Electron acoustic cnoidal waves in an electron beam plasma

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***Abstract*—A study of nonlinear electron acoustic (EA) cnoidal waves is carried out in an un-magnetized plasma, in which cold electrons and electron beam as a fluid, inertialess hot superthermal electrons and stationary ions are considered. The reductive perturbation approach is used to derive the Korteweg-de Vries (KdV) equation. Further, the solution for EA cnoidal waves is obtained with appropriate boundary conditions using the Sagdeev pseudopotential technique. The effects of electron beam and other plasma parameters have been analysed numerically to analyse the characteristic properties of EA cnoidal waves. The findings of the current study may be helpful in understanding different nonlinear excitations in space and astrophysical regions.**

***Index Terms*—Electron acoustic, Korteweg-de Vries equation, Superthermal electrons, Cnoidal waves.**

I. Introduction

Over the past many years, satellite measurements in the auroral and other areas of the magnetosphere have confirmed the existence of electron acoustic waves [1]–[3]. In plasmas with cold and hot electrons having different temperatures, electron-acoustic waves may occur. In these plasmas, inertia is provided by the cold electrons, and the hot electrons pressure provides the necessary restoring force [4], [5]. Numerous authors have investigated the electron acoustic waves in the presence of electron beam. The formation of isolated structures in the geotail of Broadband Electrostatic Noise (BEN) emissions indicates the presence of an electron beam [6]–[8]. Singh et al. [8] demonstrated how electron acoustic solitons evolved in a magnetised plasma containing nonthermal electrons embedded with an electron beam. They discovered that the electron beam and other plasma parameters have a considerable impact on negative potential solitons. Bansal and Gill [9] studied the propagation properties of EA shock waves in an electron beam magnetised plasma deriving the Korteweg–de Vries–Burgers (KdVB) equation and showed that the EA shock waves are considerably altered by the electron beam. Kaur and Saini [10] studied the role of higher order effects in EA beam plasma using a the KdV-type inhomogeneous equation. It was observed that electron beam along with higher order contributions have significantly modified the properties of different kinds of EA waves.

A number of studies have shown that when plasma particles deviate from the Maxwell-Boltzmann distribution, especially in the high energy tail, the physical velocity distribution can be described by a non-Maxwellian distribution function [11], [12]. To describe this, the Kappa (κ) distribution function, which is an extension of the Maxwell-Boltzmann distribution, describes the superthermal particles. Initially, the Kappa distribution was developed as a power-law function to describe particles in plasmas that were out of thermal equilibrium [13]–[15]. Many authors have investigated the behaviour of electron-acoustic waves with the κ distribution for hot and cold electrons. Danekhar [16] investigated the dynamics of EA solitons in an unmagnetized plasma consisting of cool inertial electrons, hot electrons obeying kappa distribution and immobile ions. The propagation properties of EA waves were analysed under the influence of superthermality (via κ) and other different plasma parameters. Danehkar [17] investigated the characteristics of EA solitary waves in an electron beam-embedded superthermal plasma. It was found that the electron beam parameters and other plasma parameters have a significant impact on the properties of EA waves.

Numerous researchers have investigated nonlinear cnoidal waves in various plasma environments. Due to its applicability in several fields of plasma physics, the study of cnoidal (periodic) waves has great importance during the past few decades. The Korteweg-de Vries (KdV) equation has an accurate periodic solution in the form of cnoidal waves (Jacobi elliptic function) [18]. Nonlinear wave transportation in various plasma environments is one of the principal applications of the dynamics of cnoidal waves [19]. The features of DA cnoidal waves in an unmagnetized dusty plasma with negatively charged dust fluid, superthermal ions, and Maxwellian electrons in the presence of polarization force were studied by Singh et al. [20] by examining the impact of various plasma parameters. Khaled and Rehman [21] reported the propagation properties of ion acoustic periodic waves in a plasma containing Maxwellian electrons and positive ions with anisotropic thermal pressure. They analysed that the pressure anisotropy has a great impact on the characteristics of ion acoustic cnoidal waves.

