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Orbit Propagator Coefficients for Earth Orbiting Satellites

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Abstract

The orbit of GNSS ephemerides are provided in RINEX format at every 15min interval for next two days at IGU website on daily basis for public use. The extraction of position and velocity information of GNSS satellites at given time require separate algorithms which depend on precision of several universal constant parameters and many coordinate transformation. In this paper we have developed an algorithm to estimate finite number of satellite Orbit Propagator Coefficients (OPC) accounting full force perturbative forces. Based on these coefficients user can derive satellite position for the required time within given period time with good accuracy. In this paper firstly we have propagated the orbit of earth orbiting satellites using in-house developed full force model, subsequently estimated 23 numbers of OPC within propagation period. Finally we have compared the position derived from OPC with high-precise RINEX files of GNSS satellites and also with operational ephemerides for LEO satellites with very close match. The estimated OPC requires very less run-time for satellite position time unlike any high precise numerical propagator and therefore it can be utilised for satellite on-board navigation for micro/nano satellites.

1. Introduction

One of the primary requirements for orbit analysis is the ability to predict future, or past, locations of a satellite. This activity is known as orbit propagation. The analytical methods are known as general perturbations methods, whereas numerical methods are known as special perturbations methods. The analytical methods were the first used and applied methods in the orbit propagation [1]. Their main advantage is computational efficiency; however, their accuracy is limited due to truncation. On the other hand, the special perturbations methods are characterized by high accuracy, especially for short-term propagation. However, their drawbacks are the limited accuracy for long-term propagation due to round-off error and high computational requirements. The semi-analytical methods were introduced as a solution to this dilemma between precision and computational cost and between the round-off and truncation errors [2]. A numerical orbit propagator consists of three main components: Differential equations, Environment model, and numerical Integrator. The importance of accurate propagation numerically: Encke's and Cowell's method. In modern days, it's common in astrodynamics to use Cowell's formulation to set up the equations of motion for numerical integration as it isthe simplest and most straightforward of all the perturbation methods. An overwhelming advantage is the fact that the solution contains all secular and periodic variations introduced by the perturbing forces [3-4].

The perturbative forces acting on the spacecraft (S/C) are either conservative or neoconservative. The total energy for conservation force (e.g., central-body, third-body gravitational, tides effects etc.) Systems is constant, whereas non-conservative force (e.g., solar radiation pressure, drag, thrust etc.) Systems may lose or gain the energy. The major perturbations that affect the motion of an Earth orbiting satellite (Fig. 1) Include Earth gravitation, atmospheric drag, lunar and solar gravitation, and solar radiation pressure (SRP). Depending on the orbital altitude and physical size of the satellite, the effect of these perturbations may be more or less important.

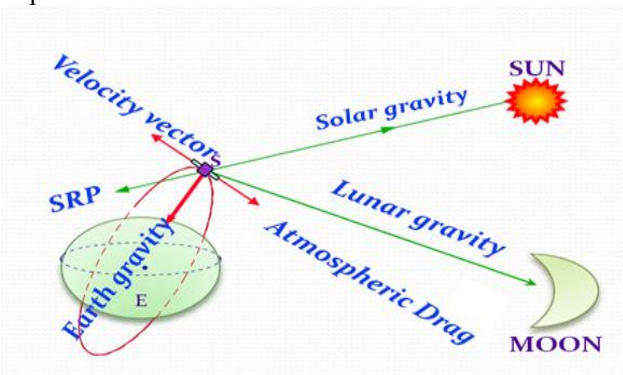

Fig. 1: Forces acting on an Earth Orbiting Satellite

The numerical propagation requires several areas such as coordinate systems, time, force models, and integration techniques. The coordinate frame is very relevant in highly accurate programs. In general, numerical techniques use fixed, variable, or regularized methods to move the satellite forward through time. The selection of one over another is generally based on the orbit type, but often on what is available. The fixed step methods are chosen mostly due to their popularity and easy implementations [3, 5]. In this

exercise Cowell Method has been used for Orbit Propagation and integrated numerically. The Dynamical Model used for in-house developed Satellite Precise Orbit Propagator (SPOP)[7] is given in Table-1:

Table-1: Satellite Precise Orbit Propagator (OP) and Orbit Determination Model

EGM-70x70 for Gravity, NRLMS-2000 for Aerodynamic Drag, Solid and Ocean Tides 4x4
SRP with GPSII Block MODEL, 3rd Body (Sun and Moon), DE430 (JPL Ephemerides)
Numerical Integrator RK7(8),Cowell Method

The rest of the paper is organized as follows; Section II describes the architecture full force orbit propagator model while Section III describes the mathematical model for generating 23 number of Orbit Propagator Coefficients (OPC) and the computation of state vector using OPC coefficients. Section IV describes accuracy assessment of the state vector with operational ephemerides of Low Earth Orbit satellites and also with RINEX file of NavIC satellites.

