and (c), it is observed that with increase in the value of e, there is increase in the amplitude of shocks. And finally, comparing plots of Figs. 3 (a) and (d), it is seen that with increase in the ratio of beam to ion number density δ_b, there is also decrease in the amplitude and steepness of the shocks. It is highlighted that the condition $|\phi_1| \geq |\phi_2|$ is extremely satisfied at all the values of ξ for the chosen set of parameters in the present investigation. Finally, it is remarked that only negative potential (rarefactive) IAShs for the first order and both positive as well as negative potential (compressive and rarefactive) IAShs due to the presence of higher order contributions are analyzed.

6. Conclusions

In this investigation, Burgers equation by employing reductive perturbation method is derived. Only negative potential IAShs are observed. Further, contributions of higher order effects of nonlinearity and dissipation are also considered which result in the formation of dressed IAShs. The inhomogeneous Burgers-type equation is also derived to describe the characteristics of dressed IAShs. It is noticed that after the insertion of contributions of higher order effects, the negative potential IA first order shocks are converted to both positive as well as negative potential dressed shocks. The characteristics of IA shocks and dressed shocks have been studied by considering the data from the environment of Earth's magnetosphere. It is observed that electron beam density ratio, q-nonextensivity and nonthermality of electrons have a great impact on the characteristics of different kinds of IA shocks. The findings may provide a new insight to understand the evolution of dressed IA shocks in space and astrophysical environments where non-Maxwellian electrons and electron beam are present.

7. References

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$$\chi_1 = \left(\frac{8}{3}T_4\frac{C}{A^2} - T_1\frac{2}{A} - \frac{20}{3}T_2\frac{C^2}{A^3} - \frac{2}{3}T_3\frac{C}{A^2}\right),$$

$$\chi_2 = \left(\frac{4}{3}T_3\frac{C}{A^2} - T_1\frac{4}{A} - \frac{8}{3}T_2\frac{C^2}{A^3} + \frac{8}{3}T_4\frac{C}{A^2}\right),$$

$$\chi_3 = \left(T_2\frac{8C^2}{A^3} - T_4\frac{4C}{A^2}\right).$$

$$T_1 = \frac{1}{A_2}\begin{bmatrix} 1 + \eta R^2 V_p A_3 - R^2 V_p C(A_1 + A_3) \\ +(1-\delta)PQ^2 C(A_5 + A_7) \end{bmatrix}$$

$$T_2 = \frac{1}{A_2}\begin{bmatrix} R^2 V_p (2A(A_2 + A_4) + 3RV_p A_4) + 9\sigma_i R^3 A_2 \\ +(1-\delta)[PQ^2(2A(A_6 + A_8 - 3\alpha PQ))] \end{bmatrix}$$

$$T_3 = \frac{1}{A_2}\begin{bmatrix} -R^2 V_p (A + V_p^2 R)(A_1 + A_3) - R^3(V_p^2 A_3 - 3\sigma_i A_1) \\ +(1-\delta)(Q^2 P(A + PQ)(A_5 + A_7)) \\ -(\alpha Q^3 (P^2 A_7 + 3\alpha\sigma_b\delta_b^2 A_5)) \end{bmatrix}$$

$$T_4 = \frac{1}{A_2}\left[-R^2 V_p A(A_1 + A_3) + 2R^3 V_p^3 A_3 - V_p^3 R^2 (A_1 + A_3) + 6\sigma_i R^3 A_1)) - 2R^2 V_p \eta A_4 - (1-\delta)(PQ^2 A(A_5 + A_7) + 2P^2 Q^3 A_7 + 6\alpha^2 \sigma_b \delta_b^2 Q^3 A_5)\right]$$