Our goal in this work is to examine how the electron beam and superthermal hot electrons affect the features of EA nonlinear cnoidal waves in a four-component plasma with stationary ions, electron beam, hot electrons that follow a superthermal distribution, and inertial cold electrons. The manuscript is organised as follows: In Section II, the fluid equations are presented. The derivation of KdV and its cnoidal solution are mentioned in Sections III and IV respectively. Parametric analysis is illustrated in Section V. Section VI is devoted to conclusions.

II. Fluid Equations

We consider an unmagnetized, collisionless plasma made up of stationary positive ions embedded in an electron beam, inertialess hot electrons, cold electron fluid, and superthermal hot electrons. The dynamics of EA cnoidal waves is characterized by the normalized fluid equations $(continuity, momentum \quad and \quad Poisson)$ [17]:

$$\frac{\partial n_r}{\partial t}+\frac{\partial (n_r u_r)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial x}=\frac{\partial \Phi}{\partial x}-\frac{\theta_{rh}}{n_r}\frac{\partial p_r}{\partial x}, \tag{2}$$

$$\frac{\partial p_r}{\partial t}+u_r\frac{\partial p_r}{\partial x}+3p_r\frac{\partial u_r}{\partial x}=0, \tag{3}$$

$$\frac{\partial^2 \Phi}{\partial x^2}+(1+\rho_h+\rho_b)-n_c-\rho_b n_b-\rho_h n_h=0, \tag{4}$$

where $(r=b,c)$ denotes electron beam and cold electrons respectively). The normalization of the above set of equations is mentioned in detail in [10]. Here, $\rho_h=\frac{n_{h0}}{n_{c0}}$, $\rho_b=\frac{n_{b0}}{n_{c0}}$, $\theta_{ch}=\frac{T_c}{T_h}$, $\theta_{bh}=\frac{T_b}{T_h}$.

The number density of superthermal hot electrons is given as [22]:

$$n_h=\left(1-\frac{\Phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+\frac{1}{2}}. \tag{5}$$

At equilibrium, the plasma is quasi-neutral, so that

$$n_{c0}+n_{b0}=n_{i0}-n_{h0}. \tag{6}$$

III. Derivation of the KdV equation and its solution

We use the reductive perturbation approach to get the KdV equation by specifying the stretched independent variables as follows:

$$\xi=\epsilon^{\frac{1}{2}}(x-\lambda t) \quad \text{and} \quad \tau=\epsilon^{\frac{3}{2}}\mathrm{t}, \tag{7}$$

where, λ is the phase velocity of EA waves. The physical quantities in equations (1-5) are expanded into powers of ϵ as:

$$S=S_0+\sum_{m=1}^{\infty}\epsilon^m S_m, \tag{8}$$

where, $S=(n_c,n_b,u_c,u_b,p_c,p_b,\Phi)$ and $S_0=(1,1,0,V_b,1,1,0)$.

After substituting Eqs. (7-8) into Eqs. (1-4), we have obtained the expressions with different powers of ϵ. For lowest order in ϵ, we obtain the following first order evolution equations:

$$n_{c1}=\frac{\Phi}{(-\lambda^2+3\theta_{ch})}+K_1, \tag{9}$$

$$u_{c1}=\frac{\Phi\lambda}{(-\lambda^2+3\theta_{ch})}, \tag{10}$$

$$p_{c1}=\frac{3\Phi}{(-\lambda^2+3\theta_{ch})}, \tag{11}$$

$$n_{b1}=\frac{\Phi}{(-(V_b-\lambda)^2+3\theta_{bh})}+K_2, \tag{12}$$

$$V_{b1}=\frac{-\Phi(V_b-\lambda)}{(-(V_b-\lambda)^2+3\theta_{bh})}, \tag{13}$$

$$p_{b1}=\frac{3\Phi}{(-(V_b-\lambda)^2+3\theta_{bh})}. \tag{14}$$

Here, K_1, K_2 are integration constants. After solving Eqs. (9-14), we get the phase velocity relation as:

$$\frac{1}{-\lambda^2+3\theta_{ch}}+\frac{\rho_b}{(-(V_b-\lambda)^2+3\theta_{bh})}=-\rho_h\left(\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\right). \tag{15}$$