2. Full Force Model Orbit Propagator (SPOP) Architecture

SPOP provides high precision orbit prediction and subsequently orbit products generation for the Earth Orbiting satellites (EOSs), especially for LEO and GEO satellites. **Fig. 2** shows the schematic computational process diagram of SPOP.

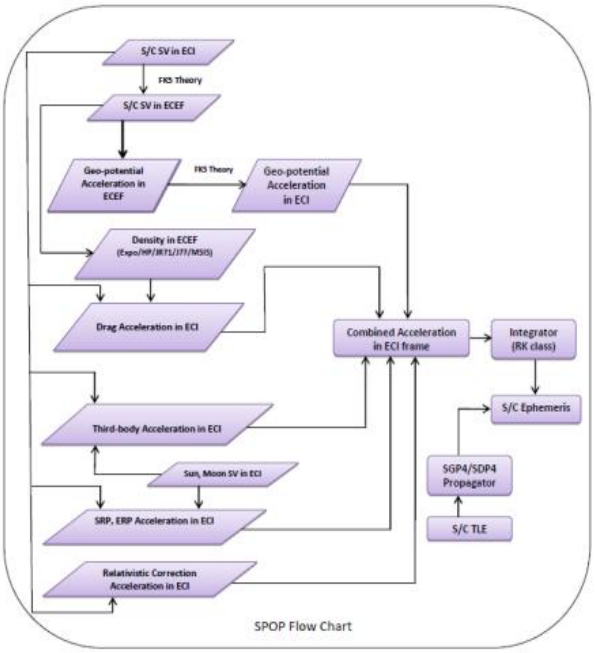

Fig. 2: SPOP computational schematic procedure for the Earth Orbiting Satellite

SPOP has features to handle the satellite initial input parameter in various form like SV in TOD/J2000/ECEF/TEME or the orbital elements given in terms of Osculating orbital elements, Mixed Spherical, Spherical, Delaunay or Equinoctial orbital elements. The SPOP predicts satellite ephemeris (state vector) in TOD, EME2000, ICRF, TEME, and ECEF frames as well as the osculating elements in TOD, EME2000, ICRF, and TEME frames.

3. OPC Estimation

Since position and velocity components of orbiting satellites follow sinusoidal signature w.r.to time, also the orbital period of LEO satellites is around 100min at nearly 800km altitude therefore a Mathematical Model is developed to estimate 23number of OPC coefficients using 23Orbits ephemerides based on full force model SPOP. The Mathematical Model for 23 OPC coefficients estimated based on linear equation of 23x23 and is given below

$$F = (A^T A)^{-1} A^T X \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$X = \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ . \\ . \\ X_{22} \end{bmatrix}$ F = $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_{22} \end{bmatrix}$ are computed OPC coefficients, A

$$= \begin{bmatrix} A_{00} & A_{01} & A_{02} & & A_{022} \\ A_{10} & A_{11} & A_{12} & & A_{122} \\ & & . & & \\ & & . & & \\ & & . & & \\ A_{220} & A_{221} & A_{221} & & A_{2222} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_{22} \end{bmatrix},$$

$$A_{I0} = 1; A_{i1} = t_i; A_{i2} = t_i^2; A_{i3} = \sin(\omega t_i); A_{i4} = A_{i1} A_{i3}; A_{i5} = A_{i2} A_{i3}; A_{i6} = \cos(\omega t_i);$$

$$A_{i7} = A_{i1}A_{i6}; A_{i8} = A_{i2}A_{i6}; A_{i9} = A_{i3}A_{i3}; A_{i10} = A_{i1}A_{i9}; A_{i11} = A_{i3}A_{i6}; A_{i12} = A_{i1}A_{i11}$$