To the next order of ϵ, the second order equations are obtained. After equating the second order equations using first order equations, the KdV type equation for EA cnoidal waves is derived as :

$$\frac{\partial \Phi}{\partial \tau}+A\Phi\frac{\partial \Phi}{\partial \xi}+B\frac{\partial^3 \Phi}{\partial \xi^3}+K\frac{\partial \Phi}{\partial \xi}=0, \tag{16}$$

here nonlinear coefficient $A=\frac{b}{a}$, dispersion coefficient $B=\frac{-1}{a}$, K is integration constant,

where,

$$a=\left(\frac{-2\lambda}{(-\lambda^2+3\theta_{ch})^2}+\frac{2(V_b-\lambda)\rho_b}{(-(V_b-\lambda)^2+3\theta_{bh})^2}\right),$$

$$b=\frac{-3\lambda^2-3\theta_{ch}}{(-\lambda^2+3\theta_{ch})^3}+\frac{(-3(V_b-\lambda)^2-3\theta_{bh})\rho_b}{(-(V_b-\lambda)^2+3\theta_{bh})^3}+\frac{\rho_h(-\kappa+\frac{1}{2})(\kappa+\frac{1}{2})}{(\kappa+\frac{3}{2})^2}.$$

IV. Cnoidal wave solution of KdV type equation

The stationary solution of equation (16) is obtained by using the transformation $\eta = \xi - U_1\tau$, here U_1 is velocity of EA cnoidal waves. By changing Eq. (16) into η coordinate, the energy balance equation is obtained as:

$$\frac{1}{2}\left(\frac{\partial\Phi}{\partial\eta}\right)^2 + W(\Phi) = 0, \tag{17}$$

where $W(\Phi)$ is the Sagdeev potential, which is given as:

$$W(\Phi) = \frac{A}{6B}\Phi^3 - \frac{U}{2B}\Phi^2 + \rho_0\Phi - \frac{1}{2}E_0^2. \tag{18}$$

ρ_0 and E_0 are the integration constants representing the charge density and the electric field, respectively. $U = U_1 - K$ and $E_0^2/2$ is the total energy of oscillations. By using the initial boundary conditions $\Phi(0) = \Phi_0$ and $\frac{\partial\Phi(0)}{\partial\eta} = 0$, the expression for electric field E_0 is obtained as

$$E_0^2 = \frac{A}{3B}\Phi_0^3 - \frac{U}{B}\Phi_0^2 + 2\rho_0\Phi_0 \tag{19}$$

Substituting Eqs. (18) and (19) in Eq. (17) and after some mathematical calculations, the periodic wave solution of Eq. (16) is given as [23]:

$$\Phi(\eta) = \Phi_1 + \Phi_{cn} cn^2(H\eta, q), \tag{20}$$

where cn is the Jacobian elliptic function and the parameters q and H are defined as $q^2 = \frac{(\Phi_0-\Phi_1)}{(\Phi_0-\Phi_2)}$ and $H = \sqrt{\frac{A}{12B}(\Phi_0 - \Phi_2)}$, where Φ_1, Φ_2 is explained in [23] and q^2 is varied as $0 \leq q \leq 1$. The amplitude and the wavelength of the cnoidal wave are defined as $\Phi_{cn} = (\Phi_0 - \Phi_1)$ and $\lambda_c = 4\sqrt{\frac{3B}{A(\Phi_0-\Phi_1)}}C(q)$, where $C(q)$ is the complete elliptic integral of the first kind.

For the case, $q \to 1$ ($E_0 = \rho_0 = 0$) at $\Phi_1 = \Phi_2 = 0$, $\Phi_{cn} = \Phi_0 = \frac{3U}{A} = \Phi_m$, $H = \left(\frac{A\Phi_0}{12B}\right)^{1/2} = \left(\frac{U}{4B}\right)^{1/2} = \frac{1}{w}$, and $cn\,\delta = sech\,\delta$, the cnoidal wave solution reduces to the following solitary wave solution [20] as:

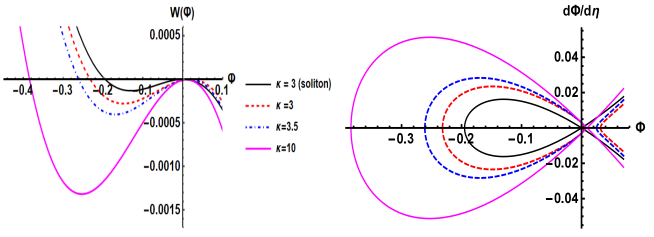
$$\Phi(\eta) = \Phi_m sech^2\left(\frac{\eta}{w}\right), \tag{21}$$

where, $\Phi_m = \frac{3U}{A}$ and $w = \sqrt{\frac{4B}{U}}$ are the peak amplitude and the width of the EA solitary waves, respectively.

V. Parametric analysis

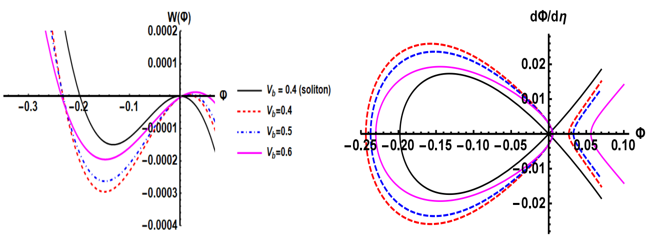
In this section, we have analysed the features of EA cnoidal waves in a superthermal plasma with an electron beam under the influence of various plasma parameters such as beam to hot electron temperature ratio (θ_{bh}) , cold to hot electron temperature ratio (θ_{ch}) and superthermal distribution parameter κ along with other normalized physical parameters : hot to cold electron number density ratio $\rho_h(= n_{h0}/n_{c0})$ and beam to cold electron number density ratio $\rho_b(= n_{b0}/n_{c0})$ beam velocity (V_b). We have taken the observations of BEN in the auroral zone [2], [3] and selected the appropriate values $n_c \sim 10$ cm^{-3}, $n_h \sim 10$ cm^{-3}, $T_h \sim 500$ eV, $T_b \sim 1$ eV .

Fig. 1 (a) depicts the variation of Sagdeev potential W(Φ) with Φ for different values of superthermality parameter (κ). It is seen that with the increase in the values of superthermality parameter the maximum amplitude and depth of Sagdeev potential increases for EA cnoidal waves. Additionally, the Sagdeev potential W(Φ) associated with the electron acoustic cnoidal waves does not vanish at $\Phi = 0$, however, W(Φ) for EA solitary waves becomes zero at $\Phi = 0$ (with $E_0 = 0$, $\rho_0 = 0$). Fig. 1 (b) presents the phase plane plots of electron acoustic cnoidal waves and solitary waves separatix for negative potential. It is observed that the nonlinear and dispersion properties of EA cnoidal waves are significantly



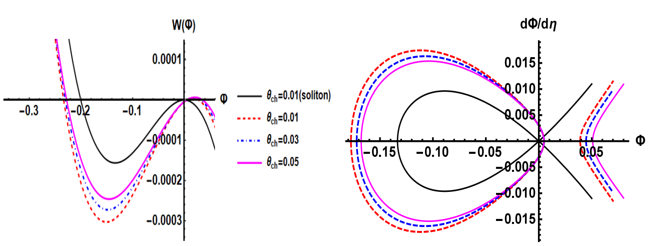
(a) (b)

Fig. 1. (Color online) Variation of (a) Sagdeev potential of EA cnoidal waves W(Φ) vs. η (b) Phase plot for EA cnoidal waves for different values of κ with $V_b = 0.45$, $\rho_b = 0.008$, $\rho_h = 1$, $\theta_{bh} = 0.004$, $U = 0.3$, $E_0 = 0.004$ and $\rho_0 = 0.001$, Solid (Black) curve: soliton with $\kappa = 3$, $\rho_0 = 0$ and $E_0 = 0$; Dashed (Red) curve: $\kappa = 3$; Dotdashed (Blue) curve: $\kappa = 3.5$; Solid (Magenta) curve: $\kappa = 10$.



(a) (b)

Fig. 2. (Color online) Variation of (a) Sagdeev potential of IA cnoidal waves V(Φ) vs. Φ (b) Phase plot for ion IA cnoidal waves for different values of V_b with $\kappa = 3$, $\rho_b = 0.008$, $\rho_h = 1$, $\theta_{bh} = 0.001$, $U = 0.3$, $E_0 = 0.004$ and $\rho_0 = 0.001$, Solid (Black) curve: soliton with $V_b = 4$, $\rho_0 = 0$ and $E_0 = 0$; Dashed (Red) curve: $V_b = 4$; Dotdashed (Blue) curve: $V_b = 5$; Solid (Magenta) curve:$V_b = 6$.