$$A_{i13} = A_{i3}A_{i9;}A_{i14} = A_{i6}A_{i9;}A_{i15} = \sin(2\omega_e t_i); A_{i16} = \cos(2\omega_e t_i);\ A_{i17} = A_{i3}A_{i15}$$

$$A_{i18} = A_{i3}A_{i16;}A_{i19} = A_{i6}A_{i15;}A_{i20} = A_{i6}A_{i16;}; A_{i21} = \sin(\omega_e t_i);\ A_{i22} = \cos(2\omega_e t_i)$$

$$i = 0,1,2, \ldots., 22$$

X are x,y,z-coordinate coefficients of state vector computed using full force model SPOP at different times t_i . ω_e is earth rotation rate and ω is orbital period of satellite derived at converged Orbit Determination of Satellite.

The major advantage of estimating OPC orbit coefficients are they can be used for fairly good accuracy within the period of two days for LEO, MEO and GEO satellites with fairly good accuracy w.r.to full force orbit propagator model. Also they can be used as on-board uplinking parameter for smaller satellites for their on-board navigation. Further these coefficients can be used by researcher/academic who have application of orbit but have less knowledge of flight dynamics. Once the coefficients are estimated then position components of earth orbiting satellite will be computed using

$$X = \sum_{j=0}^{22} \sum_{j=0}^{22} A_{ij}\, a_i \ldots\ldots\ldots\ldots (2)$$

4. OPC Coefficients Accuracy

To assess the accuracy of OPC orbit coefficients a true orbit of LEO satellite in 500km SSPO orbit is considered and propagated through in-house developed software SPOP for next four days. Subsequently using ephemerides the 23 OPC coefficients are estimated based on equation (1) and further position components are computed using equation (2) at 1sec interval within propagation period of four days and finally position difference is computed between SPOP based precise orbit and OPC coefficients based orbit. The difference plot is given in Fig-3.Also corresponding estimated OPC coefficients

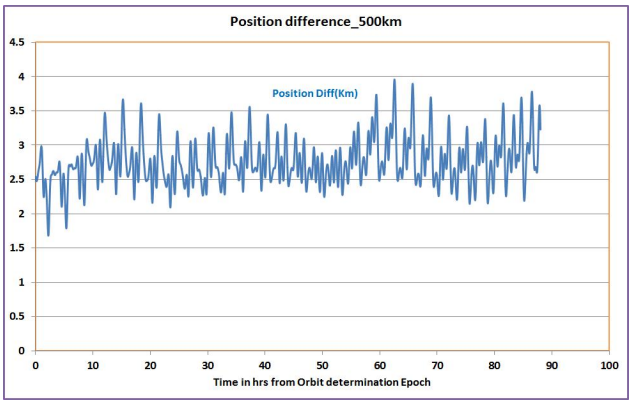

Fig. 3: Position difference between Full Force Orbit Propagator and OPC Coefficients

Similarly rinex file of NavIC satellite is used for propagating with SPOP and further OPC coefficients generation. The initial Orbit in terms RINEX format of NaVIC satellite and difference between OPC and SPOP is given below

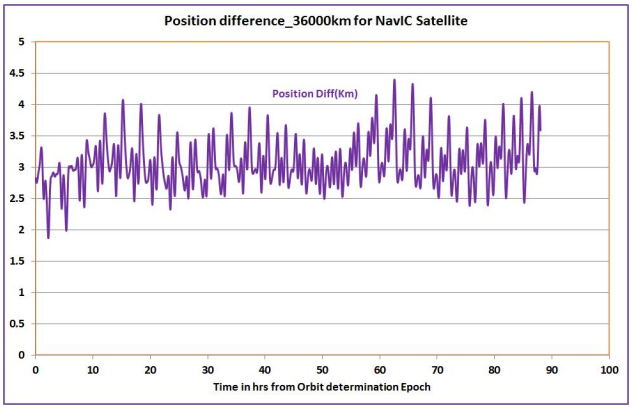

Fig. 4: Position difference between Full Force Orbit Propagator and OPC Coefficients

5. Discussion

Estimated OPC using full force model are better than 4km at height of 36000km altitude satellites, this can be further improved while estimating more number of coefficients like 40 to 50coefficients by considering more number of terms in equation(2).

6. Acknowledgements

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7. References

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