(a) (b)

Fig. 3. (Color online) Variation of (a) Sagdeev potential of IA cnoidal waves V(Φ) vs. Φ (b) Phase plot for ion IA cnoidal waves for different values of θ_{ch} with $V_b = 0.45$, $\kappa = 3$, $\rho_b = 0.006$, $\rho_h = 1$, $\theta_{bh} = 0.001$, $U = 0.3$, $E_0 = 0.004$ and $\rho_0 = 0.001$, Solid (Black) curve: soliton with $\theta_{ch} = 0.01$, $\rho_0 = 0$ and $E_0 = 0$; Dashed (Red) curve: $\theta_{ch} = 0.01$; Dotdashed (Blue) curve: $\theta_{ch} = 0.03$; Solid (Magenta) curve: $\theta_{ch} = 0.05$.

enhanced with a change in the superthermality parameter. Fig. 2 (a) shows the variation of Sagdeev potential W(Φ) with Φ for different values of beam velocity (V_b). It is found that the amplitude and width of EA cnoidal waves are decreased with an increase in the values of beam velocity (V_b). This shows that electron beam parameters have greatly modified the characteristic properties of nonlinear EA cnoidal waves. Fig. 2 (b) depicts the phase plot, which represents the similar variation in the features of EA cnoidal waves with different values of V_b. We have analysed the variation of Sagdeev potential W(Φ) versus Φ for different values cold to hot electron temperature ratio (θ_{ch}) in Fig 3 (a). As seen in the Fig. 3 (a), with the increase of cold to hot electron temperature ratio (θ_{ch}), the width and amplitude of EA cnoidal waves are decreased. Fig. 3 (b) illustrates the variation in phase plot of EA cnoidal waves with cold to hot electron temperature ratio. It is important to remark that the EA cnoidal waves are significantly influenced by cold to hot electron temperature ratio.

VI. Conclusion

In this investigation, we have studied the propagation properties of EA cnoidal waves in an unmagnetized plasma consisting of inertial cold electrons, electron beam fluid, inertialess superthermal hot electrons, and with stationary background ions. The reductive perturbation method is employed to derive the KdV equation and only negative potential EA cnoidal waves are observed in the presence of an electron beam. The effects of the electron beam and various plasma parameters on EA cnoidal waves have been analysed. The electron acoustic cnoidal waves are reduced to solitary waves in the limiting condition. The findings of this study may be helpful for comprehending the plasma that is seen in various areas of the Earth's magnetosphere.

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References

[1] R. L. Mace, S. Baboolal, R. Bharuthram and M. A. Hellberg, "Arbitrary-amplitude electron-acoustic solitons in a two-electron-component plasma," J. Plasma Phys., Vol. 45 no. 1, pp. 323–338, Feb. 1991.

[2] R. L. Tokar and S. P. Gary, "Electrostatic hiss and the beam driven electron acoustic instability in the dayside polar cusp," Geophys. Res. Lett., Vol. 11, pp. 1180–1183, Dec. 1984.

[3] N. Dubouloz, R. Pottelette, M. Malingre, G. Holmgren, and P. A. Lindqvist, "Detailed analysis of broadband electrostatic noise in the dayside auroral Zone," J. Geophys. Res., Vol. 96 no. A3, pp. 3565–3579, March 1991.

[4] K. Watanabe and T. Taniuti, " Electron-acoustic mode in a plasma of two-temperature electrons," J. Phys. Soc. Jpn., Vol. 43 no. 5, pp. 1819–1820, Nov. 1977.

[5] R. L. Mace and M. A. Hellberg, "Higher-order electron modes in a two-electrontemperature plasma," J. Plasma Phys., Vol. 43, pp. 239–255, April 1990.

[6] M. Berthomier, R. Pottelette, M. Malingre, and Y. Khotyaintsev, "Electron-acoustic solitons in an electron-beam plasma system," Phys. Plasmas, Vol. 7 no. 7, pp. 2987–2994, July 2000.

[7] W. F. El-Taibany, "Electron-acoustic solitary waves and double layers with an electron beam and phase space electron vortices in space plasmas," J Geophys Res, Vol. 110 , p. A01213, Jan. 2005.

[8] S. V. Singh, S. Devanandhan, G. S. Lakhina, and R. Bharuthram, "Electron acoustic solitary waves in a magnetized plasma with nonthermal electrons and an electron beam," Phys. Plasmas, Vol. 23, p. 082310, Aug. 2016.

[9] S. Bansal and T. S. Gill, "The Existence and Propagation of Electron Acoustic Shock Waves in Magnetized Plasma with Electron Beam," Braz. J. Phys., Vol. 51, pp. 1719—1726, 2021.

[10] R. Kaur, and N. S. Saini, "Electron acoustic dressed solitons in an electron-beam plasma with higher-order contributions," Chinese J. Phys., Vol. 77, pp. 760—772, 2022.

[11] V. M. Vasyliunas , "A Survey of Low-Energy Electrons in the Evening Sector of the Magnetosphere with OGO1 and OGO3," J. Geophys. Res., Vol. 73 no. 9, pp. 2839—2884, May 1968.

[12] E. Marsch, K. H. Muhlhauser, R. Schwenn, H. Rosenbauer, W. Pillip and F. M. Neubauer, "Solar Wind Protons: Three-Dimensional Velocity Distributions and Derived Plasma Parameters Measured Between 0.3 and 1 AU," J. Geophys. Res., Vol. 87 no. A1, pp. 52—72, Jan 1982.

[13] R. L. Mace, and M. A. Hellberg, "A dispersion function for plasmas containing superthermal particles," Phys. Plasmas, Vol. 2, pp. 2098—2109 june 1995.

[14] T. K. Baluku and M. A. Hellberg, "Dust acoustic solitons in plasmas with kappa-distributed electrons and/or ions," Phys. Plasmas, Vol. 15 , p. 123705, Dec. 2008.

[15] N. S. Saini, I. Kourakis and M. A. Hellberg, "Arbitrary amplitude ion-acoustic solitary excitations in the presence of excess superthermal electrons," Phys. Plasmas, Vol. 16, p. 062903, June 2009.

[16] A. Danehkar, N. S. Saini, M. A. Hellberg, and I. Kourakis, "Electron-acoustic solitary waves in the presence of a suprathermal electron component," Phys. Plasmas, Vol. 18, pp. 072902—072902, July 2011.

[17] A Danehkar,"Electron beam-plasma interaction and electron-acoustic solitary waves in a plasma with suprathermal electrons," Plasma Phys. Control. Fusion, Vol. 60 no. 6, p. 065010, April. 2018.

[18] T. B. Benjamin, M.J. Lighthill, "On Cnoidal Waves and Bores," Proc. R. Soc. Lond. Vol. 224, pp. 448–460, July 1954.

[19] K. Konno, T. Mitsuhashi, and Y. H. Ichikawa, "Propagation of Ion Acoustic Cnoidal Wave," J. Phys. Soc. Jpn., Vol. 46 no. 6, pp. 1907–1914, Feb. 1979.

[20] K. Singh, Y. Ghai, N. Kaur, and N. Singh Saini, "Effect of polarization force on dust-acoustic cnoidal waves in dusty plasma," Eur. Phys. J. D, Vol. 72, no. 9, p. 160, Sep. 2018.

[21] M. Khalid, and Au. Rehman, "Ion acoustic cnoidal waves in a magnetized plasma in the presence of ion pressure anisotropy" Astrophys. Space Sci., Vol. 364 no. 28, Feb 2019.

[22] T. Akter, F. Deeba and Md. K. A. Hassan, "Electron-Acoustic Solitary Waves in a Two-Temperature Plasma Having Electrons With Kappa Distribution," IEEE Trans. Plasma Sci., Vol. 44 no. 8, Aug. 2016.

[23] N. Kaur, R Kaur, and N. S. Saini, "Ion-acoustic cnoidal waves with the density effect of spin-up and spin-down degenerate electrons in a dense astrophysical plasma," Z. Naturforsch. A, Vol. 75 no. 2, pp. 103–111, 2019.